1.5:24. Calcule a inversa, se existir, de

$$A = \begin{pmatrix} 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 3 & 0 \\ 2 & 1 & 5 & -3 \end{pmatrix}.$$

O algoritmo é:

$$\left(\begin{array}{cccc|cccc} 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 3 & 0 & 0 & 0 & 1 & 0 \\ 2 & 1 & 5 & -3 & 0 & 0 & 0 & 1 \end{array}\right)$$

escalonar a matriz de forma a obter a matriz na forma escalonanada reduzida

↳ se aparecer a id. ⇒ A é invertível

$$\left(\begin{array}{cccc|cccc} 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 3 & 0 & 0 & 0 & 1 & 0 \\ 2 & 1 & 5 & -3 & 0 & 0 & 0 & 1 \end{array}\right) \underset{L_1 \leftrightarrow L_2}{\sim} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 3 & 0 & 0 & 0 & 1 & 0 \\ 2 & 1 & 5 & -3 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\underset{L_4' = L_4 - 2L_1}{\sim} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 3 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 5 & -5 & 0 & -2 & 0 & 1 \end{array}\right) \underset{L_2 \leftrightarrow L_3}{\sim} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & -5 & 0 & -2 & 0 & 1 \end{array}\right)$$

$$\underset{L_4' = L_4 - L_2}{\sim} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 8 & -5 & 0 & -2 & 1 & 1 \end{array}\right) \underset{\substack{L_3' = \frac{1}{2}L_3 \\ L_4' = L_4 - 4L_3}}{\sim} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 & -4 & -2 & 1 & 1 \end{array}\right)$$

$$L_4' = -\frac{1}{5}L_4 \sim \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 4/5 & 2/5 & -1/5 & -1/5 \end{array}\right) \begin{array}{l} L_1' = L_1 - L_4 \\ \sim \\ L_2' = L_2 + 3L_3 \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -4/5 & 3/5 & 1/5 & 1/5 \\ 0 & 1 & 0 & 0 & 3/2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 4/5 & 2/5 & -1/5 & -1/5 \end{array}\right)$$

Daí A é invertível e

$$A^{-1} = \begin{pmatrix} -4/5 & 3/5 & 1/5 & 1/5 \\ 3/2 & 0 & -1 & 0 \\ 1/2 & 0 & -1 & 0 \\ 4/5 & 2/5 & -1/5 & -1/5 \end{pmatrix}$$

Para checar se está correto: o produto:

$$\begin{pmatrix} 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 3 & 0 \\ 2 & 1 & 5 & -3 \end{pmatrix} \cdot \begin{pmatrix} -4/5 & 3/5 & 1/5 & 1/5 \\ 3/2 & 0 & -1 & 0 \\ 1/2 & 0 & -1 & 0 \\ 4/5 & 2/5 & -1/5 & -1/5 \end{pmatrix}$$

deve ser igual a I.

$\boxed{A \text{ é invertível} \Leftrightarrow \det A \neq 0}$

1.7: 22. É invertível?

$$A = \begin{pmatrix} 2 & 0 & 0 & 0 \\ -3 & -1 & 0 & 0 \\ -4 & -6 & 0 & 0 \\ 0 & 3 & 8 & -5 \end{pmatrix}.$$

Não é invertível, pois

$\det A = 2 \cdot (-1) \cdot 0 \cdot (-5) = 0.$

2.1. Calcule o determinante:

30. $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{pmatrix}$

31. $B = \begin{pmatrix} 1 & 2 & 7 & -3 \\ 0 & 1 & -4 & 1 \\ 0 & 0 & 2 & 7 \\ 0 & 0 & 0 & 3 \end{pmatrix}$

$\det A = 1 \cdot 2 \cdot 3 \cdot 4 = 24$

$\det B = 1 \cdot 1 \cdot 2 \cdot 3 = 6.$

2.2: 17. Calcule o determinante

$$A = \begin{pmatrix} 1 & 3 & 1 & 5 & 3 \\ -2 & -7 & 0 & -4 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Método 1: expansão por cofatores

$$\det A = 1 \cdot (-1)^{1+1} \cdot \begin{vmatrix} -7 & 0 & -4 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix} + (-2) \cdot (-1)^{2+1} \begin{vmatrix} 3 & 1 & 5 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix}$$

$$= 1 \cdot (-7) \cdot (-1)^{1+1} \begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} + 2 \cdot 3 (-1)^{1+1} \begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} =$$

$$= -7 (1 + 2 + 0 - 0 - 0 - 1) + 6 (1 + 2 + 0 - 0 - 0 - 1)$$

$$= -7 \cdot 2 + 6 \cdot 2 = -2.$$

Método 2:

$$\begin{vmatrix} 1 & 3 & 1 & 5 & 3 \\ -2 & -7 & 0 & -4 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix} \underset{L_4' = L_4 - 2L_3}{\overset{L_2' = L_2 + 2L_1}{=}} \begin{vmatrix} 1 & 3 & 1 & 5 & 3 \\ 0 & -1 & 2 & 6 & 8 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix} =$$

$$\underset{L_5' = L_5 - L_4}{=} \begin{vmatrix} 1 & 3 & 1 & 5 & 3 \\ 0 & -1 & 2 & 6 & 8 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 2 \end{vmatrix} = 1 \cdot (-1) \cdot 1 \cdot 1 \cdot 2 = -2.$$

3.1:25. Sejam $u = (1, -1, 3, 5)$ e $v = (2, 1, 0, -3)$. Encontre $a, b \in \mathbb{R}$ tais que $au + bv = (1, -4, 9, 18)$.

Queremos encontrar $a, b \in \mathbb{R}$ tais que

$$\begin{aligned} (1, -4, 9, 18) &= a(1, -1, 3, 5) + b(2, 1, 0, -3) \\ &= (a, -a, 3a, 5a) + (2b, b, 0, -3b) \\ &= (a + 2b, -a + b, 3a, 5a - 3b). \end{aligned}$$

Então, obtemos

$$\begin{cases} a + 2b = 1 \\ -a + b = -4 \\ 3a = 9 \\ 5a - 3b = 18 \end{cases} \sim \quad a = 3 \Rightarrow -a + b = -4 \quad b = -1.$$

Checando:

$$\begin{aligned} a + 2b &= 3 + 2(-1) = 1 \\ -a + b &= -3 + (-1) = -4 \\ 3a &= 3 \cdot 3 = 9 \\ 5a - 3b &= 5 \cdot 3 - 3(-1) = 18 \end{aligned}$$

Daí, $a = 3$ e $b = -1$ é a solução.

Alternativamente, podemos escalonar:

$$\begin{pmatrix} 1 & 2 & 1 \\ -1 & 1 & -4 \\ 3 & 0 & 9 \\ 5 & -3 & 18 \end{pmatrix} \underset{\substack{L_3' = L_3 - 3L_1 \\ L_4' = L_4 - 5L_1}}{\overset{L_2' = L_2 + L_1}{\sim}} \begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & -3 \\ 0 & -6 & 6 \\ 0 & -13 & 13 \end{pmatrix} \underset{\substack{L_3' = \frac{1}{-6} L_3 \\ L_4' = \frac{1}{-13} L_4}}{\overset{L_2' = \frac{1}{3} L_2}{\sim}} \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{pmatrix} \underset{\substack{L_3' = L_3 - L_2 \\ L_4' = L_4 - L_2}}{\overset{L_1' = L_1 - 2L_2}{\sim}} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

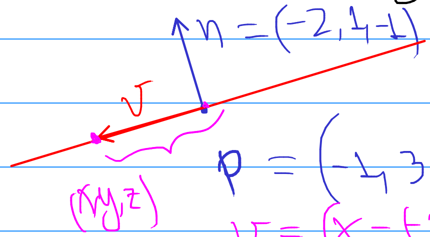
Daí a solução é $a = 3$ e $b = -1$.

3.3:9. Encontre a equação do plano que passa por $(-1, 3, -2)$ e tem $(-2, 1, -1)$ como normal.

$$0 = \langle (x - (-1), y - 3, z - (-2)), (-2, 1, -1) \rangle =$$

$$= -2(x+1) + 1 \cdot (y-3) + (-1)(z+2)$$

$$= -2x + y - z - 2 - 3 - 2 = -2x + y - z - 7.$$



$p = (-1, 3, -2)$

$v = (x - (-1), y - 3, z - (-2))$

3.3:32. Encontre a distância de $P = (1, 8)$ a $r: 3x + y = 5$.

$3x + y - 5 = 0$

$$d(P, r) = \frac{|3 \cdot 1 + 8 - 5|}{\sqrt{3^2 + 1^2}} = \frac{|6|}{\sqrt{10}} = \frac{6}{\sqrt{10}}.$$

3.3:36. Encontre a distância entre $P=(0,3,2)$ e $\Pi: x-y-z=3$.

$$\iff x-y-z-3=0$$

$$d(P,\Pi)=\frac{|0-(3)-(-2)-3|}{\sqrt{1^2+(-1)^2+(-1)^2}}=\frac{|-4|}{\sqrt{3}}=\frac{4}{\sqrt{3}}.$$

3.3:38. Determine a distância entre

$\Pi_1: 3x-4y+z=1$ $\quad$ $\Pi_2: 6x-8y+2z=3$

Vamos tomar P_1 em Π_1: Seja $P_1=(0,0,z_0)$ em Π_1. Entã

$$3\cdot 0-4\cdot 0+z_0=1 \Rightarrow z_0=1$$

Entã, $(0,0,1)$ está em Π_1.

Daí

$$d(\Pi_1,\Pi_2)=d((0,0,1),\Pi_2)=$$

$$=\frac{|6\cdot 0-8\cdot 0+2\cdot 1-3|}{\sqrt{6^2+(-8)^2+2^2}}=\frac{|-1|}{\sqrt{104}}=\frac{1}{2\sqrt{26}}$$

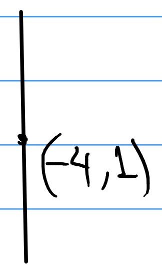
3.4: Encontre a equaçã da reta que passa por P, e é paralelo a v:

1. $P=(-4,1)$, $v=(0,-8)$

$$(x,y)=(-4,1)+t(0,-8)$$
$$=(-4,1-8t).$$

$\iff$

$x=-4$



2. $P=(2,-1)$, $v=(-4,-2)$.

$$(x,y)=(2,-1)+t(-4,-2)$$
$$=(2-4t,-1-2t)$$

$$\iff \begin{cases} x=2-4t \\ y=-1-2t \end{cases} \sim \quad t=\frac{x-2}{-4} \quad t=\frac{y+1}{-2}$$

$$\iff \frac{x-2}{-4}=\frac{y+1}{-2}$$

$\Leftrightarrow -2x+4=-4y-4$

$\Leftrightarrow -2x+4y+8=0.$

Extra. 1. Sejam A e B matrizes quadradas de mesma ordem. Calcule

$$tr\left(A^2B - 2BA^2 + ABA\right)$$

2. Seja r a reta que passa por $x_0=(1,2)$ e é paralelo a $v=(-1,1)$. Calcule a distância de $p=(-2,-1)$ à r.

1. Relembre: $\cdot\ tr(MN)=tr(NM)$

$\cdot\ tr(M+N)=tr(M)+tr(N)$

$\cdot\ tr(\lambda M)=\lambda\, tr(M)$

$$tr\left(A^2B-2BA^2+ABA\right)=tr\left(A^2B\right)+tr\left(-2BA^2\right)+tr(ABA)$$

$$=tr(A^2B)-2\,tr(BA^2)+tr(ABA)$$

$$=tr(A^2B)-2\,tr(A^2B)+tr(A^2B)=0.$$

$tr(BA^2)=tr(A^2B)$, $tr((AB)\cdot A)=tr(A\cdot(AB))=tr(A^2B)$

2. A eq. da reta é $(x,y)=(1,2)+t(-1,1)=(1-t,2+t)$.

$$\Leftrightarrow \begin{cases} x=1-t \\ y=2+t \end{cases} \Rightarrow \begin{cases} t=1-x \\ t=y-2 \end{cases} \Rightarrow 1-x=y-2 \Leftrightarrow -x-y+3=0.$$

Daí

$$d(p,r)=\frac{|-(-2)-(-1)+3|}{\sqrt{(-1)^2+(-1)^2}}=\frac{|6|}{\sqrt{2}}=\frac{6}{\sqrt{2}}.$$

2.2: 15. Calcule o determinante de

$$A = \begin{pmatrix} 2 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{pmatrix}$$

$$\det A = 2 \cdot (-1)^{1+1} \begin{vmatrix} 0 & 1 & 1 \\ 2 & 1 & 0 \\ 1 & 2 & 3 \end{vmatrix} + 1(-1)^{2+1} \begin{vmatrix} 1 & 3 & 1 \\ 2 & 1 & 0 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= 2(0+0+4-1-6-0) - (3+0+4-1-18-0)$$

$$= 2(-3) - (-12) = -6+12 = 6.$$