On the application of Gröbner basis theory to the study of certain evaluation codes

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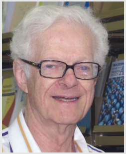
Celebrating Paulo Ribenboim's ninetieth birthday



IME-USP October 24th to 27th, 2018.

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Let $\mathcal{M}$ be the set of monomials of $k[X_1, \ldots, X_n] =: k[\mathbf{X}]$ and endow $\mathcal{M}$ with a monomial order $\prec$. Given $f \in k[\mathbf{X}] \setminus \{0\}$ the leading monomial of f ($\mathrm{lm}(f)$) is the greatest monomial appearing in f. Let $I \subset k[\mathbf{X}]$ be an ideal, we say that $\{g_1, \ldots, g_s\} \subset I$ is a Gröbner basis for I (w.r.t. $\prec$) if the leading monomial of any nonzero polynomial in I is multiple of $\mathrm{lm}(g_i)$ for some $i = 1, \ldots, s$. One can prove that such a set is a basis for I in the usual sense $I = (g_1, \ldots, g_s)$ and that any ideal has a Gröbner basis.

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be the *footprint* of I (w.r.t. $\prec$). Observe that if $I \subset J$ then $\Delta(J) \subset \Delta(I)$.

Buchberger (1965) proved that $\{M + I \mid M \in \Delta(I)\}$ is a basis for $k[\mathbf{X}]/I$ (considered as a k-vector space).

If I is homogeneous then $\{M + I \mid M \in \Delta(I), \deg(M) = d\}$ is a basis for $k[\mathbf{X}]_d/I(d)$.

We also have that $M \in \Delta(I)$ if and only if M is not a multiple of $\mathrm{lm}(g_i)$ for all $i = 1, \ldots, s$.

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Gröbner bases and coding theory

A (linear error correcting) code of length N defined over $\mathbb{F}_q$ is an $\mathbb{F}_q$-vector subspace $C \subset \mathbb{F}_q^N$. Given an N-tuple $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_N) \in \mathbb{F}_q^N$ the weight of $\boldsymbol{\alpha}$ is $w(\boldsymbol{\alpha}) = \#\{i \,|\, \alpha_i \neq 0\}$ and the minimum distance of C is $d_{\min}(C) := \min\{w(\boldsymbol{\alpha}) \,|\, \boldsymbol{\alpha} \in C \setminus \{\mathbf{0}\}\}$.

One may use a set $X = \{P_1, \ldots, P_N\} \subset \mathbb{A}^m(\mathbb{F}_q)$ to construct a code in the following way. Let $I_X \subset \mathbb{F}_q[X_0, \ldots, X_m] = \mathbb{F}_q[\mathbf{X}]$ be the ideal of X and let $\varphi : \mathbb{F}_q[\mathbf{X}]/I_X \to \mathbb{F}_q^N$ be given by $\varphi(f + I_X) = (f(P_1), \ldots, f(P_N))$. It is not difficult to show that φ is an isomorphism of $\mathbb{F}_q$-vector spaces. Thus, for any subspace $L \subset \mathbb{F}_q[\mathbf{X}]/I_X$ we have a code $C_L := \varphi(L)$.
Let d be a nonnegative integer and let
$L_d := \{f + I_X \,|\, f = 0 \text{ or } \deg(f) \leq d\}$. In this case we say that C_{L_d} is "of Reed-Muller type" and has order d. From Buchberger's result we know that the classes of the monomials in $\Delta(I_X))$ form a basis for $\mathbb{F}_q[\mathbf{X}]/I_X$ (in particular $\#(\Delta(I_X)) = N$) and one may prove that the set $\Delta(I_X)_d := \{M + I_X \,|\, M \in \Delta(I_X), \deg(M) \leq d\}$ is a basis for L_d, so that $\dim(C_{L_d}) = \#(\Delta(I_X)_d)$.

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Gröbner basis methods and the parameters of C_L

As for the minimum distance $d_{\min}(C_L)$, we would like to estimate the number of zero entries in $\varphi(f + I_X) = (f(P_1), \ldots, f(P_N))$. Let $I_{X,f} := I_X + (f)$, we want to estimate $N - \#(V(I_{X,f}))$. From $I_X \subset I_{X,f}$ we get $\Delta(I_{X,f}) \subset \Delta(I_X)$, in particular $\Delta(I_{X,f})$ is finite which implies $\#(V(I_{X,f})) \leq \#(\Delta(I_{X,f}))$ and we get $N - \#(V(I_{X,f})) \geq N - \#(\Delta(I_{X,f}))$.

From Buchberger's result we can assume that f is a linear combination of monomials in $\Delta(I_X)$ so that $\mathrm{lm}(f) \in \Delta(I_X)$. One may prove that $N - \#(\Delta(I_{X,f})) \geq \#(\{M' \in \Delta(I_X) \mid \mathrm{lm}(f) | M'\})$, so the idea now is to determine for each monomial $M \in \Delta(I_X)$ the cardinality of the set $\{M' \in \Delta(I_X) \mid M | M'\}$, and from this determine a lower bound for $d_{\min}(C_L)$. Moreover, it is true that if $\{g_1, \ldots, g_s\}$ is a Gröbner basis for I_X and $\{f, g_1, \ldots, g_s\}$ is a Gröbner basis for $I_{X,f}$ then this bound is the true value of the minimum distance.

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Let $A_1, \ldots, A_m$ be nonempty subsets of $\mathbb{F}_q$ such that $2 \leq |A_1| \leq \cdots \leq |A_m|$, and set $X := A_1 \times \cdots \times A_m$. In $\mathbb{F}_q[X_1, \ldots, X_m]$ let $f_i := \prod_{c \in A_i}(X_i - c)$ for all $i \in \{1, \ldots, m\}$ and let $I := (f_1, \ldots, f_m)$, then $X = V(I)$. One may show that I is the ideal of the set X, and clearly $|X| = |A_1|. \cdots .|A_n|$. Let $L_d := \{p + I \mid p = 0 \text{ or } \deg(p) \leq d\}$. Then $\varphi(L_d) =: C(L_d)$ is the *affine cartesian code* of order d. We have that $\{f_1, \ldots, f_m\}$ is a Gröbner basis for I w.r.t. the graded lexicographic order. Let $d_i := |A_i|$ for $i = 1, \ldots, m$.

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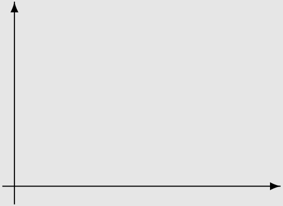

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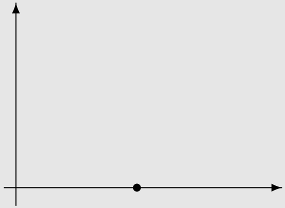

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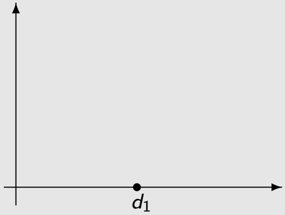

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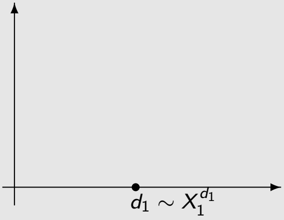

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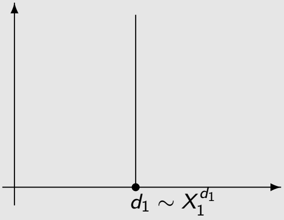

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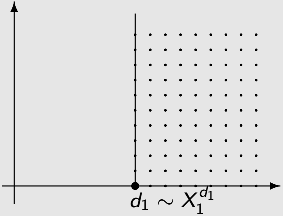

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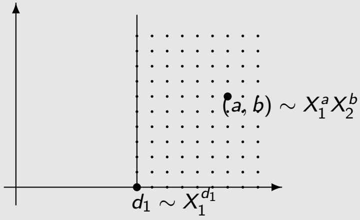

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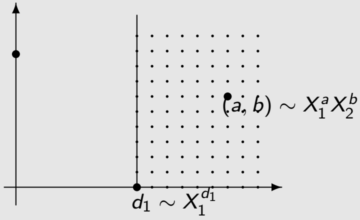

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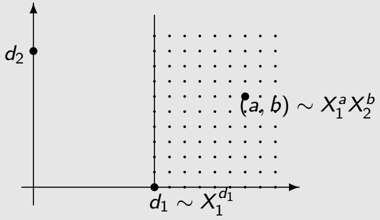

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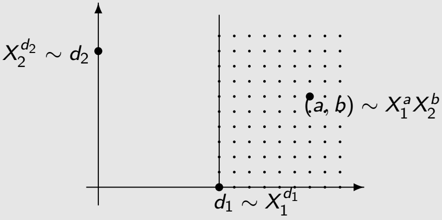

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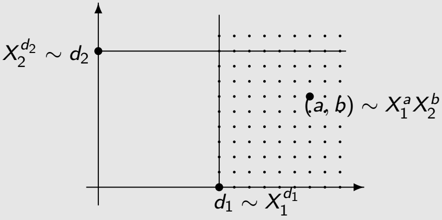

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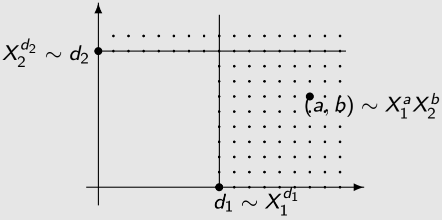

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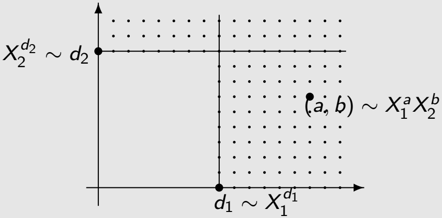

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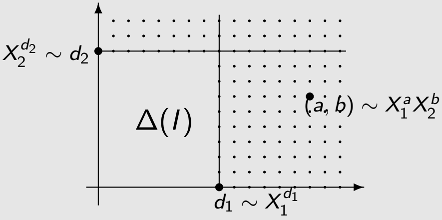

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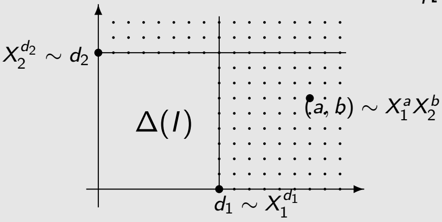

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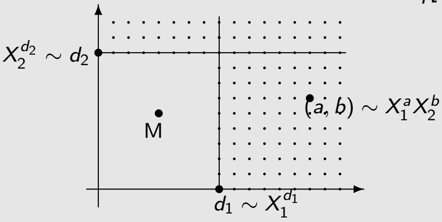

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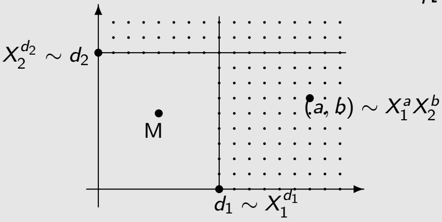

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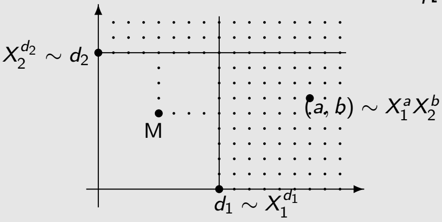

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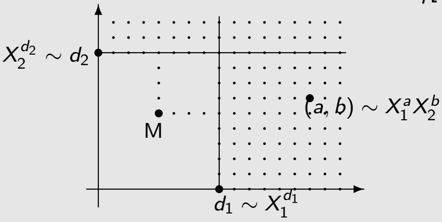

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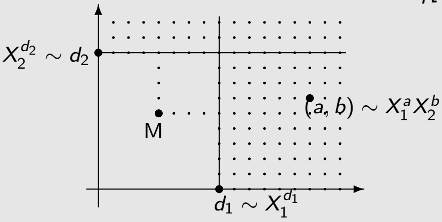

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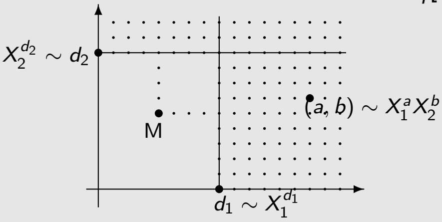

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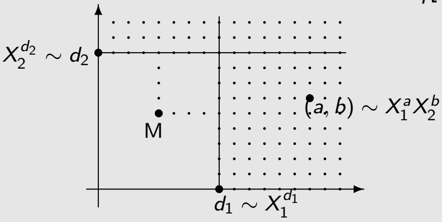

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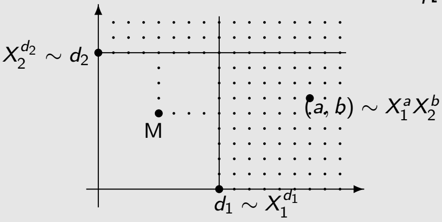

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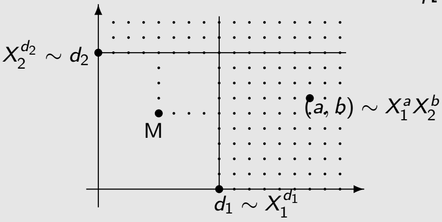

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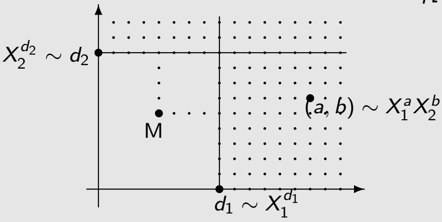

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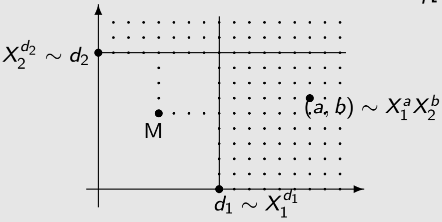

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Codes defined over projective varieties

One may also use a set $X = \{P_1, \dots, P_N\} \subset \mathbb{P}^\ell(\mathbb{F}_q)$ to construct a code in the following way.

As before let $I_X \subset \mathbb{F}_q[X_0, \dots, X_n] = \mathbb{F}_q[\mathbf{X}]$ be the ideal of X (as a set of points). In this case we have that $\mathbb{F}_q[\mathbf{X}]/I_X$ is an infinite dimensional $\mathbb{F}_q$-vector space.

Looking at $\mathbb{F}_q[\mathbf{X}]/I_X$ as a graded algebra $\mathbb{F}_q[\mathbf{X}]/I_X = \bigoplus_{d=0}^{\infty} \mathbb{F}_q[\mathbf{X}]_d/I_X(d)$ we pick a d, write the points of X in "standard form" and consider the evaluation morphism $\psi : \mathbb{F}_q[\mathbf{X}]_d/I_X(d) \to \mathbb{F}_q^N$ where $\psi(f + I_X) = (f(P_1), \dots, f(P_N))$. Now can choose an $\mathbb{F}_q$-vector subspace $L \subset \mathbb{F}_q[\mathbf{X}]_d/I_X(d)$ and define C_L to be the projective code associated to L. Observe that ψ is injective so that $\dim_{\mathbb{F}_q} C_L = \dim_{\mathbb{F}_q} L$. It is not true that ψ is an isomorphism for all $d \geq 0$ but one may prove that if $d \geq N - 1$ then ψ is an isomorphism.

On the literature usually one takes $L = \mathbb{F}_q[\mathbf{X}]_d/I_X(d)$. For example:

G. Lachaud, Projective Reed-Muller codes (1986)
A. Sørensen, Projective Reed-Muller codes (1991)
C. Rentería, H. Tapia-Recillas, Reed-Muller codes: An ideal theory approach (1997)
M. González-Sarabia et al., Reed-Muller-Type Codes Over the Segre Variety, (2002)
A. Tochimani et al., Direct products in projective Segre codes (2016)

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Let $e_0 \geq e_1 \geq e_2 \geq \ldots \geq e_n \geq 1$ be integers, and let $\ell = (e_0 + 1) + (e_1 + 1) + \ldots + (e_n + 1) - 1 = \sum_{i=0}^{n} e_i + n$. Let S be the set of zeros in $\mathbb{P}(\mathbb{F}_q)^\ell$ of the homogeneous ideal generated by the minors of

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