A topological lower bound for the energy of a unit vector field on closed Euclidean hypersurfaces

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For a unit vector field on a closed immersed Euclidean hypersurface M^{2n+1}, $n \geq 1$, we exhibit a nontrivial lower bound for its energy which depends on the degree of the Gauss map of the immersion. Two non-homotopic immersions will possess two different normal degrees; the bigger this value, the bigger the energy of a given unit vector field. When the hypersurface is the unit sphere $\mathbb{S}^{2n+1}$, this lower bound corresponds to a well established value from the literature. We introduce a list of functionals $\mathcal{B}_k$ on a compact Riemannian manifold M^m, $1 \leq k \leq m$, and show that, when the underlying manifold is a closed hypersurface, these functionals possess similar properties regarding the degree of the immersion. In addition, Hopf flows minimize $\mathcal{B}_n$ on $\mathbb{S}^{2n+1}$.

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