$f \in C^3(\mathbb{R}^3)$, $g \in C^2(\mathbb{R}^3)$

$$u(x,t) = tM_g + t\partial_t M_f + M_f$$

$$\Delta_x u = t M_{\Delta g} + t\partial_t M_{\Delta f} + M_{\Delta f} \quad (1)$$

Agora

$$\partial_t^2 u = 2\partial_t M_g + t\partial_t^2 M_g + 3\partial_t^2 M_f + t\partial_t^3 M_f$$

Lema $\partial_t M_\psi = \dfrac{1}{4\pi t^2} \displaystyle\int_{B_t(x)} \Delta\psi \qquad (t>0)$

$$\partial_t^2 M_\psi = \frac{-1}{2\pi t^3}\int_{B_t(x)} \Delta\psi + \frac{1}{4\pi t^2}\partial_t \int_{B_t(x)} \Delta\psi$$

$$\partial_t^3 M_\psi = \frac{3}{2\pi t^4}\int_{B_t(x)} \Delta\psi - \frac{1}{\pi t^3}\partial_t \int_{B_t(x)} \Delta\psi + \frac{1}{4\pi t^2}\partial_t^2 \int_{B_t(x)} \Delta\psi$$

Conclusão

$$2\partial_t M_g + t\partial_t^2 M_g = \frac{1}{4\pi t}\partial_t \int_{B_t(x)} \Delta g$$

$$3\partial_t^2 M_f + t\partial_t^3 M_f = -\frac{1}{4\pi t^2}\partial_t \int_{B_t(x)} \Delta f + \frac{1}{4\pi t}\partial_t^2 \int_{B_t(x)} \Delta f$$

Logo

$$\partial_t^2 u = \frac{1}{4\pi t}\partial_t \int_{B_t(x)} \Delta g - \frac{1}{4\pi t^2}\partial_t \int_{B_t(x)} \Delta f + \frac{1}{4\pi t}\partial_t^2 \int_{B_t(x)} \Delta f$$

Agora $\partial_t \displaystyle\int_{B_t(x)} \Delta\psi = t^2 \int_{|y|=1} \Delta\psi(x+ty)\,d\sigma(y) = t^2 4\pi M_{\Delta\psi}$

$$\Rightarrow \partial_t^2 u = t M_{\Delta g} - M_{\Delta f} + \frac{1}{4\pi t}\partial_t \left\{ t^2 \int_{|y|=1} \Delta f(x+ty)\,d\sigma(y) \right\}$$

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logo

$$\partial_t^2 u = t M_{\Delta g} - M_{\Delta f} + \frac{1}{4\pi t} \partial_t \left\{ t^2 \int_{|y|=1} \Delta f(x+ty)\, d\sigma(y) \right\}$$

$$= t M_{\Delta g} - M_{\Delta f} + 2 M_{\Delta f} + \frac{t}{4\pi} \partial_t \int_{|y|=1} \Delta f(x+ty)\, d\sigma(y)$$

$$= t M_{\Delta g} + M_{\Delta f} + t\, \partial_t M_{\Delta f} \underset{(1)}{=} \Delta_x u \quad \blacksquare$$

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