1) $R(g) = \mathbb{P}(f(X) \neq Y)$

$= \mathbb{P}(X \in [q, p])$

$= \mathbb{P}([q-p]) = |p-q|$

2) $R_n(g) = \frac{1}{n} \sum_{i=1}^{n} \mathbb{1}_{\{g(x_i) \neq y_i\}}$

$= \frac{1}{5}(0+0+0+0+0) = 0$

3) $R(g) = \mathbb{P}(g(X) \neq Y)$

$= \mathbb{P}(\mathbb{1}_{\{X \leq q\}} \neq \mathbb{1}_{\{X \leq\}} \cdot Z)$

$= \mathbb{P}(Z=0)\, \mathbb{P}(\mathbb{1}_{\{X \leq q\}} \neq \mathbb{1}_{\{X \leq p\}} \cdot Z \mid Z=0) +$

$\mathbb{P}(Z=1) \cdot \mathbb{P}(\mathbb{1}_{\{X \leq q\}} \neq \mathbb{1}_{\{X \leq p\}} \cdot Z \mid Z=1)$

$= \mathbb{P}(Z=0) \cdot \mathbb{P}(\mathbb{1}_{\{X \leq q\}} \neq 0) + \mathbb{P}(Z=1) \cdot \mathbb{P}(\mathbb{1}_{\{X \leq q\}} \neq \mathbb{1}_{\{X \leq p\}})$

$= \varepsilon \cdot \quad q \quad + (1-\varepsilon) \cdot |p-q|$

4) i) $R_n(g) - R(g) = R_n(g) - \frac{n}{n} R(g)$

$= \frac{1}{n} \sum_{i=1}^{n} \mathbb{1}_{\{g(x_i) \neq y_i\}} - \frac{1}{n} \sum_{i=1}^{n} R(g)$

$= \frac{1}{n} \sum_{i=1}^{n} \left[\mathbb{1}_{\{g(x_i) \neq y_i} - R(g) \right]$

i)
$$\mathbb{E}[R_n(g) - R(g)] = \mathbb{E}\left(\frac{1}{n}\sum_{i=1}^{n}\left[\mathbb{1}_{\{g(x_i)\neq y_i\}} - R(g)\right]\right)$$

$$= \frac{1}{n}\sum_{i=1}^{n}\mathbb{E}\left[\mathbb{1}_{\{g(x_i)\neq y_i\}} - R(g)\right]$$

$$= \frac{1}{n}\sum_{i=1}^{n}\mathbb{E}\left(\mathbb{1}_{\{g(x_i)\neq y_i\}} - R(g)\right)$$

$$= \frac{1}{n}\sum_{i=1}^{n}\underbrace{\mathbb{P}(g(x_i)\neq y_i)}_{R(g)} - R(g)$$

$$= 0$$

ii)
$$\mathrm{Var}(R_n(g) - R(g)) = \mathrm{Var}\left(\frac{1}{n}\sum_{i=1}^{n}\left[\mathbb{1}_{\{g(x_i)\neq y_i\}} - R(g)\right]\right)$$

$$= \frac{1}{n^2}\sum_{i=1}^{n}\mathrm{Var}\left[\mathbb{1}_{\{g(x_i)\neq y_i\}}\right]$$

$D_i = \mathbb{1}_{\{g(x_i)\neq y_i\}}$

$$= \frac{1}{n^2}\sum_{i=1}^{n}\mathrm{Var}(D_i) = \frac{1}{n^2}\cdot n\,(R(g) - R(g)^2)$$

$\mathbb{E}(D_i) = \mathbb{P}(g(x_i)\neq y_i) = R(g)$

$$= \frac{1}{n}R(g)(1 - R(g))$$

$\mathbb{E}(D_i^2) = \mathbb{E}(D_i)$

$\mathrm{Var}(D_i) = \mathbb{E}(D_i^2) - \mathbb{E}(D_i)^2$

$= R(g) - R(g)^2$

5) $\mathbb{P}(|R_n(g)-R(g)|>\varepsilon) \leq \dfrac{Var(R_n(g)-R(g))}{\varepsilon^2}$

$$= \frac{1}{n}\,\frac{R(g)(1-R(g))}{\varepsilon^2} \leq 1/4$$

$$\leq \frac{1/4}{n\varepsilon^2} = \delta \qquad \frac{1}{4}\cdot\frac{1}{\delta\cdot\varepsilon^2} = \bar{n}$$

$$\bar{n} = \frac{1}{4}\cdot\frac{1}{(0{,}01)\cdot(0{,}01)^2} = 25.10^4$$

6) $\mathbb{P}(|R_n(g)-R(g)|>\varepsilon) \leq \underbrace{2e^{-2n\varepsilon^2}}_{\delta}$

$$\delta = 2e^{-2n\varepsilon^2}$$

$$\frac{\delta}{2} = e^{-2n\varepsilon^2} \Rightarrow \ln\left(\frac{2}{\delta}\right) = 2\bar{n}\varepsilon^2$$

$$\frac{1}{2\varepsilon^2}\ln\left(\frac{2}{\delta}\right) = \bar{n}$$

$$\bar{n} = \frac{1}{2(0{,}01)^2}\ln\left(\frac{2}{0{,}01}\right) = 2{,}6492.10^4$$

7) i) $\eta(X) = \mathbb{P}(Y=1|X\leq x) = \mathbb{1}_{\{x\leq p\}} \left|\begin{array}{l} 0 \\ 1 \leftarrow >1/2 \end{array}\right.$

$$f^*(X) = \mathbb{1}_{\{\eta(x)\geq 1/2\}} = \mathbb{1}_{\{X\leq p\}}$$

ii) $\eta(X) = \mathbb{P}(Y=1|X\leq x) = \mathbb{1}_{\{X\leq p\}}\mathbb{P}(Z=1)$

$$f^*(X) = \mathbb{1}_{\{X\leq p\}} = \mathbb{1}_{\{X\leq p\}}(1-\varepsilon) \left|\begin{array}{l} 0 \\ 1-\varepsilon \leftarrow >1/2 \end{array}\right.$$