OPERATORS ON SUBSPACES OF HEREDITARILY INDECOMPOSABLE BANACH SPACES

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Abstract

We show that if X is a complex hereditarily indecomposable space, then every operator from a subspace Y of X to X is of the form $\lambda I + S$, where I is the inclusion map and S is strictly singular.

1. *Definitions and notation*

In the following, by *space* (respectively *subspace*), we shall always mean infinite dimensional Banach space (respectively closed subspace). A *hereditarily indecomposable* (or HI) Banach space is a space that does not contain any topological direct sum of two (infinite dimensional) subspaces. In other words, for all $\varepsilon > 0$, and all subspaces Y and Z of X, there exist two unit vectors y in Y and z in Z such that $\|y - z\| \leqslant \varepsilon$. This notion was defined by Gowers and Maurey in [**2**], in which they actually proved the existence of HI spaces.

An operator S from Y to X is said to be *strictly singular* if the restriction of S to a subspace is never an isomorphism into. This is equivalent to saying that for any $\varepsilon > 0$, and any $Z \subset Y$, there exists z in Z such that $\|S(z)\| \leqslant \varepsilon \|z\|$. By Proposition 2.c.4 of [**3**], S is strictly singular if and only if for every $Z \subset Y$ and every $\varepsilon > 0$, there exists $Z' \subset Z$ such that $\|S_{|Z'}\| \leqslant \varepsilon$. Let $\mathscr{S}(Y, X)$ denote the space of strictly singular operators from Y to X. We recall that for any strictly singular operator S, and any operators T and U for which TS and SU are defined, the operators TS and SU are strictly singular.

Let Y and Z be two subspaces of X. We say that an operator T from Y to Z is an $\mathrm{Id} + S$-*isomorphism* if it is an isomorphism of the form $\mathrm{Id} + S$, where S is strictly singular from Y to X. If T is an $\mathrm{Id} + S$-isomorphism, then so is T^{-1}. If T and U are $\mathrm{Id} + S$-isomorphisms, then so is TU when it is defined. The subspaces Y and Z of X are said to be $\mathrm{Id} + S$-*isomorphic* if there exists an $\mathrm{Id} + S$-isomorphism from Y onto Z. We denote by Id_Y the inclusion map from Y to X, and by G_X the set of subspaces of the Banach space X.

Let X be a complex HI space. It was shown in [**2**] that every operator from X to X is of the form $\lambda \,\mathrm{Id}_X + S$, where λ is complex and S is strictly singular. We generalize this result by showing that for every subspace Y of X, every operator from Y to X is of the form $\lambda \,\mathrm{Id}_Y + S$, where λ is complex and S is strictly singular (this was proved in [**2**] in the particular case of the Gowers–Maurey space). It is easy to show that this property is, in fact, equivalent to the HI property: indeed, if X is not HI, and $Y \oplus Z$ is a direct sum in X, then the canonical projection from $Y \oplus Z$ onto Y is not of the form $\lambda \,\mathrm{Id}_{Y \oplus Z} + S$.

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Even if this result is about complex HI spaces, we shall write the main part of our proof in the case of a real or complex Banach space. More precisely, we shall show that if X is HI real or complex, then for $Y \subset X$, the space $\mathscr{L}(Y,X)/\mathscr{S}(Y,X)$ with a suitable norm embeds in a Banach algebra E which is a division ring. Our main theorem follows from this property in the complex case and from the Gelfand–Mazur theorem.

2. *A filter on the set of subspaces of X*

In this section, we show that the subspaces of an HI space form a filter. First, we notice that the relation of $\mathrm{Id}+S$-isomorphism is an equivalence relation on G_X. We now give a definition.

DEFINITION 1. Let Y and Z be in G_X. We say that $Z \leqslant Y$ if Z is $\mathrm{Id}+S$-isomorphic to a subspace of Y.

LEMMA 1. *Let X be HI. Then the relation $\leqslant$ defines a filter on G_X.*

Proof. It is clear that $\leqslant$ is a pre-ordering. We prove that it is also a filter. Let Y and Z be in G_X, and let us find a space W such that $W \leqslant Y$ and $W \leqslant Z$. Passing to subspaces, we may assume that Y (respectively Z) is spanned by a basis with constant 2; by support of a vector in Y (respectively Z), we shall mean support with respect to this basis. Then by the HI property, there exist two unit vectors y_0 in Y and z_0 in Z, which we may assume finitely supported, with $\|y_0-z_0\| \leqslant 1/16$. Let Y_1 (respectively Z_1) be the space of vectors successive to y_0 (respectively z_0)—by this we mean the vectors y (respectively z) such that $\min\operatorname{supp}(y) > \max\operatorname{supp}(y_0)$ (respectively $\min\operatorname{supp}(z) > \max\operatorname{supp}(z_0)$). By the HI property, we may find finitely supported unit vectors y_1 in Y_1, and z_1 in Z_1, such that $\|y_1-z_1\| \leqslant 1/32$. Repeating this procedure, we find two basic sequences $(y_n)_{n\in\mathbb{N}}$ in Y and $(z_n)_{n\in\mathbb{N}}$ in Z such that for all n, $\|y_n-z_n\| \leqslant (1/16)\,2^{-n}$. Now let $Y' = \operatorname{span}\{y_n, n\in\mathbb{N}\}$, and let $Z' = \operatorname{span}\{z_n, n\in\mathbb{N}\}$. The operator T from Y' to Z' defined by $T(y_n) = z_n$ is of the form $\mathrm{Id}+K$, where K is compact, and it is an isomorphism since $\|T-\mathrm{Id}\| \leqslant 1/2$, so it is an $\mathrm{Id}+S$-isomorphism. Then $Y' \subset Y$, so $Y' \leqslant Y$, and as Y' is $\mathrm{Id}+S$-isomorphic to the subspace Z' of Z, we have that $Y' \leqslant Z$.

3. *A semi-norm on $\mathscr{L}(Y,X)$*

DEFINITION 2. Let X be a Banach space. For $Y\in G_X$, let $\|\cdot\|_Y$ be defined on $\mathscr{L}(Y,X)$ by

$$\|T\|_Y = \sup_{Z\subset Y}\inf_{Z'\subset Z}\|T_{|Z'}\|.$$

LEMMA 2. *Let X be a Banach space, and let $Y\in G_X$. Then the function $\|\cdot\|_Y$ is a semi-norm on $\mathscr{L}(Y,X)$. Furthermore, its kernel is the space of strictly singular operators $\mathscr{S}(Y,X)$.*

Proof. It is a direct consequence of Proposition 2.c.4 of [**3**] that $\|T\|_Y = 0$ if and only if T is strictly singular. Now, to show that $\|\cdot\|_Y$ is a semi-norm, it is enough to check the triangle inequality. Let T and U belong to $\mathscr{L}(Y,X)$, and let $\varepsilon > 0$. Let Z_0 be such that the supremum in the definition of $\|T+U\|_Y$ is attained in Z_0 up to ε. It follows that

$$\|T+U\|_Y \leqslant \|(T+U)_{|Z}\| + \varepsilon \leqslant \|T_{|Z}\| + \|U_{|Z}\| + \varepsilon,$$

for all $Z \subset Z_0$.

Now let $Z_1 \subset Z_0$ be such that $\inf_{Z \subset Z_0} \|T_{|Z}\|$ is attained up to ε in Z_1; then it is also attained up to ε in any $Z \subset Z_1$. It follows that

$$\|T+U\|_Y \leqslant \inf_{Z \subset Z_0} \|T_{|Z}\| + \|U_{|Z}\| + 2\varepsilon \leqslant \|T\|_Y + \|U_{|Z}\| + 2\varepsilon,$$

for all $Z \subset Z_1$. So

$$\|T+U\|_Y \leqslant \|T\|_Y + \inf_{Z \subset Z_1} \|U_{|Z}\| + 2\varepsilon \leqslant \|T\|_Y + \|U\|_Y + 2\varepsilon,$$

and this holds for any ε, so the triangle inequality is satisfied.

LEMMA 3. *Let X be an HI Banach space, let $Y \in G_X$, and let $T \in \mathscr{L}(Y,X)$. Then the quantity $\inf_{Z' \subset Z} \|T_{|Z'}\|$ does not depend on the choice of the subspace Z of Y.*

Proof. Let Z_1 and Z_2 be two subspaces of Y. It is enough to prove that for any $\varepsilon > 0$, and any $Z'_2 \subset Z_2$, there is a subspace $Z'_1 \subset Z_1$ such that $\|T_{|Z'_1}\| \leqslant \|T_{|Z'_2}\| + \varepsilon$.

Let $\varepsilon > 0$, and let $Z'_2 \subset Z_2$. Some subspace Z'_1 of Z_1 is of the form $(\mathrm{Id}_W + s)(W)$, where W is a subspace of Z'_2 and $\mathrm{Id}_W + s$ is an $\mathrm{Id} + S$-isomorphism on W; and passing to further subspaces, we may assume that s has norm at most ε and $(\mathrm{Id}_W + s)^{-1}$ has norm at most $1+\varepsilon$. Then $T_{|Z'_1} = T(\mathrm{Id}_W + s)(\mathrm{Id}_W + s)^{-1}$. Now notice that W and Z'_1 are $\mathrm{Id} + S$-isomorphic in the HI space Y, so s takes values in Y; so Ts exists, and it follows that

$$\|T_{|Z'_1}\| \leqslant \|T_{|W} + Ts\| \, \|(\mathrm{Id}_W + s)^{-1}\| \leqslant (\|T_{|Z'_2}\| + \varepsilon \|T\|)(1+\varepsilon),$$

so

$$\|T_{|Z'_1}\| \leqslant \|T_{|Z'_2}\| + \varepsilon(2+\varepsilon)\, \|T\|.$$

COROLLARY 1. *Let X be an HI Banach space, and let $Y \in G_X$. Then for all T in $\mathscr{L}(Y,X)$, and for all $Z \subset Y$, $\|T\|_Y = \inf_{Z' \subset Z} \|T_{|Z'}\|$.*

From now on, X is assumed to be an HI space.

LEMMA 4. *Let Y, Z be in G_X, let $T \in \mathscr{L}(Y,X)$, and let $U \in \mathscr{L}(Z,Y)$. Then*

$$\|TU\|_Z \leqslant \|T\|_Y \, \|U\|_Z.$$

Proof. Let Y, Z be in G_X, let $T \in \mathscr{L}(Y,X)$, and let $U \in \mathscr{L}(Z,Y)$. If U is strictly singular, then TU is strictly singular, and $\|TU\|_Z = 0$, so $\|TU\|_Z \leqslant \|T\|_Y \, \|U\|_Z$. Now if U is not strictly singular, let Z' be a subspace of Z on which U is an isomorphism. For any $W \subset Z'$, $\|TU\|_Z = \inf_{W' \subset W} \|TU_{|W'}\|$, by Corollary 1, so

$$\|TU\|_Z \leqslant \|T_{|UW}\| \inf_{W' \subset W} \|U_{|W'}\| \leqslant \|T_{|UW}\| \, \|U\|_Z.$$

The isomorphism U induces a bijection between the set of subspaces of Z' and the set of subspaces of UZ', and the above inequality is true for any $W \subset Z'$, so

$$\|TU\|_Z \leqslant \inf_{V \subset UZ'} \|T_{|V}\| \, \|U\|_Z,$$

and by Corollary 1,

$$\|TU\|_Z \leqslant \|T\|_Y \, \|U\|_Z.$$

Definition 3. We denote by E_Y the quotient space of $\mathscr{L}(Y, X)$ by the kernel of the semi-norm $\|\cdot\|_Y$. We shall denote the norm on E_Y by $\|\cdot\|_Y$; the space E_Y is not necessarily complete. We shall denote an element of E_Y by α_Y, and for $T \in \mathscr{L}(Y, X)$, we denote by $\tilde{T}$ the class of T in E_Y.

With this new definition, the aim of the article is now to show that if X is complex, then for all $Y \in G_X$, E_Y is isometric to $\mathbb{C}$. We recall that we already know by a result of **[2]** that if X is complex, E_X is isometric to $\mathbb{C}$.

Definition 4. Let Z, Y be in G_X such that $Z \leqslant Y$. There exists a subspace Y' of Y such that $Y' = (\mathrm{Id} + s) Z$, where s is strictly singular. We define a linear operator p_{YZ} from E_Y into E_Z by

$$p_{YZ}(\tilde{T}) = \widetilde{T(\mathrm{Id} + s)}.$$

It is clear that the result does not depend on the choice of s or on the representative T, so that p_{YZ} is well defined. Furthermore, if $W \leqslant Z \leqslant Y$, we have the relation $p_{YW} = p_{ZW} p_{YZ}$. We have also the following lemma.

Lemma 5. *Let Z, Y be in G_X such that $Z \leqslant Y$. Then p_{YZ} is a linear isometry.*

Proof. The application p_{YZ} is clearly linear. Now if $\mathrm{Id} + s$ is an $\mathrm{Id} + S$-isomorphic embedding of Z into Y, and T is a representative for α_Y, then

$$\|p_{YZ}(\alpha_Y)\|_Z = \|T(\mathrm{Id} + s)\|_Z \leqslant \|T\|_Y \, \|\mathrm{Id} + s\|_Z = \|\alpha_Y\|_Y,$$

so $\|p_{YZ}\| \leqslant 1$. Now if Z and Y are $\mathrm{Id} + S$-isomorphic, it follows from what we have just shown and from the fact that $p_{YZ} p_{ZY} = \mathrm{Id}$ that p_{YZ} is an isometry. If $Z \subset Y$, it follows from Corollary 1 that p_{YZ} is an isometry. The general case is then a consequence of these two assertions and of the definition of $\leqslant$.

We now define a notion of product for elements of E_Y similar to the composition of linear operators.

Definition 5. Let Y, Z be in G_X. We denote by E_{ZY} the space of elements of E_Z that have a representative T such that $\operatorname{Im} T \subset Y$.

Definition 6. Let Y, Z be in G_X. We define a linear mapping from $E_Y \times E_{ZY}$ into E_Z by $\tilde{T} \circ \tilde{U} = \widetilde{TU}$, where T is any representative in $\mathscr{L}(Y, X)$, and U any representative in $\mathscr{L}(Z, X)$ such that $\operatorname{Im} U \subset Y$. It is clear that this mapping is well defined. Furthermore, it follows from Lemma 4 that $\|\circ\| \leqslant 1$.

Lemma 6. *Let Y, Z be in G_X, and let $Y' \leqslant Y$. Then $E_{ZY'} \subset E_{ZY}$.*

Proof. Let $\mathrm{Id}+s$ be an $\mathrm{Id}+S$-isomorphism mapping Y' into Y. Let $\alpha_Z \in E_{ZY'}$, and let T be a representative for α_Z with $\mathrm{Im}\, T \subset Y'$. Then $(\mathrm{Id}+s)\,T$ satisfies $\widetilde{(\mathrm{Id}+s)\,T} = \alpha_Z$ and $\mathrm{Im}\,((\mathrm{Id}+s)\,T) \subset Y$, so $\alpha_Z \in E_{ZY}$.

LEMMA 7. *Let Y, Z be in G_X, and let $\alpha_Y \in E_Y$. Then there exists $Y' \leqslant Y$ such that $p_{YY'}(\alpha_Y) \in E_{Y'Z}$.*

Proof. If $\alpha_Y = 0$, then it belongs to E_{YZ}. If $\alpha_Y \neq 0$, then let T in $\mathscr{L}(Y, X)$ be a representative for α_Y. It is not strictly singular, so there is a subspace Y' of Y on which T is an isomorphism, and passing to a subspace, we may assume that there is an $\mathrm{Id}+S$-embedding $\mathrm{Id}+s$ of TY' into Z. The operator $(\mathrm{Id}+s)\,T_{|Y'}$ has its image in Z, and satisfies $\widetilde{(\mathrm{Id}+s)\,T_{|Y'}} = \widetilde{T_{|Y'}} = p_{YY'}(\alpha_Y)$, so $p_{YY'}(\alpha_Y) \in E_{Y'Z}$.

4. *A limit space E*

DEFINITION 7. An element $(\alpha_Y)_{Y \in G_X}$ of $l_\infty((E_Y)_{Y \in G_X})$ is said to be *coherent* if there exists $Y_0 \in G_X$ such that for all $Y \leqslant Y_0$, $\alpha_Y = p_{Y_0 Y}(\alpha_{Y_0})$. Let $\mathscr{E}$ be the set of coherent elements of $l_\infty((E_Y)_{Y \in G_X})$. The space $\mathscr{E}$ is clearly a linear space.

By Lemma 5, for such an element, $\|\alpha_Y\|_Y$ is constant and equal to $\|\alpha_{Y_0}\|_{Y_0}$ for $Y \leqslant Y_0$; so $\lim_Y \|\alpha_Y\|_Y$ is defined and is a semi-norm on $\mathscr{E}$. Let $\mathscr{K}$ be the space of elements of $\mathscr{E}$ such that $\lim_Y \|\alpha_Y\|_Y = 0$. We let E be the quotient space of $\mathscr{E}$ by $\mathscr{K}$. By abuse of notation, we shall denote by $\alpha = (\alpha_Y)_{Y \in G_X}$ an element of E. The space E is, in fact, the algebraic inductive limit of the system (E_Y, p_{ZY}) (see **[1]** for a general definition), but remember that we shall finally prove that, at least in the complex case, the situation is trivial, that is, $E_Y = \mathbb{C}$ for all Y and $E = \mathbb{C}$.

DEFINITION 8. Let $\alpha = (\alpha_Y)_{Y \in G_X}$ and $\beta = (\beta_Y)_{Y \in G_X}$ be elements of E. We define an element $\alpha\beta$ of E by

$$\alpha\beta = \lim_Y\, (\alpha_Y \circ \beta_Z)_{Z \in G_X}.$$

We show that this element is well defined. Let Y_0 (respectively Z_0) be such that the sequence $(\alpha_Y)_{Y \leqslant Y_0}$ (respectively $(\beta_Z)_{Z \leqslant Z_0}$) is coherent. In the following proof, we shall always consider elements lower than Y_0 and Z_0, without necessarily saying so.

By Lemma 7, there exists Z_1 such that β_{Z_1} is in $E_{Z_1 Y_0}$, and it follows that for $Z \leqslant Z_1$, β_Z is in E_{ZY_0}. So for $Z \leqslant Z_1$, $\alpha_{Y_0} \circ \beta_Z$ is defined; as $(\alpha_{Y_0} \circ \beta_Z)_{Z \leqslant Z_1}$ is clearly coherent, this defines an element in E.

Furthermore, let $Y \leqslant Y_0$, and let Z be such that $\beta_Z \in E_{ZY}$. By Lemma 6, we have also $\beta_Z \in E_{ZY_0}$. For all $Z' \leqslant Z$, the elements $\alpha_{Y_0} \circ \beta_{Z'}$ and $\alpha_Y \circ \beta_{Z'}$ are defined; moreover, it follows easily from the definition of $\circ$ that they are equal. This means that $(\alpha_Y \circ \beta_Z)_{Z \in G_X}$ and $(\alpha_{Y_0} \circ \beta_Z)_{Z \in G_X}$ are equivalent modulo $\mathscr{K}$.

As Y is arbitrary, it follows that $(\alpha_Y \circ \beta_Z)_{Z \in G_X}$ is constant for $Y \leqslant Y_0$. So the limit in the definition is well defined, and is an element of E.

REMARK. By definition, for α, β in E, $\|\alpha\beta\| = \lim_Y \lim_Z \|\alpha_Y \circ \beta_Z\|_Z$. It then follows from the fact that $\|\circ\| \leqslant 1$ that

$$\|\alpha\beta\| \leqslant (\lim_Y \|\alpha_Y\|_Y)(\lim_Z \|\beta_Z\|_Z) \leqslant \|\alpha\|\,\|\beta\|,$$

so the product on E has norm at most 1.

DEFINITION 9. Let $Y \in G_X$, and let $T \in \mathscr{L}(Y, X)$. We denote by $e(T)$ the element $(p_{YZ}(\tilde{T}))_{Z \leqslant Y}$ of E. Every element of E is of the form $e(T)$ for some T defined on some Y.

LEMMA 8. *The space E is a division ring.*

Proof. The space E is clearly a ring, with neutral element $\tilde{1} = e(\mathrm{Id}_X)$. Now let $\alpha \neq 0$, and let $T \in \mathscr{L}(Y, X)$ be such that $e(T) = \alpha$. The operator T is not strictly singular, otherwise $\alpha = 0$, so passing to a subspace, and considering the restriction of T to this subspace, we may assume that T is an isomorphism onto some space W. For $Z \leqslant W$, let $\beta_Z = p_{WZ}(\widetilde{T^{-1}})$; $\beta = e(T^{-1})$ is the class of $(\beta_Z)_{Z \leqslant W}$ in E. Then $\alpha_Y \circ \beta_W = \widetilde{\mathrm{Id}_W}$, so $(\alpha_Y \circ \beta_Z)_{Z \leqslant W} = \tilde{1}$ and $\alpha\beta = \tilde{1}$. In the same way, $\beta\alpha = \tilde{1}$. So every non-zero element is invertible, and as $\tilde{1} \neq \tilde{0}$, E is a division ring.

DEFINITION 10. Let $\alpha \in E$. Given $Y \in G_X$, we say that an element T of $\mathscr{L}(Y, X)$ is a 2-*minimal representative for* α *on* Y if $\alpha = e(T)$ and $\|T\| \leqslant 2\|\alpha\|$.

LEMMA 9. *The space E is complete.*

Proof. It is enough to show that any normally converging series of E converges in E.

Let $(\alpha_n)_{n \in \mathbb{N}}$ be a normally converging series in E. We may find a 2-minimal representative T_0 for α_0 on some Y_0. Given T_{n-1}, a 2-minimal representative for α_{n-1} on Y_{n-1}, we may find by definition of the semi-norm $\|\cdot\|_Y$ a 2-minimal representative T_n for α_n on a subspace Y_n of Y_{n-1}. We can then build by induction a basic sequence $(y_n)_{n \in \mathbb{N}}$ such that for all n, $y_n \in Y_n$. Let Y be the space generated by $\{y_n, n \in \mathbb{N}\}$, let $Y_{\geqslant n}$ be the space generated by $\{y_k, k \geqslant n\}$, and let $Y_{<n}$ be the finite dimensional space generated by $\{y_k, k < n\}$. For any n, we define an operator T'_n on Y as follows:

$$T'_n(y) = T_n(y) \text{ if } y \in Y_{\geqslant n}, \quad T'_n(y) = 0 \text{ if } y \in Y_{<n}.$$

We have that $\|T'_n\| \leqslant C\|T_n\| \leqslant 2C\|\alpha_n\|$, where C is a constant associated with the basis $(y_n)_{n \in \mathbb{N}}$. Furthermore, $e(T'_n) = e(T_{n|Y_{\geqslant n}}) = e(T_n) = \alpha_n$.

Now the series $\sum_{k=0}^{+\infty} T'_k$ converges normally, so converges to some $H \in \mathscr{L}(Y, X)$. If we let $h = e(H)$, we have that for all n,

$$\left\| h - \sum_{k=0}^{n} \alpha_k \right\| = \left\| e\left(H - \sum_{k=0}^{n} T'_k\right) \right\| \leqslant \left\| H - \sum_{k=0}^{n} T'_k \right\|,$$

so $\sum_{k=0}^{+\infty} \alpha_k$ converges in E to h.

5. *Conclusion*

Assume now that X is complex. As E is a division ring, a Banach algebra, the product has norm 1 and $\|\tilde{1}\| = 1$, it follows by the Gelfand–Mazur theorem that E is isometric to $\mathbb{C}$.

Our theorem follows. Indeed, let Y be in G_X, and let T be in $\mathscr{L}(Y, X)$. Let $\lambda = e(T)$, where λ is considered as an element of $\mathbb{C}$. Then

$$e(T - \lambda\,\mathrm{Id}_Y) = e(T) - \lambda e(\mathrm{Id}_Y) = 0.$$

This means that $0 = \|e(T-\lambda\,\mathrm{Id}_Y)\| = \|T-\lambda\,\mathrm{Id}_Y\|_Y$, that is, $T-\lambda\,\mathrm{Id}_Y$ is strictly singular.

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