# Workshop

# Geometry in Algebra and Algebra in Geometry

IME—USP, Sao Paulo, Brazil December 1—2, 2015

> List of speakers, Schedule and Abstracts





Workshop

## Geometry in Algebra and Algebra in Geometry

Schedule and Abstracts

December 01–02, 2015, São-Paulo, Brazil

## LIST OF SPEAKERS

Cristian Ortiz	 IME–USP
Elizaveta Vishnyakova	 IME-USP
Eugenia Martin	 IME-USP
German Alonso Benitez	 IME-USP
Ivan Struchiner	 IME-USP
Jeffrey Carlson	 IME-USP
James Waldron	 Newcastle University
Kostiantyn Iusenko	 IME-USP

### Schedule

1st of December		
10.00-11.00	Kostiantyn Iusenko	
11.00-12.00	German Alonso Benitez	
12.00-14.00	Lunch	
14.00-15.00	Cristian Ortiz	
15.00-15.15	Coff ee-break	
15.15-16.15	Ivan Struchiner	
2st of December		
09.00-10.00	Jaffrey Carlson	
10.00-11.00	James Waldron	
11.00-11.15	Coff ee-break	
11.15-12.15	Eugenia Martin	
12.15-13.15	Elizaveta Vishnyakova	

#### 1st of December

#### Multiplicative vector fields on Lie groupoids

CRISTIAN ORTIZ

The talk will discuss the notion of multiplicative vector field on a Lie groupoid, describing the algebraic structure on the space of all such vector fields. If time permits, I will explain the connection of this talk with study of vector fields on singular spaces, e.g. orbifolds.

#### Derived bracket formalism and invariant *n*-ary algebras

ELIZAVETA VISHNYAKOVA

The derived bracket approach is used in different areas of mathematics: in Poisson geometry, in the theory of Lie algebroids and Courant algebroids, BRST and BV formalism, in the theory of Loday algebras and different types of Drinfeld doubles. The idea of the formalism is the following. One fixes an algebra L, usually a Lie superalgebra, and constructs another multiplication on the same vector space (or some subspace) using derivations of L and the iterated multiplication in L. One obtains a class of new algebras, which properties can be studied using original algebra L. For example, using this formalism we can obtain all Poisson structures on a manifold M from the canonical Poisson algebra on  $T^*M$ .

Our talk is devoted to a construction of different types of invariant (n-ary) algebras using the derived bracket formalism. We will focus in particular on commutative n-ary superalgebras with a non-degenerate skew-symmetric form.

#### Variedades de Álgebras de Jordan

#### EUGENIA MARTIN

Pretendemos descrever geometricamente as álgebras de Jordan de dimensőes pequenas sobre um corpo k algebricamente fechado de  $char(k) \neq 2$ . Para isso, introduziremos a definição de variedade das álgebras de Jordan, assim como também as noções de deformação de uma álgebra e de rigidez. Apresentaremos uma lista de invariantes que determinarão a não existencia de deformação entre um par de álgebras dadas como, por exemplo, a dimensão do grupo de automorfismos, do aniquilador, do radical nilpotente, das potencias de uma álgebra, entre outros. Provaremos que, cada componente irredutível da variedade das álgebras de Jordan, ou é o fecho de Zariski da órbita de uma álgebra rígida ou é o fecho da união infinita de uma família de órbitas de álgebras de uma mesma classe.

Ilustraremos esses fatos determinando as componentes irredutíveis da variedade Jor<sub>4</sub> das álgebras de Jordan de dimensão 4 e JorN<sub>5</sub> das álgebras de Jordan Nilpotentes de dimensão 5. No primeiro caso provaremos que a variedade possui 73 órbitas sob a ação do grupo linear geral GL(V) e cada componente irredutível é o fecho de Zariski da órbita de uma álgebra rígida. Já a segunda, representa o primeiro caso estudado de variedade de álgebras de Jordan que possui um número infinito de órbitas, portanto é de grande interesse determinar se existe nesta variedade uma componente irredutível que corresponda ao fecho da união das órbitas de alguma das 6 classes infinitas de álgebras não isomorfas que possui esta variedade.

#### Gelfand–Tsetlin varieties and Kostant–Wallach map

German Alonso

We will discuss the equidimensionality of Gelfand-Tseltin varieties for certain Lie algebras. Also we aim to discuss the relation between Konstant-Wallch map and such varieties.

#### **Stability Problems for Lie Algebras**

IVAN STRUCHINER

I will show how to use elementary techniques from differential geometry to solve some classical stability problems for Lie algebras. The talk will be based on joint work with Marius Crainic and Florian Schätz (arXiv:1307.7979).

#### Equivariant formality in rational cohomology and K-theory

#### JEFFREY CARLSON

In certain geometric settings connected with group actions (particularly symplectic geomtery), an integral of an invariant function can be reduced to an integral over a lower-dimensional (or even finite) fixed-point set. This occurs when a differential form admits an "equivariant extension," and an action is said to be "equivariantly formal" in cases where every form admits an equivariant extension.

Equivariant formality can be phrased as the purely algebraic question of whether a map  $H_K^*(G/K) \to H^*(G/K)$  of cohomology rings is surjective, and this determination is already subtle and incomplete for the fundamental example of the isotropy action of a Lie group K on a homogeneous space G/K.

I will summarize all that is known about equivariant formality, including a reduction to the torus case, a classification in the event K is of rank one, and a number of natural representation-theoretic criteria that arise from asking the analogous question for equivariant K-theory. Some of this work is joint with Chi-Kwong Fok.

#### Orbifolds, Crossed Product Algebras, and Quantization

JAMES WALDRON

Given a group G acting on a space X, one can construct the crossed product algebra  $G \ltimes C[X]$  of G with the algebra of functions on X. This generally noncommutative algebra can be seen as a replacement for the algebra of functions on the quotient X/G, which may be badly behaved.

I will speak about deformations of these algebras - both formal deformations (in the sense of Gerstenhaber) and strict deformations (in the sense of Rieffel). Via some examples, I will explain how certain geometric and algebraic properties of these deformations are related to certain questions in representation theory, and to the geometry of the original action of G on X. In particular, I will explain how the notion of Morita equivalence becomes relevant.

If there is time I will also describe the relationship to certain constructions in Poisson geometry.

#### Moduli spaces of posets

Kostiantyn Iusenko

Representations of finite partilly ordered sets is a huge field of investigation. One of the fundamental tasks is the classification up to isomorphism of indecomposables ones. But "most" of the posets are wild, in the sense that the problem of classification of their representations is as difficult as the classification of representations of free algebras. We try to approach the classification problem geometrically. So we consider the moduli spaces associated with given dimension vector and an integer weight (both associated to a given poset). We will discuss the characterization of the points in these moduli spaces, and when they are non-empty.