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Approach to equilibrium in the symmetric simple exclusion process

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Abstract

We obtain an upper bound for the rate of convergence to equilibrium of the $d-$dimensional simple symmetric exclusion process starting either with a periodic configuration or with a stationary mixing probability distribution. More precisely, calling η_t^* the process, we show that for all t large enough

$$\left| P\left\{ \eta_t^* \in \mathbf{Y}_N \right\} - \rho^N \right| \leq c_d \; N^2 \left(\frac{\log t}{\sqrt{t}} \right)^d \;,$$

where $\mathbf{Y}_N$ is the cylindrical set in which all the N sites of a fixed box are occupied.

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1 Introduction

In this paper we obtain an upper bound for the rate of convergence to equilibrium of the simple symmetric exclusion process starting either with a periodic configuration or with a stationary exponentially mixing probability distribution. We prove that the distance between the law of the process at time t and the Bernoulli invariant measure, with the same density as the initial configuration, decreases faster than $(\log t/\sqrt{t})^d$, where d is the dimension of the lattice.

The exclusion process was introduced by Spitzer in his seminal paper [16]. It can be informally described as follows. Particles are distributed on the lattice $\mathbb{Z}^d$, each site being occupied by at most one particle. Associated to each pair of nearest neighbor sites $\{x, y\}$ there is an exponential clock independent from the others. When it rings the contents of sites x and y are interchanged. Hence if both sites are occupied or both sites are empty, nothing happens but if one of the sites is occupied and the other is empty, the interchange is seen as a jump of the particle to the empty site.

The basic ergodic properties of the symmetric simple exclusion process were discussed by Spitzer and Liggett (see [17] and [14]; this last text is the basic reference on the subject). The extremal invariant probability measures for this system are the product measures ν_ρ, $\rho \in [0, 1]$. Under the measure ν_ρ, the probability that a site is occupied by a particle is ρ and the occupation variables of different sites are independent random variables.

The question of the rate of convergence to equilibrium is now a standard question in the literature (see, for instance, [1], [8] and [9] for sharp results in the case of finite Markov chains). An upper bound for spectral gap for the finite symmetric simple exclusion was obtained in [15]. Unfortunately these results do not apply for the infinite case we consider here. For infinite spin-flip systems exponential upper bounds for the rate of convergence were obtained by several authors (see for instance [13] for a clear review of the results concerning the stochastic Ising model, including the Martinelli-Olivieri theorem). For conservative processes, however the rate of convergence is expected to be much slower since it must be the consequence of a sequence of successive rearrangements of the particles present in the initial configuration. In the case of a system with infinitely many independent random walks, a recent paper by [12] shows that with suitable initial configurations this rate is bounded above by $(1/\sqrt{t})^d$, for $d \geq 2$, and is precisely $1/\sqrt{t}$ for $d = 1$. For

$d \geq 3$, the same type of bounds appears in Deuschel's study of L^2-decay of attractive processes on the lattice [7]. These results suggest that the $(\log t)^d$ factor appearing in our upper bound is not optimal.

As it was first shown by Spitzer in [16], the symmetric simple exclusion process has the property of being self-dual. This means that if we start with an initial configuration ζ, then the probability that at time t all the points of a fixed finite set F are occupied is equal to the probability that the initial configuration ζ contains the random set obtained by letting the sites of F evolve as a particles performing a simple exclusion process during a time interval of length t. This translates a question about a system with infinitely many particles in a question about a system with a finite number of particles. Therefore, a natural approach to showing convergence results for the symmetric simple exclusion process is by studying the distance between the law of a finite system with N particles interacting by exclusion and the law of N independent random walks in $\mathbb{Z}^d$. Results in this direction have been obtained by by Bertein and Galves [4], De Masi and Presutti [6] De Masi *et al.* [5] and Andjel [3] (who quotes also an unpublished result by N. Konno). In a related direction, Ferrari *et al.* [10] obtain an upper bound for the truncated correlation function of the symmetric exclusion process with a fixed initial configuration. Our result can be deduced from [10] (for $d = 1$) or from [3] (for$d = 1, 2$), adding estimate (11) below in the case of periodic initial configuration, and adding decorrelations considerations like the ones leading to inequalities (21) and (24) in the mixing case. Here we use a different approach which goes straightforward to the goal, taking full advantage of the properties of the initial distribution. As a consequence, we obtain a sharper in dimension $d \geq 3$. Moreover, compared with the proofs presented in the above mentioned papers, ours is much shorter and direct.

The idea of the proof can be sketched in a few words. We want to obtain an estimate for the probability of having all the sites of a fixed finite set Λ occupied by particles of the system at time t. We consider the dual system obtained by letting evolve backwards in time the sites of Λ and we look at their positions at an intermediate time Δ. If the distance between Δ and t is big enough, then the position of each dual paths is typically far away from the positions of the others. If moreover Δ is close enough to time 0, then the interval of time $[0, \Delta]$ the backward evolution of the dual paths will be contained in disjoint regions of the space and, therefore, will behave as independent random walks. The main technical ingredients of the proof are

Harris' pathwise version of duality and Liggett's correlation inequality.

In the next section we will present the definitions and state the theorem. The proof is given in section 3.

2 Definitions and statement of the theorem.

We use Harris [11] graphical construction to define the process. Call $\mathcal{P}$ the family of Poisson point processes constructed as follows. Two points $x = (x^1, \cdots, x^d)$ and $y = (y^1, \cdots, y^d)$ of $\mathbb{Z}^d$ are nearest neighbors if $|x - y| = \sum_{i=1}^d |x^i - y^i| = 1$. To each nearest neighbor pair of sites $\{x, y\} \subset \mathbb{Z}^d$, a Poisson point process of rate 1 is attached. These processes are mutually independent. Call $(\Omega, \mathcal{F}, P)$ the probability space where these processes are defined. Denote by $\tau_k^{\{x,y\}}$ the time at which the k-th event of the Poisson process attached to $\{x, y\}$ occurs.

For each $\omega \in \Omega$ we say that there is a path from (x, s) to (z, t), where x, z are sites and $s < t$ are times, if there exist $x_1, \ldots, x_n$ and $t_1, \ldots, t_{n+1}$ such that

(1) $x = x_1$, $|x_i - x_{i+1}| = 1$, $x_n = z$,

(2) $t_1 = s < t_2 < \cdots < t_{n+1} = t$,

(3) for $1 \le i < n$, $t_{i+1} = \tau_k^{\{x_i, x_{i+1}\}}$ for some k

(4) there are no events involving x_i in the time interval (t_i, t_{i+1}).

This defines a bijection map $\phi_{\omega,s,t} : \mathbb{Z}^d \to \mathbb{Z}^d$ in the following way: $\phi_{\omega,s,t}(x) = y$ if and only if there is a path in ω from (x, s) to (y, t).

Given an initial configuration $\zeta \in \{0,1\}^{\mathbb{Z}^d}$ we define the simple exclusion process at time t starting at time $s < t$ with configuration ζ by

$$\eta_t^{\zeta,s,\omega}(x) = \zeta(\phi_{\omega,s,t}^{-1}(x)).$$

When $s = 0$ we omit it in the notation. Usually ω is also omitted. This construction implies that

$$P\{\eta_t^{\zeta,s}(x) = 1, \text{ for some } x \in F\} = P\{\zeta(y) = 1, \text{ for some } y \in \phi_{\omega,s,t}^{-1}(F)\},$$

where

$$\phi_{\omega,s,t}^{-1}(F) = \{\phi_{\omega,s,t}^{-1}(x) : x \in F\}.$$

This property is usually called *Duality*.

For any $\rho \in [0,1]$, let ν_ρ be the product measure defined by

$$\nu_\rho(\eta(x) = 1 : x \in F) = \rho^{|F|}, \text{ for } F \subset \mathbb{Z}^d \text{ finite.}$$

The set of extremal invariant probability measures for the simple exclusion process is $\{\nu_\rho : \rho \in [0,1]\}$ (see theorem 1.12 ch. VIII in [14]).

When the initial configuration ζ is chosen at random with probability μ we indicate the process by η_t^μ. Therefore we use the notation

$$P\{\eta_t^\mu \in A\} = \int d\mu(\zeta) P\{\eta_t^\zeta \in A\}.$$

Let μ be a stationary probability measure defined on $\{0,1\}^{\mathbb{Z}^d}$ with density ρ. We shall say that μ is *exponentially mixing* if there exist two positive real constants C and γ such that $\forall k$

$$\left|\mu\left(\{\eta : \eta(x_i) = 1, i = 1, \cdots, k\}\right) - \rho^k\right| \leq C\ k^2\ \exp\{-\gamma \min_{i,j} |x_i - x_j|\}\ . \quad (1)$$

We shall call a configuration $\zeta \in \{0,1\}^{\mathbb{Z}^d}$ *periodic* if there exist positive integers $m_1, \cdots, m_d$ such that $\zeta(x) = \zeta(x + (k_1 m_1, k_2 m_2, \cdots, k_d m_d))$ for any choice of integers $k_1, ..., k_d$. In what follows, to simplify the presentation, we shall only consider the case in which $m_1 = \cdots = m_d$. The period of the configuration is the smallest positive integer m for which this property holds.

For each $n \geq 1$ let $\Lambda_n = \{1, \ldots, n\}^d$. To simplify the notation let us denote by N the cardinality of Λ_n (*i.e.* $N = n^d$) and define the cylindrical event

$$\mathbf{Y}_N = \{\eta : \eta(i) = 1, i \in \Lambda_n\}.$$

Our theorem gives an upper bound for the rate of convergence to equilibrium of the process starting either with a stationary and mixing initial distribution, or with a fixed periodic initial configuration.

Theorem. *For any periodic configuration ζ and any stationary mixing distribution μ, the following inequality holds for all t large enough and all $N = n^d$*

$$\left|P\left\{\eta_t^* \in \mathbf{Y}_N\right\} - \rho^N\right| \leq c_d\ N^2 \left(\frac{\log t}{\sqrt{t}}\right)^d\ , \quad (2)$$

where ρ is the density of the initial configuration and η_t^* means either η_t^ζ or η_t^μ. The constant c_d depends on the period m or on the mixing constants γ and C.

3 Proof of the Theorem.

For a given initial configuration ζ consider the set

$$A_t^\zeta = \left\{\omega : \ \eta_t^{\zeta,\omega}(i) = 1 \ \ \forall i \in \Lambda_n\right\} \tag{3}$$

Let us call S_N the set of all subsets of $\mathbb{Z}^d$ with cardinality N. In what follows $\mathbf{x} = \{x_1, \cdots, x_N\}$ will always denote a generic element of S_N. Let us define also $X_l = \{\mathbf{x} \in S_N : \ |x_i - x_j| > 4l \ , i \neq j\}$. By duality (3) can be written as the union of disjoint sets

$$A_t^\zeta = \bigcup_{\mathbf{x}\in S_N} \left\{\omega : \ \phi^{-1}_{\omega,\Delta,t}(\Lambda_n) = \mathbf{x}, \ \zeta\left(\phi^{-1}_{\omega,0,\Delta}(\mathbf{x})\right) = 1\right\}, \tag{4}$$

where $\zeta(\mathbf{x}) = \prod_{x\in\mathbf{x}} \zeta(x)$. Using the Markov property of the process we have

$$\begin{aligned} P\{A_t^\zeta\} &= \sum_{\mathbf{x}\in X_l} P\{\phi^{-1}_{\omega,\Delta,t}(\Lambda_n) = \mathbf{x}\} P\{\zeta\left(\phi^{-1}_{\omega,0,\Delta}(\mathbf{x})\right) = 1\} \\ &+ \sum_{\mathbf{x}\in X_l^c} P\{\phi^{-1}_{\omega,\Delta,t}(\Lambda_n) = \mathbf{x}\} P\{\zeta\left(\phi^{-1}_{\omega,0,\Delta}(\mathbf{x})\right) = 1\} \end{aligned} \tag{5}$$

Let $B^{x_i}_{\Delta,l}$ be the set of realizations in which the dual random walk starting at x_i remains inside a ball of radius l during the time interval $[0, \Delta]$

$$B^{x_i}_{\Delta,l} = \{\omega : \sup_{0\le s\le\Delta} |\phi^{-1}_{\omega,s,\Delta}(x_i) - x_i| < l\}$$

and let us use the shorthand notation $B_{\Delta,l}(\mathbf{x}) = \cap_{x_i\in\mathbf{x}} B^{x_i}_{\Delta,l}$. We can then write

$$\begin{aligned} P\{A_t^\zeta\} &= \sum_{\mathbf{x}\in X_l} P\left\{\phi^{-1}_{\omega,\Delta,t}(\Lambda_n) = \mathbf{x}\right\} P\left\{\zeta\left(\phi^{-1}_{\omega,0,\Delta}(\mathbf{x})\right) = 1, \ B_{\Delta,l}(\mathbf{x})\right\} \\ &+ \sum_{\mathbf{x}\in X_l} P\left\{\phi^{-1}_{\omega,\Delta,t}(\Lambda_n) = \mathbf{x}\right\} P\left\{\zeta\left(\phi^{-1}_{\omega,0,\Delta}(\mathbf{x})\right) = 1, \ B^c_{\Delta,l}(\mathbf{x})\right\} \\ &+ \sum_{\mathbf{x}\in X_l^c} P\left\{\phi^{-1}_{\omega,\Delta,t}(\Lambda_n) = \mathbf{x}\right\} P\left\{\zeta\left(\phi^{-1}_{\omega,0,\Delta}(\mathbf{x})\right) = 1\right\} \end{aligned} \tag{6}$$

Let $\mathcal{P}_1 \cdots \mathcal{P}_N$ be N independent copies of the family $\mathcal{P}$ of Poisson point processes, and indicate by ω_i a realization of $\mathcal{P}_i$. Using the independence of the Poisson point processes, the first term of (6) can be written as

$$\sum_{\mathbf{x}\in X_l} P\left\{\phi^{-1}_{\omega,\Delta,t}(\Lambda_n)=\mathbf{x}\right\}\prod_{x_i\in\mathbf{x}} P\left\{\left(\zeta\left(\phi^{-1}_{\omega_i,0,\Delta}(x_i)\right)=1,\ B^{x_i}_{\Delta,l}\right)\right\}\ . \tag{7}$$

We now proceed bounding (6) from below and from above.

Lower bound. Let us begin with the case of an initial configuration ζ with period m. Expression (7) is a lower bound for $P\{A^{\zeta}_t\}$. It can be bounded from below by

$$\sum_{\mathbf{x}\in X_l} P\left\{\phi^{-1}_{\omega,\Delta,t}(\Lambda_n)=\mathbf{x}\right\}\left[P\left\{\zeta\left(\phi^{-1}_{\omega_i,0,\Delta}(x_i)\right)=1\ \forall x_i\in\mathbf{x}\right\}-P\left\{B^{c}_{\Delta,l}(\mathbf{x})\right\}\right]. \tag{8}$$

Using the independence property of the Poisson point processes and adding and subtracting ρ we get the following lower bound for (8)

$$\begin{aligned}
&\sum_{\mathbf{x}\in S_N} P\left\{\phi^{-1}_{\omega,\Delta,t}(\Lambda_n)=\mathbf{x}\right\}\prod_{x_i\in\mathbf{x}}\left[\rho-\left|P\left\{\zeta\left(\phi^{-1}_{\omega,0,\Delta}(x_i)\right)=1\right\}-\rho\right|\right]\\
&-\sum_{\mathbf{x}\in X_l} P\left\{\phi^{-1}_{\omega,\Delta,t}(\Lambda_n)=\mathbf{x}\right\}P\left\{B^{c}_{\Delta,l}(\mathbf{x})\right\}\\
&-\sum_{\mathbf{x}\in X_l^c} P\{\phi^{-1}_{\omega,\Delta,t}(\Lambda_n)=\mathbf{x}\}P\left\{\zeta\left(\phi^{-1}_{\omega,0,\Delta}(\mathbf{x})\right)=1\right\}.
\end{aligned} \tag{9}$$

Let us define the torus $T^d_m=\mathbb{Z}^d/(\sim m)$, where $(\sim m)$ is the equivalence relation that identifies x with $x+(k_1m,k_2m,\cdots,k_dm)$ for any choice of integers $k_1,...,k_d$ and let Z^x_Δ be the position at time Δ of the simple random walk, on the finite state space T^d_m, started in x. The following equality holds

$$\left|P\left\{\zeta\left(\phi^{-1}_{\omega,0,\Delta}(x)\right)=1\right\}-\rho\right|=\left|\sum_{y\in T^d_m}\zeta(y)\left[P\{Z^x_\Delta=y\}-\frac{1}{|T^d_m|}\right]\right|\ . \tag{10}$$

It follows from known results on random walks on finite groups (cf. [1]), that there exist constants c and a_m ($a_m\sim 1/m^2$) such that right hand side of (10) can be bounded by

$$\sum_{y\in T^d_m}\zeta(y)\left|P\{Z^x_\Delta=y\}-\frac{1}{|T^d_m|}\right|\le c\ \exp\{-a_m\Delta\}=\epsilon_m(\Delta). \tag{11}$$

This allows us to obtain the following lower bound for the first term in (9)

$$\rho^N - C_1 \; N \; \epsilon_m(\Delta). \tag{12}$$

We now need to get upper bounds for the second and third terms in (9). The second term can be bounded by

$$\sum_{\mathbf{x}\in X_l} P\left\{\phi^{-1}_{\omega,\Delta,t}(\Lambda_n) = \mathbf{x}\right\} \sum_{x_i\in\mathbf{x}} P\left\{(B^{x_i}_{\Delta,l})^c\right\}$$

$$\leq \sum_{\mathbf{x}\in X_l} P\left\{\phi^{-1}_{\omega,\Delta,t}(\Lambda_n) = \mathbf{x}\right\} \sum_{x_i\in\mathbf{x}} \sum_{k=1}^{d} P\{R^{x_i,k}_{\Delta} \geq l\}, \tag{13}$$

where $R^{x_i,k}_{\Delta}$ is the maximum value attained by the distance between the $k-$coordinate of x_i and $\phi^{-1}_{\omega,0,s,}(x_i)$ for $s \in [0,\Delta]$. Since

$$P\{R^{x_i,k}_{\Delta} \geq l\} \leq 2P\{\mathcal{N}_{\Delta} \geq l\},$$

where $\mathcal{N}_{\Delta}$ is the number of occurrences of a rate 1 Poisson point process during the time interval $[0,\Delta]$, using the exponential Chebichev inequality, for $l = e\Delta$, we obtain the following upper bound for (13)

$$2d \; N \; \exp\{-\Delta\}. \tag{14}$$

Let study now the last term of (9). Using the translation invariance of the process and defining

$$Y^u_l = \{v \in \mathbb{Z}^d : \; |v-u| \leq 4l\}$$

we have

$$\sum_{\mathbf{x}\in X^c_l} P\{\phi^{-1}_{\omega,\Delta,t}(\Lambda_n) = \mathbf{x}\} P\left\{\zeta\left(\phi^{-1}_{\omega,0,\Delta}(\mathbf{x})\right) = 1\right\}$$

$$\leq \; \frac{N^2}{2} \sum_{u\in\mathbb{Z}^d} \sum_{v\in Y^u_l} P\left\{\phi^{-1}_{\omega,\Delta,t}(0) = u, \; \phi^{-1}_{\omega,\Delta,t}(a) = v\right\}, \tag{15}$$

where a is a nearest neighbor of 0.

Liggett's correlation inequality for the symmetric exclusion process (see [14] and [2] for an extension) provides the following upper bound for (15)

$$\leq c\ N^2 \sum_{u\in\mathbb{Z}^d}\sum_{v\in Y_l^u} P\left\{\phi^{-1}_{\omega,\Delta,t}(0)=u\right\}\ P\left\{\phi^{-1}_{\omega,\Delta,t}(a)=v\right\}\ . \tag{16}$$

Let Z_t^0 and Z_t^a be two independent simple random walks on $\mathbb{Z}^d$ starting respectively in 0 and a. By construction

$$P\{\phi^{-1}_{\omega,\Delta,t}(0)=u\}=P\{Z^0_{t-\Delta}=u\},\ P\{\phi^{-1}_{\omega,\Delta,t}(a)=v\}=P\{Z^a_{t-\Delta}=v\}$$

so that (16) is bounded by

$$c\ N^2\ \left(\frac{l}{\sqrt{t-\Delta}}\right)^d\ . \tag{17}$$

Inequalities (12), (14) and (17) provides the final lower bound for the case of periodic initial configuration when $l=e\ \Delta$

$$P\{A_t^\zeta\}\geq \rho^N - C_1N\epsilon_m(\Delta)-2d\ N\ \exp\{-\Delta\}-C_3N^2\ \left(\frac{l}{\sqrt{t-\Delta}}\right)^d\ . \tag{18}$$

Let us now consider the case of a mixing initial distribution. Using the independence of the Poisson point processes $\mathcal{P}_i,\ i=1,\cdots,N$ we can write (7) as

$$\sum_{\mathbf{x}\in X_l}\prod_{x_i\in\mathbf{x}} P\left\{\zeta\left(\phi^{-1}_{\omega_i,0,\Delta}(x_i)\right)=1,\ B^{x_i}_{\Delta,l}\right\}P\left\{\phi^{-1}_{\omega,t,\Delta}(\Lambda_n)=\mathbf{x}\right\}\ . \tag{19}$$

We then have, with the same notation introduced for the periodic case,

$$P\{A_t^\zeta\}\geq\sum_{\mathbf{x}\in X_l}\left[\prod_{x_i\in\mathbf{x}}\sum_{|y-x_i|<l}\zeta(y)P\{Z^{x_i}_\Delta=y\}\right]P\left\{\phi^{-1}_{\omega,t,\Delta}(\Lambda_n)=\mathbf{x}\right\}. \tag{20}$$

Now we take the expectation with respect to the probability distribution μ of the initial configuration. Following the same steps of the periodic case, but replacing (11) with the mixing property (1), we find (for $l=e\Delta$)

$$\int d\mu(\zeta) P\{A_t^\zeta\} \geq \rho^N - C_1 N^2 \exp\{-2\gamma\ l\}$$
$$- \quad 2d\ N\ \exp\{-\Delta\} - C_3 N^2 \left(\frac{l}{\sqrt{t-\Delta}}\right)^d . \qquad (21)$$

Upper bound. Let us start with the case of a configuration of period m. We have to bound the three terms in (6). We obtain the following upper bound for the first one

$$\sum_{\mathbf{x}\in S_N} P\left\{\phi^{-1}_{\omega,\Delta,t}(\Lambda_n) = \mathbf{x}\right\} \prod_{x_i\in\mathbf{x}} \left[\rho + \left|P\left\{\zeta\left(\phi^{-1}_{\omega,0,\Delta}(x_i)\right) = 1\right\} - \rho\right|\right]$$
$$\leq \rho^N + C_1\ N\ \epsilon_m(\Delta), \qquad (22)$$

where we have used (10) and (11) to get the last inequality.

For the second and the third ones, we can use (14) and (17). We thus have

$$P\{A_t^\zeta\} \leq \rho^N + C_1 N \epsilon_m(\Delta) + 2d\ N\ \exp\{-\Delta\} + C_3 N^2 \left(\frac{l}{\sqrt{t-\Delta}}\right)^d , \quad (23)$$

for $l = e\Delta$.

When the initial configuration is chosen with a mixing probability μ, we can use property (1) to get the upper bound for the first term in (6). The estimates for the other terms are given by (14) and (15). This finally gives the final upper bound (for $l = e\Delta$)

$$\int d\mu(\zeta) P\{A_t^\zeta\} \leq \rho^N + C_1 N^2 \exp\{-2\gamma\ l\}$$
$$+ \quad 2d\ N\ \exp\{-\Delta\} + C_3 N^2 \left(\frac{l}{\sqrt{t-\Delta}}\right)^d . \qquad (24)$$

Then, choosing $\Delta \sim b \log t$, where b is a suitable constant, which depends on m or on γ and C, from (18)-(23) and (21)-(24), we get the result.

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