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Geometry in Algebra and Algebra in Geometry

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ABSTRACTS

Geometry in Algebra and Algebra in Geometry V

Schedule and Abstracts

October 21–25, 2019, Medellin, Colombia

List of Speakers

Alberto Medina	—	Universite de Montpellier, France
Alexander Quintero Velez	—	UNAL, Colombia
Aline Viela Andrade	—	UFF, Brazil
Andrés Franco	—	Universidad de Antioquia, Colombia
Bernardo Uribe	—	Universidad del Norte, Colombia
Camilo Arias Abad	—	UNAL, Colombia
Charles Almeida	—	IME-USP, Brazil
Daniele Sepe	—	UFF, Brazil
Elizabeth Gasparim	—	Universidad Catolica del Norte, Chile
Hernan Giraldo	—	Universidad de Antioquia, Colombia
Ivan Struchiner	—	IME-USP, Brazil
Manuel Rivera	—	Purdue University
Maria Amelia Salazar	—	UFF, Brazil
Matias del Hoyo	—	UFF, Brazil
Omar Saldarriaga	—	Universidad de Antioquia, Colombia
Valeriano Lanza	—	UFF, Brazil

ORGANIZERS

Cristian Ortiz	—	IME–USP, Brazil
Kostiantyn Iusenko	—	IME–USP, Brazil
Pedro Hernandez Rizzo	—	UdeA, Colombia

ABSTRACTS

Mini-courses:

ALEXANDER QUINTERO VELEZ (UNAL, COLOMBIA).

Quivers and noncommutative resolutions

In this mini-course, participants will be introduced to the mathematical machinery behind the study of noncommutative resolutions of algebraic varieties. This machinery find its foundations in the theory of quiver representantions and formal noncommutative geometry. Thus, we will give a survey of these areas, emphasizing those aspects that use algebraic geometry as tool.

MARÍA AMELIA SALAZAR (UNIVERSIDADE FEDERAL FLUMINENSE) AND DANIELE SEPE (UNIVERSIDADE FEDERAL FLUMINENSE)

Jacobi geometry via Lie algebroids and Lie groupoids

A *Jacobi structure* on a (real) line bundle is a Lie bracket on the space of smooth sections that is local. Jacobi structures extend and unify several well-known geometric objects, ranging from Poisson bivectors to contact distributions. The aim of this minicourse is to provide an introduction to the theory of Jacobi geometry, *i.e.* line bundles endowed with a Jacobi structure, using the language of *Lie groupoids* and *Lie algebroids*. Lie groupoids and Lie algebroids are geometric objects that, while generalising Lie groups and Lie algebras, allow to deal with (certain families of) infinite dimensional Lie groups and Lie algebras in a finite dimensional setting. Using this perspective, we will show that there is an intimate relation between Jacobi geometry and *real projective* geometry, that generalises (and is connected to!) the relation between Poisson and affine structures. Throughout the minicourse, examples are used to illustrate the theory that we develop.

1 M. Crainic and M. A. Salazar, *Jacobi structures and Spencer operators*, J. Math. Pures Appl. (9), Vol. 103, (2015), 504 – 521.

2 M. Crainic and C. Zhu, *Integrability of Jacobi and Poisson structures*, Ann. Inst. Fourier (Grenoble), Vol. 57, (2007), 1181 – 1216.

3 M. A. Salazar and D. Sepe, *Contact isotropic realisations of Jacobi manifolds via Spencer operators*, SIGMA 13, (2017), 033, 44 pages.

Talks:

ALBERTO MEDINA (INSTITUT A. GROTHENDIECK-UNVERSIDAD DE ANTIOQUIA)

Groups of flat affine transformations

A well understanding of the category of lagrangian manifolds suppose a good knowledge of the category of flat affine manifolds. These last manifolds appear naturally in integrable systems and Mirror Symetry.

Given a real n-dimensional manifold M endowed with a linear (hence affine) connection ∇

1. We give a characterization of the fact that ∇ will be affinely flat by a natural homomorphism of Lie groups

$$\rho : \mathsf{Aff}(M, \nabla) \longrightarrow \mathsf{Aff}(\mathbb{R}^n, \nabla^0)$$

2. If $\dim(A(M), \nabla)) \geqslant n$ and ∇ is flat affine we show the existence of an affine representation of $\mathsf{Aff}(M, \nabla)$ in $(\mathbb{R}^n, \nabla^0)$ having an open orbit. Moreover if $\dim(\mathsf{Aff}(M, \nabla)) = n$ this orbit has discret isotropy.

3. Given a Lie subgroup of $\mathsf{Aff}(M, \nabla)$ then there exists an associative envelope of Lie(H) relative to ∇.

4. We exhibit some cases where $\mathsf{Aff}(M, \nabla)$ admits a flat affine (or projective) structure deduced from ∇.

Keywords: Flat affine manifolds, Infinitesimal affine transformations, Sheaves of Lie algebras, Associative Envelope.

Collaboration with O. Saldarriaga and A. Villabón.

1. C. Ehresman. *Sur les espaces localement homogènes*, Enseign. Math. 35 (1936) 317—333.

2. M. Kontsevich; Y. Soibelman. *Affine structures and non-Archimedean analytic spaces.* The unity of mathematics, Progr. Math., 244, Birkhäuser Boston, Boston, MA, (2006), 321—385.

3. J. L. Koszul. *Variétés localement plates et convexité,* Osaka J. Math. 2 (1965) 285-290.

4. A. Medina, O. Saldarriaga and H. Giraldo. *Flat Left Invariant or Projective Geometries on Lie Groups*. Journal of Algebra 455 (2016) 183–208

5. A. Weinstein. *Symplectic Manifolds and their Lagrangian submanifolds.* Adv. in Math, 6, (1971), 329-346

6. K. Yagi. *On compact homogeneous affine manifolds.* Osaka J. Math. 7 (1970), no. 2, 457–475.

ALINE V. ANDRADE (UNIVERSIDADE FEDERAL FLUMINENSE - UFF)

Moduli space of orthogonal instanton bundles of higher rank on $\mathbb{P}^n$

In order to obtain existence criteria for orthogonal instanton bundles on $\mathbb{P}^n$ $(n \geqslant 3)$, we provide a bijection between equivalence classes of orthogonal instanton bundles with no global sections and symmetric forms. Using such correspondence we are able to provide explicit examples of orthogonal instanton bundles with no global sections on $\mathbb{P}^n$ and prove that every orthogonal instanton bundle with no global sections on $\mathbb{P}^n$ and charge $c \geqslant 2$ has rank $r \leqslant (n-1)c$. We also prove that when the rank r of the bundles reaches the upper bound, $\mathcal{C}^{\mathcal{O}}_{\mathbb{P}^n}(c, r)$, the coarse moduli space of orthogonal instanton bundles with no global sections on $\mathbb{P}^n$, with charge $c \geqslant 2$ and rank r, is affine, smooth, reduced and irreducible. Last, we construct Kronecker modules to determine the splitting type of the bundles in $\mathcal{C}^{\mathcal{O}}_{\mathbb{P}^n}(c, r)$, whenever is non-empty. This is a joint work with Simone Marchesi (Universidade Estadual de Campinas/UNICAMP) and Rosa Maria Miró-Roig (Universidad de Barcelona/UB).

1 A. V. Andrade, S. Marchesi, R. M. Miró-Roig. *Irreducibility of the moduli space of orthogonal instanton bundles on* $\mathbb{P}^n$, Revista Matematica Complutense. (2019) [Accepted for publication - available online].

ANDRÉS FRANCO (UNIVERSIDAD DE ANTIOQUIA)

String and band complexes over SAG algebras

In this talk I will explain a combinatorial description of a family of indecomposable objects in the bounded derived categories of string almost gentle algebras. These indecomposable objects are, up to isomorphism, the string and band objects introduced by V. Bekkert and H. Merklen in its well-known classification of the indecomposable complexes over the bounded derived category of a gentle algebra. With this description, we give a sufficient condition for a given string complex to have infinite minimal projective resolution, and we extend this condition for the case of string algebras.

BERNARDO URIBE (UNIVERSIDAD DEL NORTE)

Pontrjagin duality in Multiplicative gerbes

Multiplicative gerbes are group objects in the monoidal category of gerbes. In this talk I will introduce these objects and I will show how to classify them using Segal-Mitchison cohomology. Some futher properties will be presented.

1 Waldorf, Konrad. Multiplicative bundle gerbes with connection. Differential Geom. Appl. 28 (2010), no. 3, 313–340.

CAMILO ARIAS ABAD (UNIVERSIDAD NACIONAL DE COLOMBIA)

Singular chains on Lie groups and the Cartan relations

If G is a Lie group, the space C(G) of singular chains on G is a dg-Hopf algebra. We show that if G is simply connected, the category of representations of this algebra is equivalent to the category of representation of the dg-Lie algebra Tg, which is universal for the Cartan Relations. In the case where G is compact, we use Alekseev and Meinrenken's noncommutative Weil algebra and Gugenheim's A-infinity de Rham theorem to extend the theorem to an equivalence of dg-categories. We will explain the relationship of these results to Chern-Weil theory. Partly based on joint work with A. Quintero.

CHARLES ALMEIDA (IME-USP)

Geometry of moduli spaces of representation of posets

In this talk we will present the notion of semistability conditions for the moduli space of representation of posets, studied by Futorny and Iusenko in [1], and then introduce the notion of wall system for the stability conditions. Next, we will characterize all possible walls for a given vector dimension. Finally we will discuss the wall crossing phenomena in the stability conditions of the moduli spaces of semistable representations of posets. *This is a joint work with Kostiantin Iusenko*.

1 V. Futorny, and K. Iusenko, *Stable representations of posets*, in press (2019), Journal of Pure and Applied Algebra; Online version https://doi.org/10.1016/j.jpaa.2019.03.020

ELIZABETH GASPARIM - UNIVERSIDAD CATÓLICA DEL NORTE, CHILE

The effects of deformations on mirrors and moduli

I will discuss some effects that deformations of complex structures provoque on various geometric objects in both algebraic and symplectic geometry, such as:

- Mirrors of Landau–Ginzburg Models,
- Moduli of Vector Bundles and Instantons,
- Lagrangian Submanifolds.

This is joint work with a team of collaborators: E. Ballico (Italy), S. Barmeier (Germany), L. Grama (Brazil), F. Rubilar (Chile), T. Köppe (UK), L. San Martin (Brazil), B. Suzuki (Chile), C. Varea (Brazil), and F. Valencia (Chile).

1 E. Gasparim, F. Valencia, C. Varea, *Invariant generalized geometry on maximal flag manifolds*, in preparation.

2 E. Ballico, E. Gasparim, B. Suzuki, *Infinite dimensional families of Calabi–Yau threefolds and moduli of vector bundles*, preprint.

3 F. Rubilar, L. Schultz, *Adjoint orbits of* $\mathfrak{sl}(2,\mathbb{R})$ *and their geometry*, preprint.

4 E. Gasparim, F. Rubilar, *Deformations of non-compact Calabi–Yau manifolds, families and diamonds*, arXiv: 1908.09045.

5 E. Gasparim, S. Barmeier, *Quantization of local surfaces and rebel instantons*, arXiv:1904.09455.

6 E. Gasparim, L. San Martin, F. Valencia, *Infinitesimally tight Lagrangian orbits*, arXiv:1903.03717.

7 E. Ballico, E. Gasparim, L. San Martin, F. Rubilar, *KKP conjecture for minimal adjoint orbits*, arXiv:1901.07939.

8 E. Ballico, S. Barmeier, E. Gasparim, L. Grama, L. San Martin, *A Lie theoretical construction of a Landau-Ginzburg model without projective mirrors*, Manuscripta Math. **158** no. 1-2 (2019) 85–101.

9 E. Gasparim, S. Barmeier, *Classical deformations of local surfaces and their moduli of instantons*, J. Pure and Applied Algebra. **223** no. 6, (2019), 2543–2561.

10 E. Gasparim, T. Köppe, F. Rubilar, B. Suzuki, *Deformations of noncompact Calabi–Yau threefolds*, Rev. Colombiana de Matemáticas, **52** no. 1 (2018) 41–57.

HERNÁN GIRALDO (UNIVERSIDAD DE ANTIOQUIA)

Derived tame Nakayama algebras

We determine derived representation type of Nakayama algebras and prove that a derived tame Nakayama algebra without simple projective module is gentle or derived equivalent to some skewed-gentle algebra and as a consequence, we determine its singularity category.

This is a joint-work with Viktor Bekkert and José A. Vélez-Marulanda.

IVAN STRUCHINER (IME-USP)

Stability of Lie Group Homomorphisms

We will show how to use a Moser type argument to prove that any homomorphism from a compact Lie group to any other Lie group is stable under deformations. The talk will be based on joint [1] work with Cristian Camilo Cárdenas (UFF).

1 C. C. Cárdenas, and I. Struchiner, *Stability of Lie Group Homomorphisms and Lie Subgroups*, in press (2019), Journal of Pure and Applied Algebra; Online version available at https://doi.org/10.1016/j.jpaa.2019.07.017

MANUEL RIVERA (UNIVERSITY OF MIAMI)

Singular chains and the fundamental group

In this talk I will explain the sense in which the fundamental group of a path connected space, as well as higher dimensional homotopical aspects, can be determined directly and in complete generality from algebraic structure of the singular chains on the space. To give a precise and transparent explanation of this fact we 1) extend a classical result of F. Adams from 1956 regarding an algebraic model for the based loop space of a simply connected space and 2) use the homotopical symmetry of the chain approximations to the diagonal map on a space.

MATIAS DEL HOYO (UFF)

Simplicial vector bundles as Lie groupoid representations

Lie groupoids are categorified manifolds, they provide a unified framework for classic geometries, and they can be used to model stacks in differential geometry. As Lie groups are represented by symmetries of a vector space, a convenient way to represent Lie groupoids, introduced by C. Abad and M. Crainic, is by symmetries of a graded vector bundle. I will present a new approach to the theory, developed jointly with G. Trentinaglia, by means of a semi-direct product construction, which sets a correspondence between simplicial vector bundles and Lie groupoid representations. As an application, I will mention a project developed jointly with C. Ortiz and F. Studzinski, on Morita invariance of Lie groupoid cohomology.

OMAR SALDARRIAGA (UNIVERSIDAD DE ANTIOQUIA)

About flat affine manifolds and their transformaions

The talk will start by recalling basic notions of flat affine manifolds, flat affine Lie groups and their transformations. We will give examples of affine manifolds whose group of affine transformations admit a flat (or projective) structure deduced of those of the manifold. gives a partial answer to a question posed by A. Medina during a congress in Vigo in the year 2000. We determine the group of affine transformations of all connected flat affine surfaces. Finally given a connected surface M and a Galoisian (or regular) covering M' of M we describe relations between $\mathsf{Aff}(M,\nabla)$ and $\mathsf{Aff}(M',\nabla')$ where ∇ is naturally determined by a flat affine connection on M'.

Work in collaboration with A. Medina y A. Florez.

1 D. Arrowsmith, P. Furness. Locally symmetric spaces. J. London Math. Soc. (2) 10 (1975), no. 4, 487-499.

2 L. Auslander and L. Markus, Holonomy of Flat Affinely Connected Manifolds. Annals of Math. Vol. 62, No. 1 1955, pp. 139-151

3 Y. Benoist. Tores affines. Contemporary Math; 262; Crystallographic Groups and Their Generalizations Workshop; (2000) 1-37

4 M. Bordemann, M and A. Medina. Le groupe des transformations affines d'un groupe de Lie a structure affine bi-invariante. Research and Exposition in Mathematics, (2002), no 25, 149-179

5 P. Furness and D. Arrowsmith. Flat Affine Klein Bottles Geometriae Dedicata, (5) 1 (1976), 109-115.

6 W. Goldman. Geometric structures on manifolds. University of Maryland, Lecture Notes. (2018)

7 A. Medina. Flat left invariant connections adapted to the automorphism structure of a Lie group. Journal of differential Geometry. Vol 16 (1981), no 3, 445–474.

8 A. Medina, O. Saldarriaga and H. Giraldo. Flat Left Invariant or Projective Geometries on Lie Groups. Journal of Algebra 455 (2016) 183–208

9 J. Milnor. On fundamental groups of complete affinely flat manifolds. Advances in Math., 25 (2) (1977) 178–187

10 T. Nagano and K. Yagi. The affine structures on the real two-torus. I Osaka Journal of Mathematics, 11 (1974), no. 1, 181-210.

11 A. Onishchik and E. Vinberg. Lie Groups and Lie Algebras II. Encyclopedia of mathematical sciences Vol 21; Springer Verlag; (1988)

12 M. Postnikov. Geometry VI. Riemannian Geometry. Encyclopedia of mathematical sciences Vol 91; Springer Verlag; (2000)

13 J. Smillie. An obstruction to the existence of affine structures, Invent. Math. 64 no 3, (1981) 411–415.

VALERIANO LANZA (UFF)

Hilbert schemes and irreducibility of certain quiver varieties

In [1], the Hilbert scheme of points of the total space of $\mathcal{O}_{\mathbb{P}^1}(-n)$ was described as an irreducible component of the moduli space of semistable representations of a suitable framed quiver $\mathcal{Q}_n^{\mathrm{fr}}$. We review and complete that result, by showing that actually the Hilbert scheme coincides with the whole moduli space of representations of $\mathcal{Q}_n^{\mathrm{fr}}$, which in particular turns out to be irreducible.

1 C. Bartocci, U. Bruzzo, V. Lanza and C. L. S. Rava, *Hilbert schemes of points of* $\mathcal{O}_{\mathbb{P}^1}(-n)$ *as quiver varieties*. Journal of Pure and Applied Algebra 221 (2017), 2132–2155.