Balanced metrics on the blow-up of $\mathbb{C}^2$ at the origin

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Abstract

An interesting open question in Kähler geometry is the characterization of projectively induced Kähler metrics. An important class of projectively induced Kähler metrics are the so called *balanced metrics* in the sense of Donaldson S. In this talk, after a briefly introduction of Balanced metrics, we see the blowing-up operation and we introduce two important complete Kähler metrics defined on the blow-up of $\mathbb{C}^2$ at the origin: the celebrated *Simanca metric* and the *Eguchi-Hanson metric*. The Simanca metric is an important example (both from mathematical and physical point of view) of non homogeneous complete, zero constant scalar curvature metric. The Eguchi-Hanson metric is a well-know example of non homogeneous complete, Ricci-flat Kähler metric. In this talk we discuss on the balanced condition for these two Kähler metrics. The main results are the following theorems:

Theorem 1. *Let $\tilde{\mathbb{C}}^2$ be the blow-up of $\mathbb{C}^2$ at the origin endowed with the Eguchi–Hanson metric g_{EH}. Then $(\tilde{\mathbb{C}}^2, mg_{EH})$ is not balanced for any positive integer m.*

Theorem 2. *Let $\tilde{\mathbb{C}}^2$ be the blow-up of $\mathbb{C}^2$ at the origin endowed with the Simanca metric g_S. Then $(\tilde{\mathbb{C}}^2, mg_S)$ is balanced for any positive integer m.*

We also prove a result on Berezin's quantization on the dense subset $\mathbb{C}^2 \setminus \{0\} \subset \tilde{\mathbb{C}}^2$ equipped with the restriction of the Simanca Kähler form ω_S associated to the Simanca metric g_S. This is expressed by the following corollary:

Corollary 3. *$(\mathbb{C}^2 \setminus \{0\}, \omega_S)$ admits a Berezin quantization.*

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