GA AG— IV

Geometry in Algebra and Algebra in Geometry
IME-USP

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Abstracts

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FAPESP

INSTITUTO DE MATEMÁTICA E ESTATÍSTICA
UNIVERSIDADE DE SÃO PAULO

Organizers — Cristian Ortiz, Ivan Struchiner, Kostiantyn Iusenko

Geometry in Algebra
and
Algebra in Geometry
IV

Schedule and Abstracts

November 05–09, 2018, São Paulo, Brasil

List of Speakers

Abdelmoubine Amar Henni	—	UFSC
Alejandro Cabrera	—	UFRJ
Alessia Mandini	—	PUC-Rio
David Martinez Torres	—	PUC-Rio
Eugenia Martin	—	UFPR
Jean Valles	—	UPPA-LMA
Luca Vitagliano	—	Universitá di Salerno
Mark Colarusso	—	University of South Alabama
Matias del Hoyo	—	UFF
Oliver Lorscheid	—	IMPA
Peter Arndt	—	University of Düsseldorf
Simone Marchesi	—	UNICAMP

Organizers

Cristian Ortiz	—	IME–USP
Ivan Struchiner	—	IME–USP
Kostiantyn Iusenko	—	IME–USP

ABSTRACTS

Mini-courses:

LUCA VITAGLIANO (UNIVERSITÁ DI SALERNO)

Calculus up to homotopy on leaf spaces

Foliations are important objects in differential geometry. The main reason for their importance is that they are distinguished instances of more general, and ubiquitous, objects: differentiable stacks on one side, and solution spaces of PDEs on the other side. Given a foliation, its leaf space is of a particular interest, for instance, in reduction problems. However, leaf spaces are often highly non-regular, even in the case of a regular foliation, and it is natural to ask the question: how can we define a calculus on a leaf space? In this mini-course, I will propose an answer inspired by Homotopical/Derived Geometry. It turns out that functions, vector fields, differential forms, etc. on a leaf space can be understood as elements of suitable homotopy algebras. Accordingly, every leaf space supports a calculus up to homotopy.

PETER ARNDT (UNIVERSITY OF DÜSSELDORF)

Abstract motivic homotopy theory

Motivic homotopy theory is a fusion of homotopy theory and algebraic geometry. In analogy to the homotopy category of topological spaces, obtained by making the unit interval contractible, one obtains a homotopy category of schemes by making the affine line contractible. This category has a rather topological flavour; one can for example talk about suspensions, loop spaces and classifying spaces and represent cohomology theories by spectra. The approach has led to a multitude of results, for example on quadratic forms, vector bundles over schemes, algebraic cycles, algebraic K-theory and computations of stable homotopy groups of spheres in topology.

The basic construction steps of motivic homotopy theory out of the category of schemes have been repeated for complex and non-archimedean analytic geometry, as well as derived algebraic geometry, and these new settings are being profitably applied. Further geometric settings suggest themselves as input for variants of motivic homotopy theory, in particular the many proposals for geometry over the field with one element.

In this minicourse I will present an abstract setup for motivic homotopy theory, encompassing most known variants and analogues of algebraic geometry. We start with a reminder of classical homotopy theory and then introduce motivic homotopy theory in a parallel fashion. Then I will motivate what follows by presenting some notions of "schemes over deeper bases" and other alternative settings of algebraic geometry. In the main part of the course I will start from a very general input: a cartesian closed, presentable infinity category, replacing the category of motivic spaces, and a commutative group object therein, replacing the multiplicative group scheme. For this starting data I will present constructions and results generalizing those of motivic homotopy theory. Among these are a representation theorem for line bundles, a Snaith type algebraic K-theory spectrum, Adams operations, rational splittings and a rational motivic Eilenberg-MacLane spectrum.

MARK COLARUSSO (UNIVERSITY OF SOUTH ALABAMA)

Integrable systems and Lie theory

In this minicourse, we study the interaction between Lie theory and the theory of integrable systems. Integrable systems first appeared in the 19th century as classical mechanical systems for which the equations of motion could be solved by a finite sequence of algebraic operations, integration, and function inversion. Such classical integrable systems include the motion of certain spinning tops, the free motion of a particle on an ellipsoid, and the motion of a rigid body in an ideal fluid.

In the last 40 years, Lie theory has played a large role in the study of integrable systems. The symplectic geometry of Lie algebras is a natural place to understand certain integrable systems such as the Toda lattice. Aside from physics, integrable systems have proven themselves to be useful in Lie theory. In the 1980's Guillemin and Sternberg showed that a certain integrable system on conjugacy classes of $n \times n$ Hermitian matrices can be thought of as a geometric analogue of the classical Gelfand-Zeitlin basis for irreducible representations of the unitary group. One of our main goals of the course will be to understand a complexifed version of the integrable system developed by Guillemin and Sternberg and its interaction with the representation theory of infinite dimensional Gelfand-Zeitlin modules introduced by Drozd, Futorny, and Ovseinko.

We will begin discussing some ideas from Hamiltonian mechanics and symplectic geometry and then use these ideas to analyze some classical integrable systems. In the following talks, we will see how Lie theory and Poisson geometry can be used to describe some modern integrable systems including the Toda lattice and Gelfand-Zeitlin integrable systems. Finally, we will discuss how the geometry of Gelfand-Zeitlin integrable systems.

SIMONE MARCHESI (UNICAMP)

An introduction to instanton bundles

The instanton bundles were introduced on the 4-dimensional sphere S^4 by Atiyah, Drinfeld, Hitchin and Manin in [ADHM78] and generalized in algebraic geometry by Okonek and Spindler in [OS86]. The instant bundles represent a connection between algebraic and physical geometry: in fact, instant bundles on $\mathbb{P}^3$ are related to the solutions of the Yang-Mills equations in the 4 - compact S^4 euclidean space. Since then, the research of this family of fibrados and its space of modules has had great interest in the mathematical community. For example, recall that Spindler and Trautmann in [ST90] studied modulos of instanton bundles; Costa and Ottaviani demonstrate in [CO02] that the moduli space of symplectic bundle on $\mathbb{P}^{2n+1}$ is affin. In [JV14], Jardim and Verbitsky prove the smoothness of the moduli space of instanton bundles of rank 2 over $\mathbb{P}^3$, in [Tik12, Tik13] Tikhomirov demonstrates the irreducibility in $\mathbb{P}^3$ for instantons.

In this mini-course we will introduce the concept of instanton sheaves [J02], presenting some of the properties mentioned above. Then we will focus on some special family of instanton bundles (symplectic and orthogonals [AMS18, FFM09, JMW14]) describing ways to find examples and also describing their moduli space. Finally, we will see possible extensions of the concept of instanton bundles for non-projective varieties [F14, MMP17] and describe a relation between instanton and quivers theory.

[AMS18] A. Andrade, S. Marchesi, R.M. Miró-Roig Orthogonal instanton bundles on $\mathbb{P}^n$, arXiv preprint (2018)

[ADHM78] M. Atiyah, V. Drinfeld, N. Hitchin, and Yu Manin. Construction of instantons. Phys. Lett., 65A:185–187, 1978

[CO02] L. Costa and G. Ottaviani. Nondegenerate multidimensional matrices and instanton bundles. Trans. Amer. Math. Soc., 355:49–55, 2002.

[F14] D. Faenzi: Even and odd instanton bundles on Fano threefolds of Picard number one. Manuscripta Math. 144 (2014), 199–239.

[FFM09] Farnik, L., Frapporti, D., and Marchesi, S. On the non-existence of orthogonal instanton bundles on $\mathbb{P}^{2n-1}$. Le Matematiche Catania, 2 (2009), 81–90.

[J02] Jardim, M. Instanton sheaves on complex projective spaces. Collectanea Mathematica 57 (2005), 69–91.

[JMW14] Jardim, M., Marchesi, S., and Wißdorf, A. Moduli of autodual instanton bundles. Bulletin of the Brazilian Mathematical Society, New Series 47, 3 (2016), 823–843.

[JV14] M. Jardim, , and M. Verbitsky, Trihyperkähler reduction and instanton bundles on CP3. Compositio Mathematica 150, 11 (2014), 1836–1868.

[MMP17] F. Malaspina, S.Marchesi, J. Pons, Instanton bundles on the flag variety F(0,1,2), arXiv preprint (2017)

[OS86] C. Okonek and H. Spindler. Mathematical instanton bundles on P2n+1. J. Reine Angew. Math., 364:35–50, 1986.

[ST90] H. Spindler and G Trautmann. Special instanton bundles on P2n+1, their geometry and their moduli. Math. Ann., 286:559–592, 1990.

[Tik12] A. Tikhomirov, Moduli of mathematical instanton vector bundles with odd on projective space. Izvestiya: Mathematics 76, 5 (2012), 991.

[Tik13] A. Tikhomirov, Moduli of mathematical instanton vector bundles with even on projective space. Izvestiya: Mathematics 77, 6 (2013), 1195.

Talks:

ABDELMOUBINE AMAR HENNI (UFSC)

The fixed rank 2 Instanton sheaves on $\mathbb{P}^3$

We give some properties of the fixed rank 2 Instanton sheaves on $\mathbb{P}^3$ under the natural action of the 3-dimensional torus. This allows us to relate then to Pandharepande-Thomas stable pairs. Moreover, we classify all the supports and give a lower bound on the number of irreducible components of the fixed locus.

ALEJANDRO CABRERA (UFRJ)

Formal symplectic realizations and beyond

We will review a construction of a formal symplectic realization for any Poisson structure on $\mathbb{R}^{\mathrm{d}}$. We will show how weighted sums over graphs naturally appear in this problem, and how they coincide with the semi-classical part of the Kontsevich star product formula. This is based on joint work with B. Dherin. We speculate about possible extensions of such interpretation to the full star product formula.

ALESSIA MANDINI (PUC-RIO)

Hyperpolygons and parabolic Higgs bundles

Hyperpolygons spaces are a family of (finite dimensional, non-compact) hyperkaehler spaces, that can be obtained from coadjoint orbits by hyperkaehler reduction. Jointly with L. Godinho, we show that these space are diffeomorphic (in fact, symplectomorphic) to certain families of parabolic Higgs bundles. In this talk I will describe this relation and use it to analyse the fixed points locus of a natural involution on the moduli space of parabolic Higgs bundles. I will show that each connected components of the fixed point locus of this involution is identified with a moduli spaces of polygons in Minkowski 3-space.

DAVID MARTINEZ TORRES (PUC-RIO)

Coadjoint orbits and standard symplectic structures

We will survey on some results on describing the symplectic structure of some (non-compact) coadjoint orbits as a standard one, providing an alternative approach to some known constructions.

EUGENIA MARTIN (UFPR)

Irreducible components of varieties of Jordan algebras

In 1968, F. Flanigan [1] proved that every irreducible component of a variety of structure constants must carry an open subset of nonsingular points which is either the orbit of a single rigid algebra or an infinite union of orbits of algebras which differ only in their radicals.

In the context of the variety $\mathtt{Jor}_n$ of Jordan algebras, it is known that, up to dimension four, every component is dominated by a rigid algebra. In this work, we show that the second alternative of Flanigan's theorem does in fact occur by exhibiting a component of $\mathtt{JorN}_5$ which consists of the Zariski closure of an infinite union of orbits of five-dimensional nilpotent Jordan algebras, none of them being rigid.

This is a joint work with I. Kashuba.

[1] Francis Flanigan. Algebraic Geography: Varieties of Structure Constants. Pacific Journal of Mathematics 27, 1 (1968), pp. 71–79

JEAN VALLES (UPPA-LMA)

Free curves in a pencil

A plane curve is called free when its associated logarithmic sheaf is a sum of two line bundles. In this talk, I will explain how to produce free curves from a pencil of curves with same degree and smooth base locus.

MATIAS DEL HOYO (UFF)

The general linear 2-groupoid

When working with Lie groupoids, representations up to homotopy arise naturally, and they are useful, for instance, to make sense of the adjoint and coadjoint representations. The idea behind them is to use graded vector bundles and allow non-associativity. This brings up our interest in understanding the symmetries of a graded vector bundle, and in particular, of a 2-term graded vector space. In a joint work with D. Stefani we show that they can be encoded in a general linear 2-groupoid, a higher analogue of the classic general linear group. I will present our contributions and discuss several lines for further research.

OLIVER LORSCHEID (IMPA)

Representation type via quiver Grassmannians

The representation type of a quiver Q can be characterized by the geometric properties of the associated quiver Grassmannians: (a) Q is representation finite if all quiver Grassmannians are smooth and have cell decompositions into affine spaces; (b) Q is tame if all quiver Grassmannians have cell decompositions into affine spaces and if there exist singular quiver Grassmannians; (c) Q is wild if every projective variety occurs as a quiver Grassmannian.

With the exception of (extended) Dynkin type E, this result is proven in joint work with Thorsten Weist, based on previous results by Haupt, Reineke, Hille and Ringel. The result for type E was recently completed by Cerulli Irelli-Esposito-Franzen-Reineke. In this talk, we will give an introduction to quiver Grassmannians, explain this result and outline its proof.