Research Project:
Packing Problems in Combinatorial Optimization

submitted to the Probral DAAD–CAPES Program

1 Introduction

In this project we plan to develop new approaches and algorithms to solve hard combinatorial optimization problems. Many of these problems have important applications in practice. For instance, many problems that arise in the design of VLSI circuits can be formulated as combinatorial optimization problems (see [26]).

We are specially interested in packing problems. Such problems have been broadly investigated not only because of their interesting theoretical aspects, but also due to their many practical applications, such as in cloth, glass, paper and wood industries. They also have applications in design of VLSI circuits, warehouse storage, newspapers paging, production of alcohol in the sugar cane industry, and in several other contexts.

In this project we intend to develop new algorithms for a variety of packing problems. All participants have ongoing experience on this subject, and this project is a natural attempt to combine their skills to solve new hard problems.

2 Theoretical foundations

Our goal is to design and implement new algorithms for the solution of hard combinatorial optimization problems. In the next sections we outline some of the main topics we plan to focus in this project.

2.1 Multiple Knapsack problems with additional constraints

The Multiple Knapsack problem occurs as a subproblem in many different applications such as design of VLSI circuits [12], production of sugar cane alcohol [15], matrix decomposition [1] and compiler design problems [13, 14].

In 1996, Ferreira, Martin, and Weismantel [15] developed a branch-and-cut algorithm to solve the Multiple Knapsack problem. IN this project we intend to extend this algorithm to provide solutions to problems where additional constraints occur, and Multiple Knapsack is a subproblem. This

research can give rise to new interesting questions on the structure of the polyhedra corresponding to these new problems.

2.2 Multidimensional bin and cylinder packing

The problem of packing a set of given objects in a certain region in order to maximize the number of packed objects or to maximize the occupied area is known as the Bin Packing problem. It has many important real world applications such as pallet loading, container packing and in the manufacturing of many different products (e.g. clothes and furniture).

Birgin et al. [3] studied the problem of packing a set of identical cylinders in a rectangular container, using nonlinear optimization techniques. One of the techniques is the use of the so called sentinels. A sentinel set of a given set of objects is some subset of points of the objects that can be used to detect whenever two or more of the given objects overlap. This approach was used to solve the problem of packing objects in arbitrary convex regions [2]. The packing problem is then to find a global minimum for a nonlinear problem.

Our main goals in this topic are to extend the ideas in [3] to pack non-uniform cylinders, to find sentinel sets for three-dimensional objects and apply the approach to the container packing problem and also to develop new approaches for rectangle packing.

In the Brazilian side, some results were achieved on column generation based algorithms for two-dimensional packing problems [4]. Such algorithms were also developed by the German colleagues for the solution of set covering problems with additional constraints [18]. We plan to attack other packing problems considered in the literature with this approach taking advantage of both sides previous experiences.

2.3 Approximation algorithms

The hardness of many combinatorial optimization problems stimulated the study and development of approximation algorithms. An approximation algorithm is not simply a heuristic: it finds efficiently a solution whose value is guaranteed to be within a factor of the value of an optimal solution. Several members of this project have worked with approximation algorithms [5, 6, 8, 27, 9, 10, 11, 30, 31, 32].

In the pioneering works of Johnson [24, 25], asymptotic approximation algorithms are proposed for one-dimensional packing problems. Since then,

several improvements have been achieved, specially in the development of approximation algorithms for the two-dimensional case [28].

In the context of approximation algorithms, we have worked in the two-dimensional bin packing problem, in the two-dimensional strip packing problem, in the three-dimensional packing problem and in the container packing problem [16, 29, 30, 31, 32]. Our goal in this project is to improve these results and possibly to extend them to other versions of the problem, when, for example, rotations of the objects are allowed.

2.4 Packing of Steiner trees

Steiner trees can be used to model several problems in the design of VLSI circuits and are in the core of several algorithms for a variety of problems. Some members of this project from the Brazilian side have worked on approximation algorithms for variants of the Steiner tree problem, such as the prize-collecting Steiner tree problem [27] and the group Steiner tree problem [17], and have also used ideas of classic Steiner tree algorithms for other problems [6]. Recently, they invested some time on trying to get approximation algorithms or hardness results for the problem of packing Steiner trees. Particular cases were considered and some partial results were obtained. This is an on-going work.

Prof. Martin has a large experience on the problem of packing Steiner, especially on the use of polyhedral approaches to attack this problem to solve real application instances [19, 20, 21, 22, 23]. To solve such instances, one has to consider heuristic methods and reduction techniques in a preprocessing phase. It is our plan to work on these approaches too and to share the experiences of both sides.

3 Main goals

Our main goal is to strengthen and start new collaborations among the participants of the project and make progress on the resolution of the problems listed above.

Another goal is to give to our Ph.D. students an opportunity to take part of an international cooperation, to initiate international contacts, to be exposed to a different research environment, and to benefit from the expertise and the experience of different researchers.

As concrete results, we expect publications in scientific journals, proceedings of international well-known conferences and participations in some

of the main workshops in the area. We also expect that some students will get their degree working on topics within the scope of this project.

4 Methodology

There will be visits of members of the project and some of their Ph.D. students to the partner institution. These scientific missions will provide the conditions for the interaction between the two groups to occur. The project will be developed mainly in the scientific missions of the participants to the partner institution.

We also intend to take part in good conferences of the area, which have a very strict policy for the selection of the presented works. Such participations, besides helping disseminate the work that is being done within the project, may bring some feedback on the work from the academic community.

Our group at IME-USP is responsible for a weekly seminar where ongoing research is presented and discussed. The seminar is attended by all member of this project, besides other faculty members from related areas and our students. Naturally the results obtained in this project will be presented at this seminar.

As with other projects, we would maintain a webpage with information on the project (participant institutions, members, project proposal, reports, publications, meetings, etc).

5 Plans toward the graduation of students

Our group at IME-USP has around 10 Ph.D. and M.Sc. students working on topics addressed by this project or related topics. A similar number of students is being currently supervised by Prof. Martin at the TU Darmstadt. Part of these students will graduate during the period covered by this project and all of them will benefit somehow from it.

We plan one student mission per year per institution. The mission would last from one up to three months.

6 Available infrastructure

Both institutions have good infrastructure for the development of the project: computer laboratories, libraries, and offices.

For the past three decades, the Department of Computer Science at IME-USP has made a great effort in the areas of Combinatorial Optimization, Combinatorics and Theoretical Computer Science. In terms of research, it is one of the main centers in these areas in Brazil. The past activities and projects within the Combinatorics and Combinatorial Optimization Research Group (`http://www.ime.usp.br/~yoshi/index_combinatorics.html`) of this department show the importance given to the research proposed in this project and show also the results obtained from previous research projects involving this group. Several different current research projects in the department provide counterpart to this project:

- Edital 2001 CNPq Proc. 47.0608/2001-3 "Problemas de Otimização Combinatória: Algoritmos e Aplicações";
- FAPESP grant (Proc. 2003/09169-6) "Desenvolvimento e Aplicação de Métodos Numéricos para Otimização Contínua de Grande Porte";
- Individual grants of each of the participants (CNPq Proc. 30452/89-0, 300752/94-6, 301174/97-0 and 302266/2002-0);
- Complexity of Discrete Structures, a ProNEx project (1998 to 2004), funded by the Ministry of Science and Technology (Brazil) through FINEP.

7 Further research after the project conclusion

The topics addressed in this project deal with the main research interests of the participants and therefore there will be a natural follow-up to this project. Further research might include the consideration of other variants of the problems considered so far, application of the techniques in other contexts, and the study of related problems. As the cooperation settles, other problems in combinatorial optimization might be considered.

8 Schedule of the missions and budget

From the Brazilian side, we plan the following missions and scholarships:

1. Fernando Mario de Oliveira Filho, May 2005, for four months.
2. Prof. Dr. Ernesto Birgin, July 2005, for 21 days.
3. Prof. Dr. Cristina G. Fernandes, July 2005, for one month.

4. Prof. Dr. Yoshiko Wakabayashi, February 2006, for one month.

5. Prof. Dr. Carlos E. Ferreira, July 2006, for one month.

6. Marina Andretta, September 2006, for four months.

The schedule of the missions for the German side is as follows:

1. Prof. Dr. Alexander Martin, March 2005, for 21 days.

2. Susanne Moritz, September 2005, for one month.

3. Armin and Marzena Fügenschuh, March 2006, for one month.

4. Prof. Dr. Alexander Martin, August 2006, 14 days.

5. Daniel Junglas, August 2006, for one month.

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