

Melhores momentos

AULA PASSADA

Alfabeto, símbolos e cadeias

Um **alfabeto** é conjunto finito não vazio.

Os elementos de um alfabeto são chamados de **símbolos**.

Exemplos:

- $\Sigma_1 = \{0, 1\}$

- $\Sigma_2 = \{a, b, \dots, z\}$

Uma **cadeia** sobre um alfabeto é uma sequência finita de símbolos do alfabeto.

Exemplos:

- 01001 é uma cadeia sobre Σ_1

- $abracadabra$ é uma cadeia sobre Σ_2 .

Comprimento de cadeias

O **comprimento** de uma cadeia w sobre um alfabeto Σ , denotado por $|w|$, é o **número de símbolos** em w , contando multiplicidades.

Exemplos:

- 01001 tem comprimento 5
- abracadabra tem comprimento 11

A **cadeia vazia** é denotada por ε e tem comprimento **zero**.

Se w tem comprimento n escrevemos

$$w = w_1 w_2 \dots w_n,$$

onde cada w_i está em Σ .

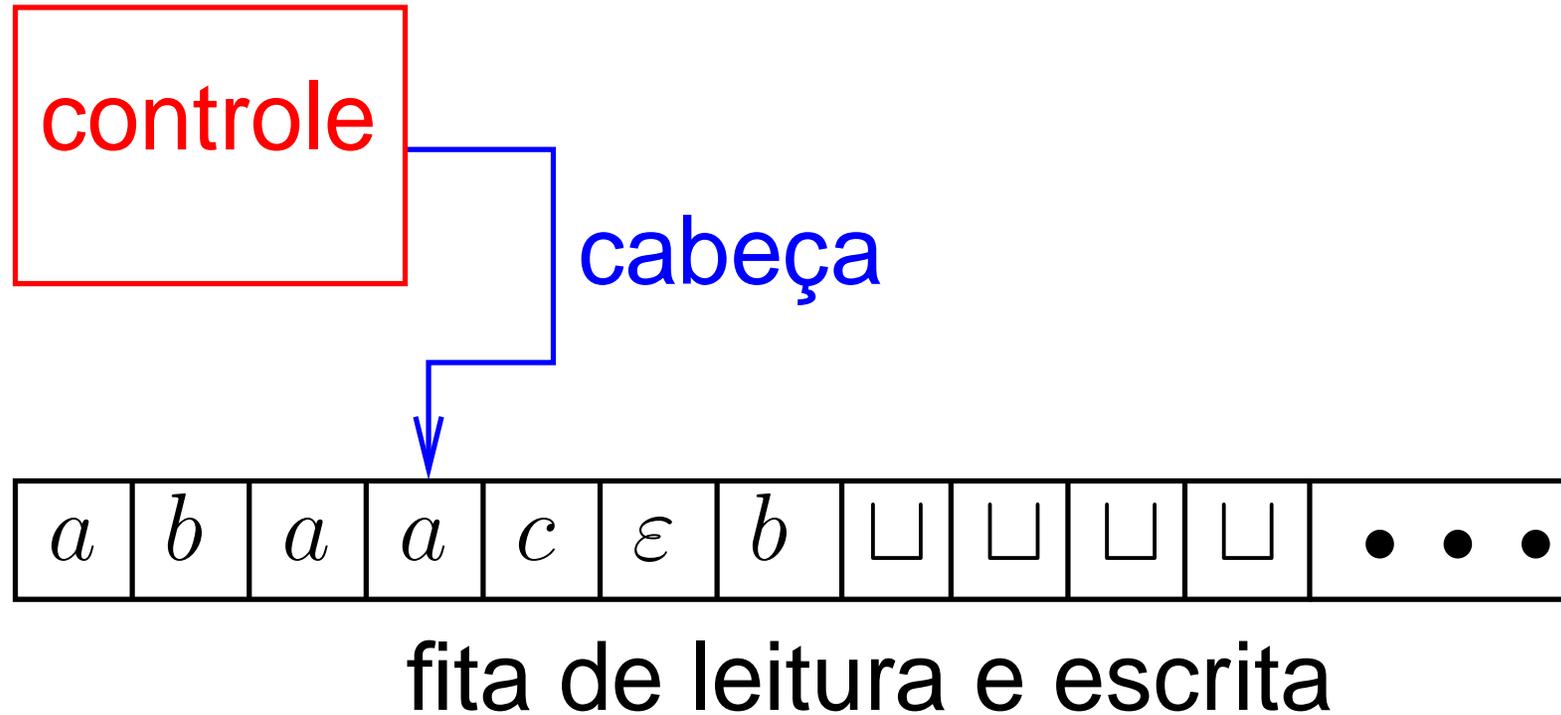
Linguagem

Uma **linguagem** é um conjunto de cadeias sobre um alfabeto.

Exemplos:

- conjunto das cadeias que codificam grafos
- conjunto das cadeias que codificam grafos bipartidos que possuem um emparelhamento perfeito
- conjunto das cadeias que codificam grafos hamiltonianos
- conjunto de cadeias da forma $w\#w^R$ para alguma cadeia w sobre Σ
- ...

Máquinas de Turing



Componentes:

1. controle
2. cabeça
3. fita

Sobre os componentes

Componentes de uma máquina de Turing:

Controle: possui um conjunto finito de **estados**, dentre eles há 3 estados especiais: *inicial*, *aceitação* e *rejeição*.

Inicialmente está em um **estado inicial**.

Cabeça: **lê** o símbolo na posição sobre a qual está, **escreve** um símbolo nessa posição e **move** uma posição para a **direita** ou **esquerda**;

Inicialmente está sobre a **primeira posição** da fita;

Fita: **infinita** à direita.

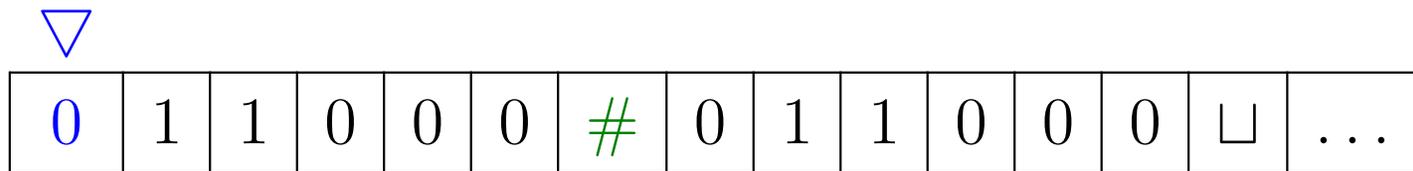
Inicialmente contém a cadeia representando a entrada do problema nas primeiras posições, as demais posições contém \square s

Exemplo de máquina de Turing

Descrição alto nível de uma máquina de Turing que decide se uma dada cadeia w está na linguagem

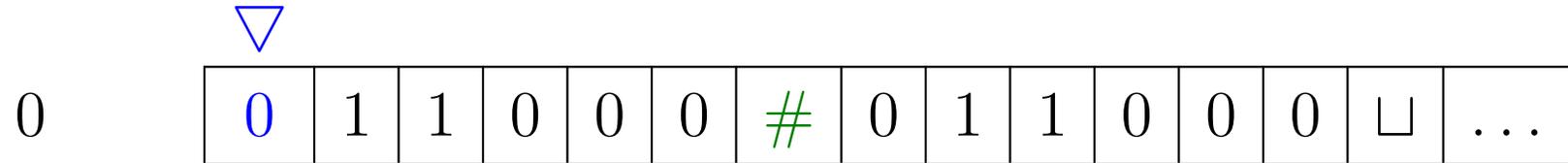
$$\{z\#z : z \in \{0, 1\}^*\}.$$

M_1 = “Com entrada w :



Simulação de M_1

passo



Simulação de M_1

passo

0

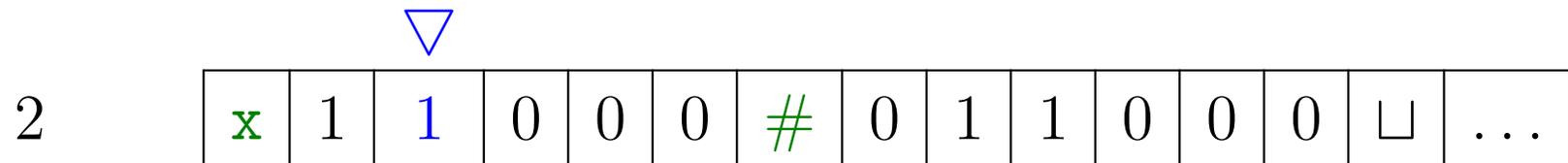
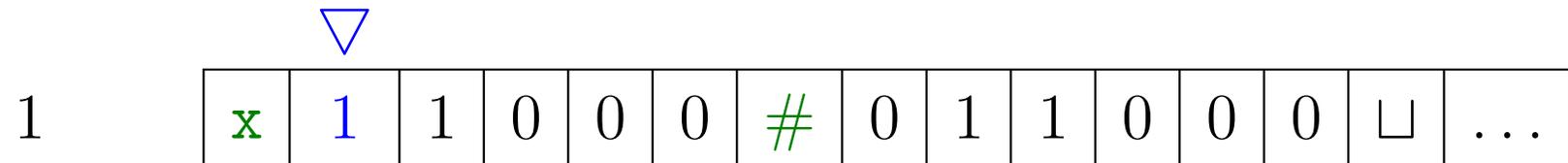
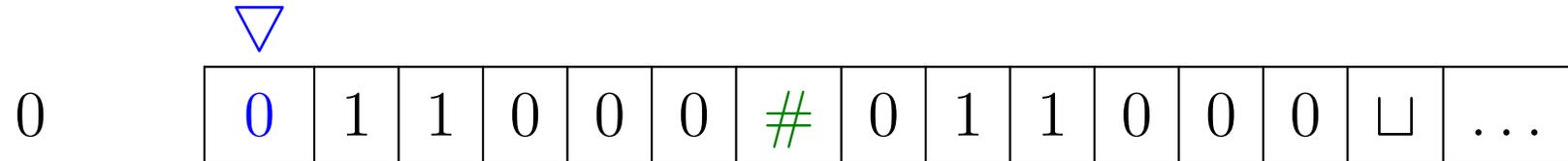
0	1	1	0	0	0	#	0	1	1	0	0	0	□	...
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1

x	1	1	0	0	0	#	0	1	1	0	0	0	□	...
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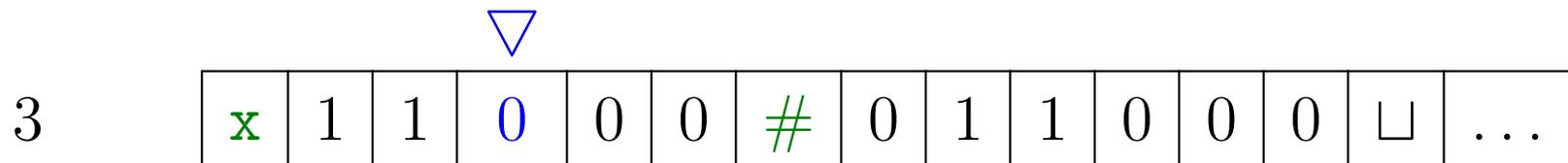
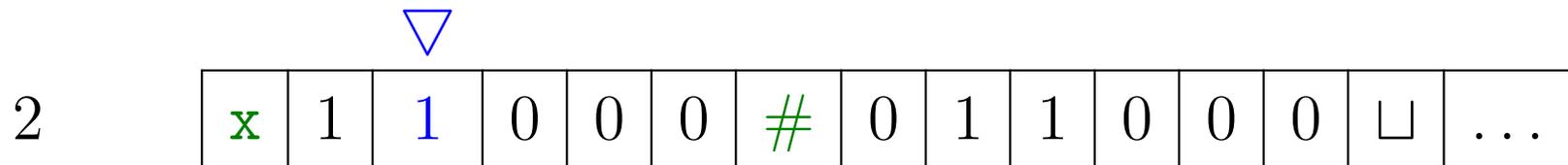
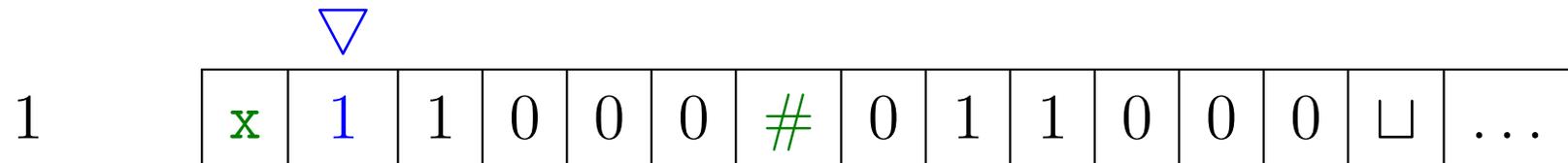
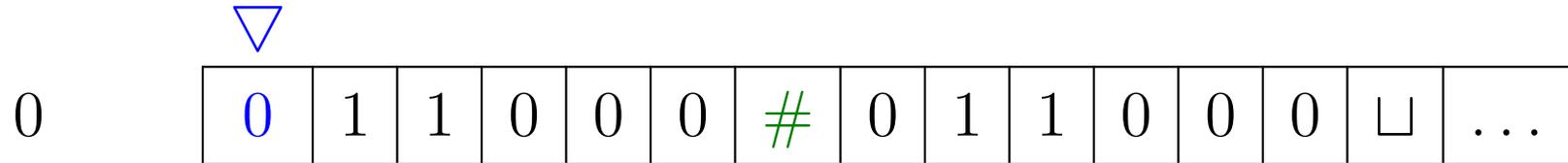
Simulação de M_1

passo



Simulação de M_1

passo



Simulação de M_1

passo

4

				▽										
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Simulação de M_1

passo

4

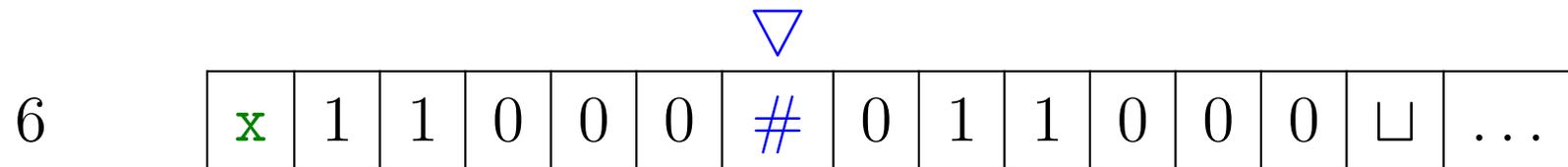
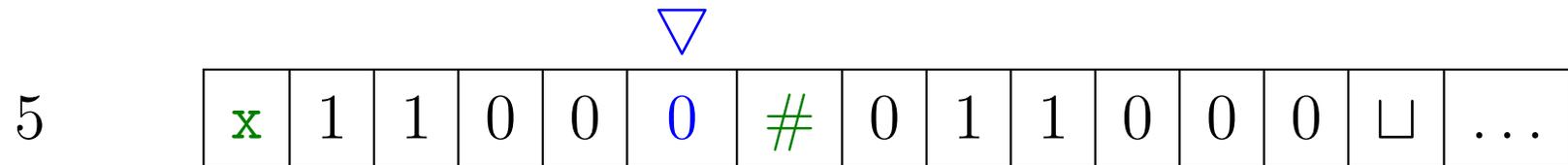
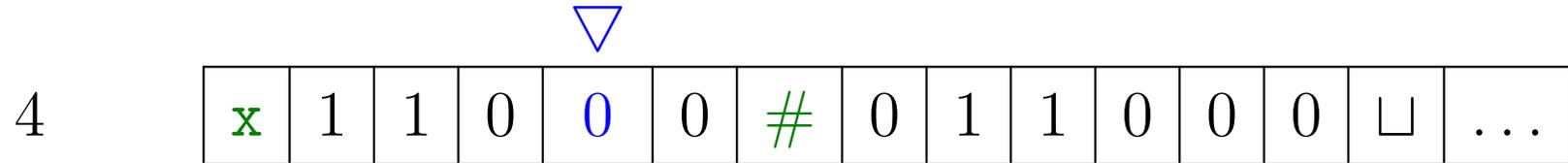
				▽										
x	1	1	0	0	0	#	0	1	1	0	0	0	□	...

5

					▽									
x	1	1	0	0	0	#	0	1	1	0	0	0	□	...

Simulação de M_1

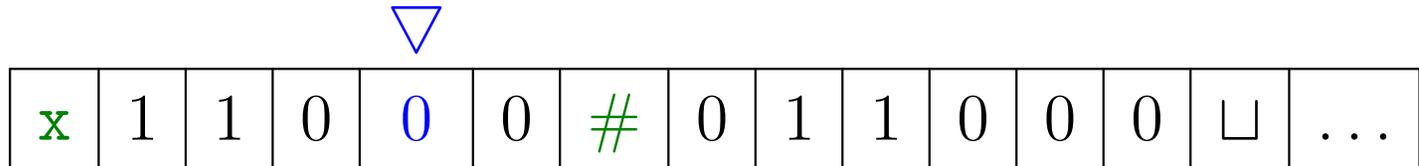
passo



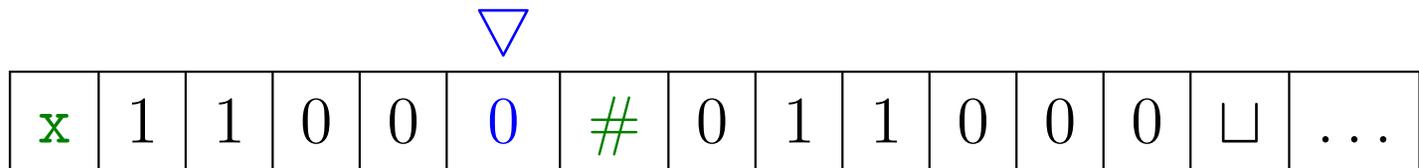
Simulação de M_1

passo

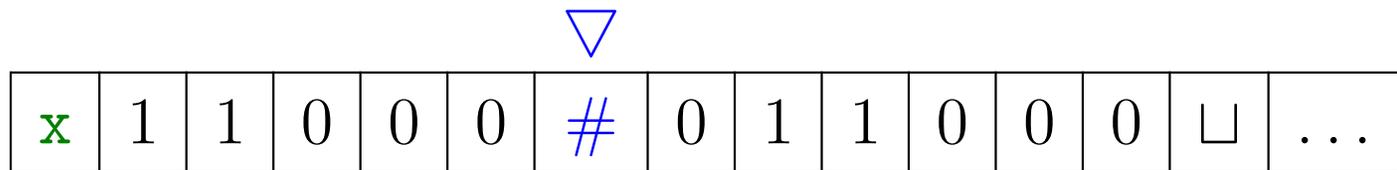
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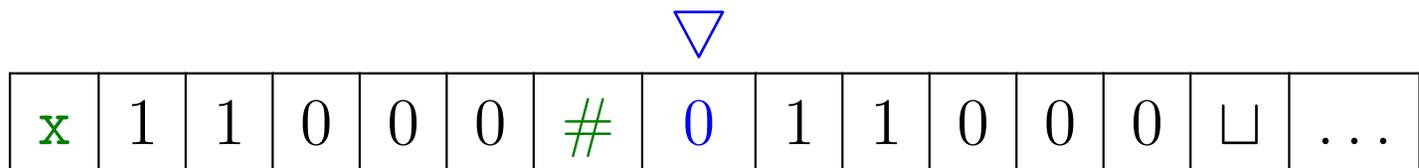
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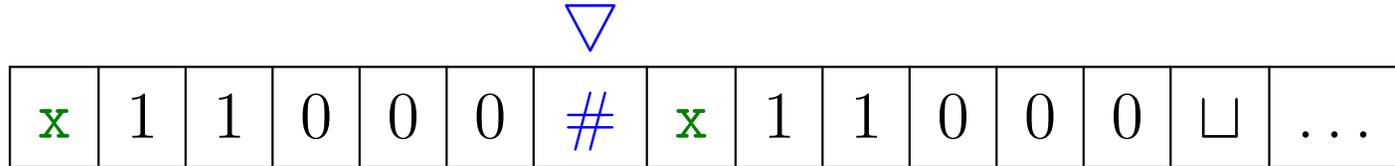
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Simulação de M_1

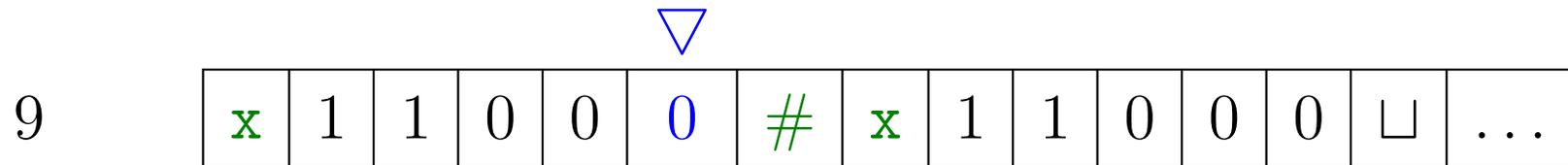
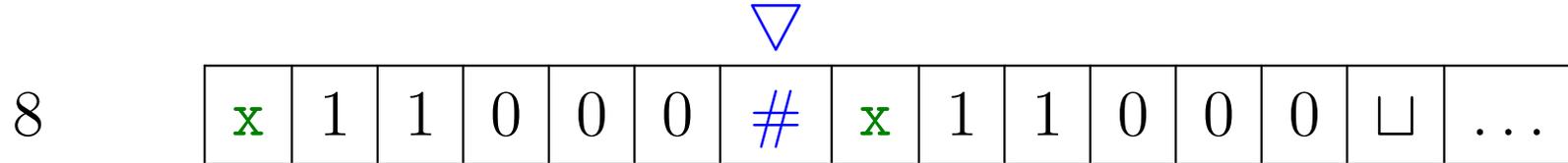
passo

8



Simulação de M_1

passo



Simulação de M_1

passo

8

x	1	1	0	0	0	#	x	1	1	0	0	0	□	...
---	---	---	---	---	---	---	---	---	---	---	---	---	---	-----



9

x	1	1	0	0	0	#	x	1	1	0	0	0	□	...
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10

x	1	1	0	0	0	#	x	1	1	0	0	0	□	...
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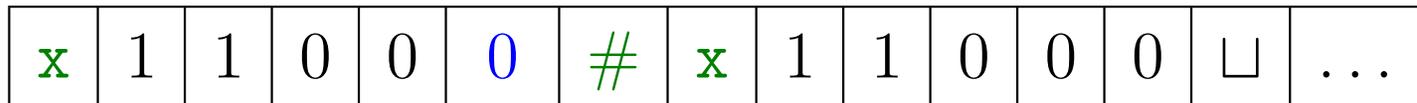
Simulação de M_1

passo

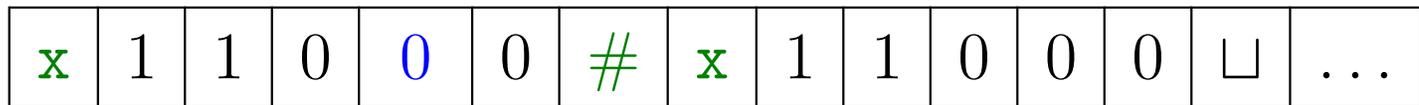
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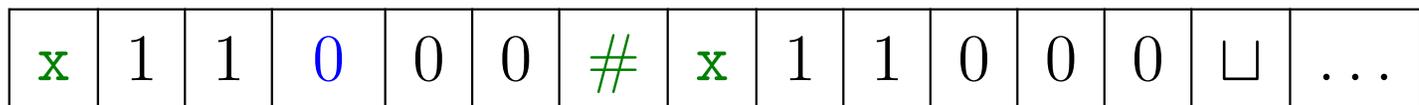
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10



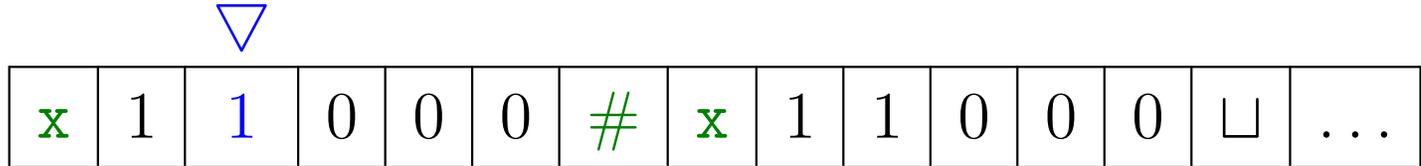
11



Simulação de M_1

passo

12



Simulação de M_1

passo

12

		▽												
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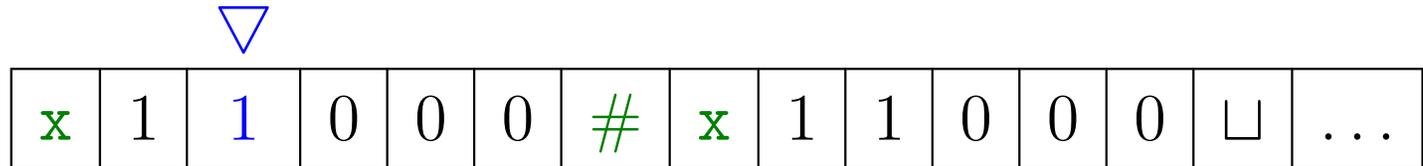
13

		▽												
x	1	1	0	0	0	#	x	1	1	0	0	0	□	...

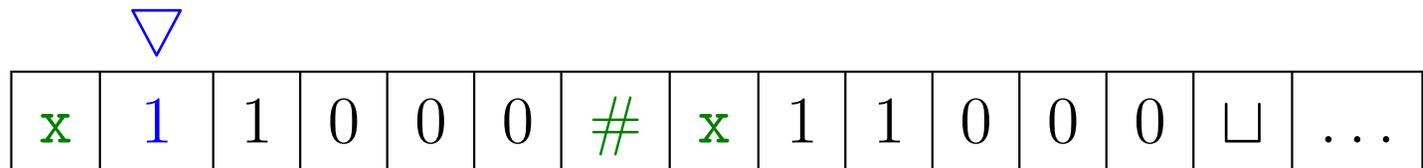
Simulação de M_1

passo

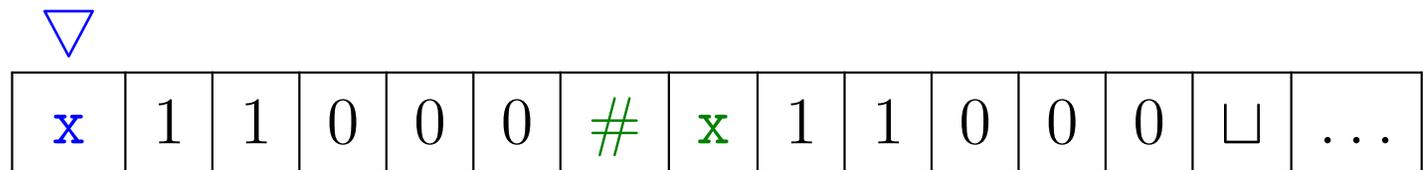
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13



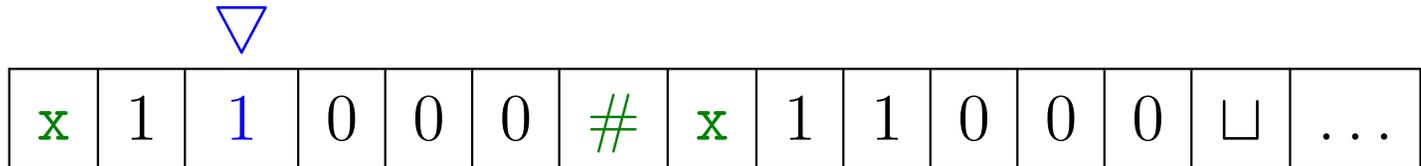
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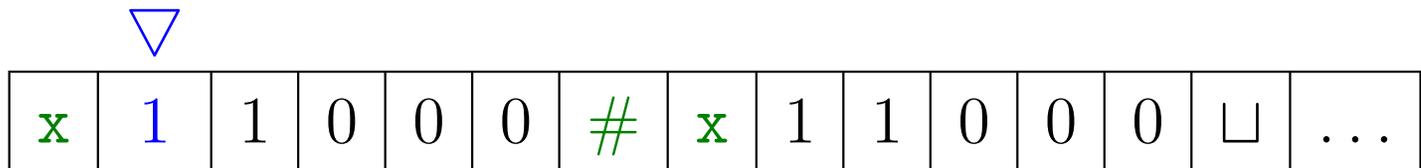
Simulação de M_1

passo

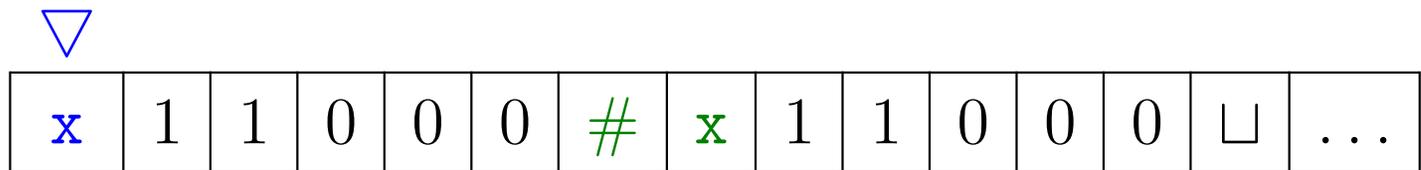
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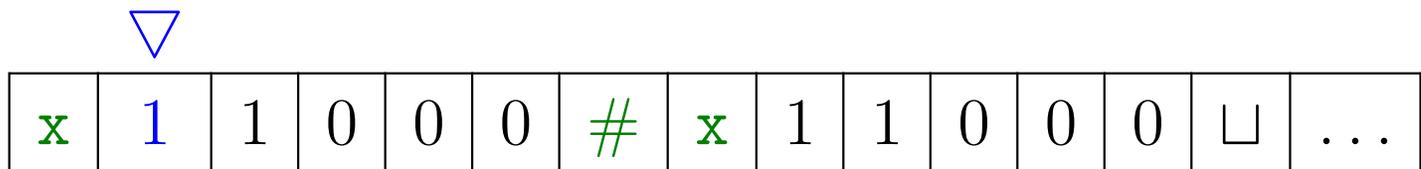
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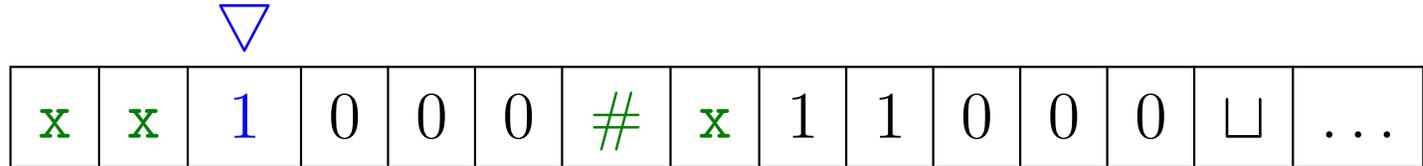
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Simulação de M_1

passo

16



Simulação de M_1

passo

16

x	x	1	0	0	0	#	x	1	1	0	0	0	□	...
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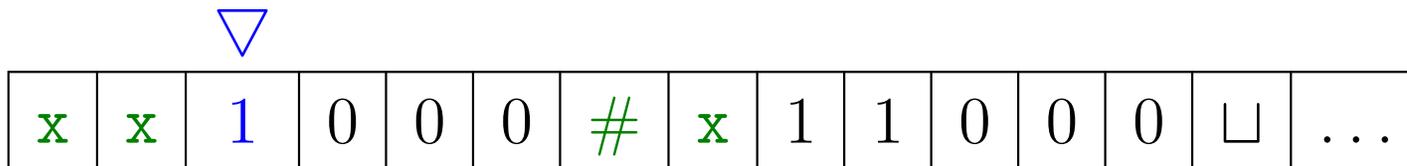
17

x	x	1	0	0	0	#	x	1	1	0	0	0	□	...
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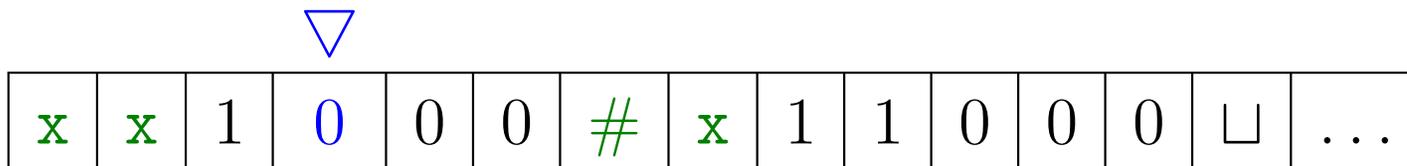
Simulação de M_1

passo

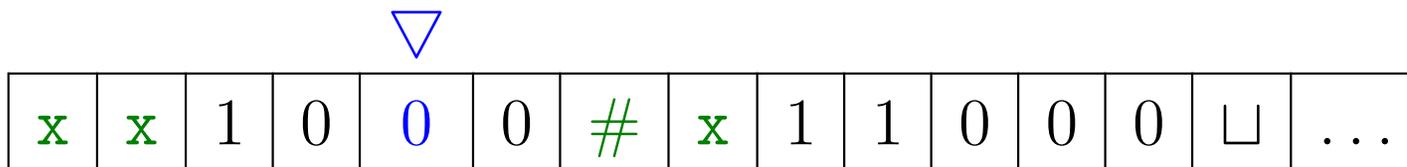
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17



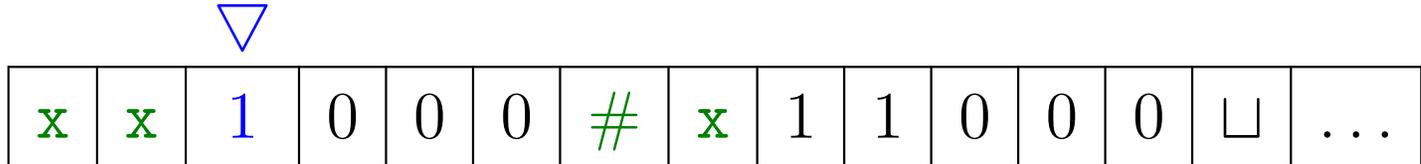
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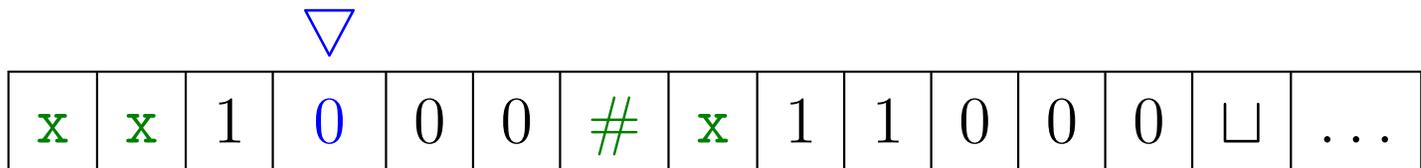
Simulação de M_1

passo

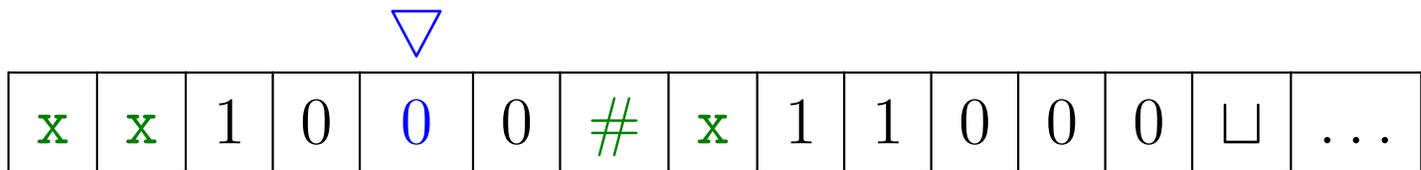
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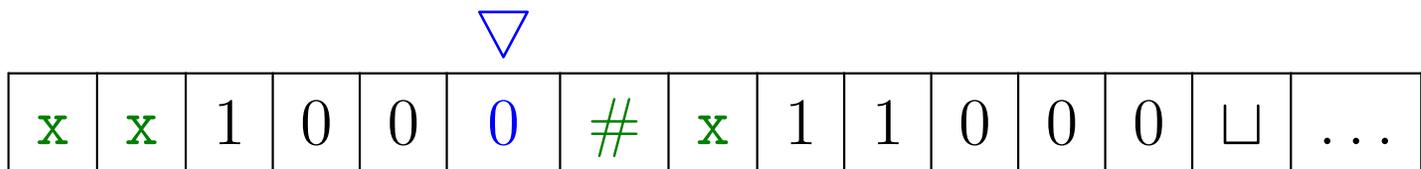
17



18



19



Simulação de M_1

passo

20

x	x	1	0	0	0	#	x	1	1	0	0	0	□	...
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Simulação de M_1

passo

20

x	x	1	0	0	0	#	x	1	1	0	0	0	□	...
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21

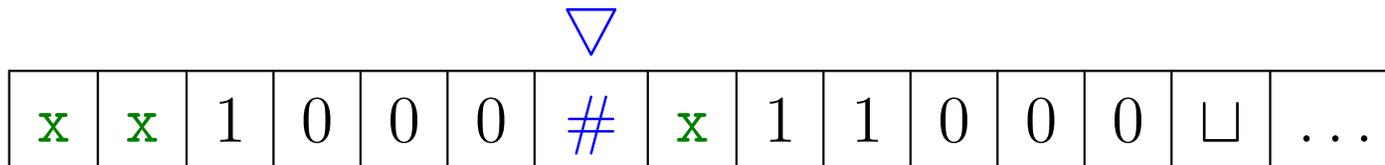
x	x	1	0	0	0	#	x	1	1	0	0	0	□	...
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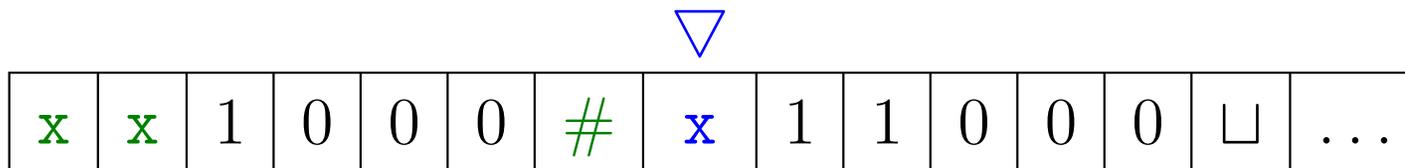
Simulação de M_1

passo

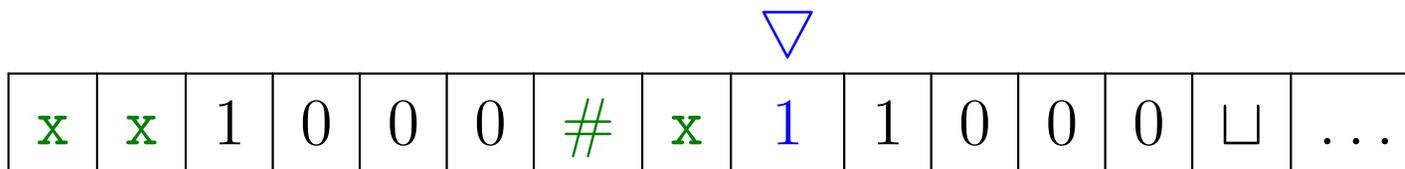
20



21



22



Simulação de M_1

passo

20

x	x	1	0	0	0	#	x	1	1	0	0	0	□	...
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21

x	x	1	0	0	0	#	x	1	1	0	0	0	□	...
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22

x	x	1	0	0	0	#	x	1	1	0	0	0	□	...
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23

x	x	1	0	0	0	#	x	x	1	0	0	0	□	...
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Simulação de M_1

passo	conteúdo da fita														
0	<u>0</u>	1	1	0	0	0	#	0	1	1	0	0	0	□	...
1	x	<u>1</u>	1	0	0	0	#	0	1	1	0	0	0	□	...
8	x	1	1	0	0	0	#	<u>x</u>	1	1	0	0	0	□	...
15	<u>x</u>	1	1	0	0	0	#	x	1	1	0	0	0	□	...
16	x	<u>x</u>	1	0	0	0	#	x	1	1	0	0	0	□	...
60	x	x	x	x	x	x	#	x	x	x	x	x	x	<u>□</u>	...

aceita

Exemplo de máquina de Turing

Descrição alto nível de uma máquina de Turing que decide se uma dada cadeia w está na linguagem

$$\{z\#z : z \in \{0, 1\}^*\}.$$

M_1 = “Com entrada w :

1. **Vá e volte** na fita em torno de $\#$ **verificando** se posições correspondentes contém o mesmo símbolo. Caso negativo, ou no caso em que $\#$ não é encontrado, **rejeite**. **Marque** os símbolos já verificados.
2. quando todos os símbolos à esquerda de $\#$ foram verificados, **examine** se sobraram símbolos do lado direito de $\#$. Se sobrou algum símbolo, **rejeite**; senão, **aceite**.”

AULA 2

Máquinas de Turing

MS 3.1

Definição

Uma **máquina de Turing** (MT) é uma 7-upla $(Q, \Sigma, \Gamma, \delta, q_0, q_{\text{aceitação}}, q_{\text{rejeição}})$ onde:

1. Q é o conjunto finito de **estados**;
2. Σ é o **alfabeto de entrada** não contendo o símbolo branco \sqcup ;
3. Γ é o **alfabeto da fita**, onde $\sqcup \in \Gamma$ e $\Sigma \subset \Gamma$;
4. $\delta : (Q \setminus \{q_{\text{aceitação}}, q_{\text{rejeição}}\}) \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$ é a **função de transição**;
5. $q_0 \in Q$ é o **estado inicial**,
6. $q_{\text{aceitação}} \in Q$ é o **estado de aceitação** e
7. $q_{\text{rejeição}} \in Q$ é o **estado de rejeição**, onde $q_{\text{aceitação}} \neq q_{\text{rejeição}}$.

Computação

Máquina de Turing $(Q, \Sigma, \Gamma, \delta, q_0, q_{\text{aceitação}}, q_{\text{rejeição}})$:

1. **cabeça** na primeira posição da fita;
2. **controle** está no estado inicial;
3. computação segue de acordo com a **função de transição**;
4. se cabeça tenta andar à esquerda da primeira posição, ela fica parada;
5. computação prossegue até atingir estado de **aceitação** ou estado de **rejeição**.

Exemplo de MT

Máquina M_0

$$Q = \{q_0, q_1, q_2, q_3, q_{\text{aceitação}}, q_{\text{rejeição}}\}$$

$$\Sigma = \{0, 1\} \quad \Gamma = \{0, 1, \sqcup\}$$

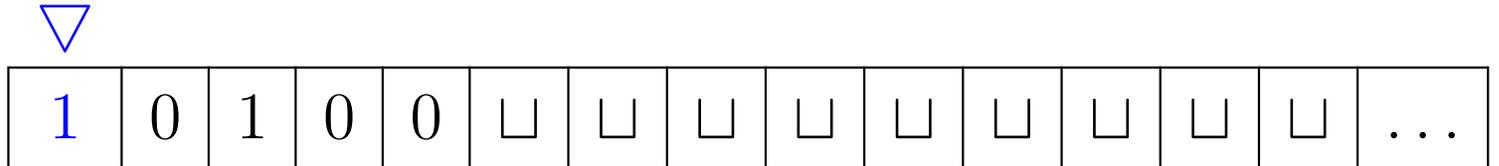
	0	1	\sqcup
q_0	$(q_0, 0, R)$	$(q_0, 1, R)$	(q_1, \sqcup, L)
q_1	(q_2, \sqcup, L)	(q_3, \sqcup, L)	(q_r, \sqcup, L)
q_2	(q_a, \sqcup, L)	(q_r, \sqcup, L)	(q_r, \sqcup, L)
q_3	(q_r, \sqcup, L)	(q_r, \sqcup, L)	(q_r, \sqcup, L)

Função de transição $\delta(q, s)$

Simulação de M_0

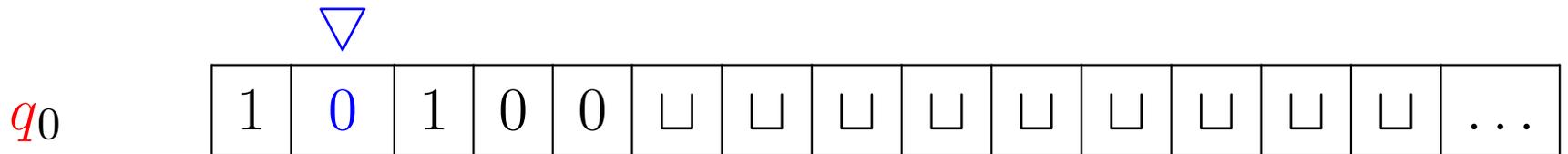
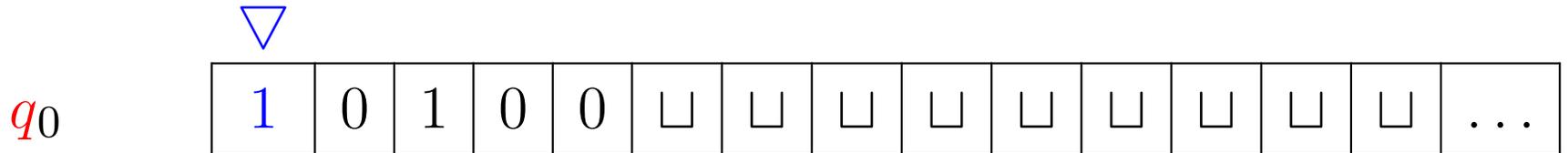
estado

q_0



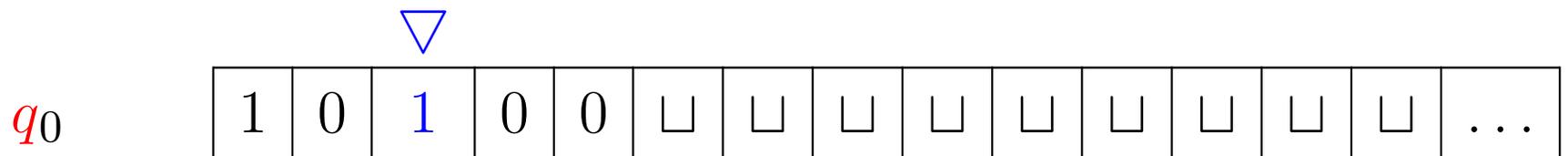
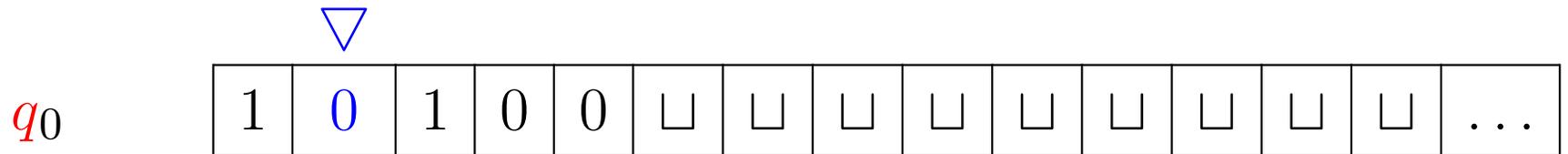
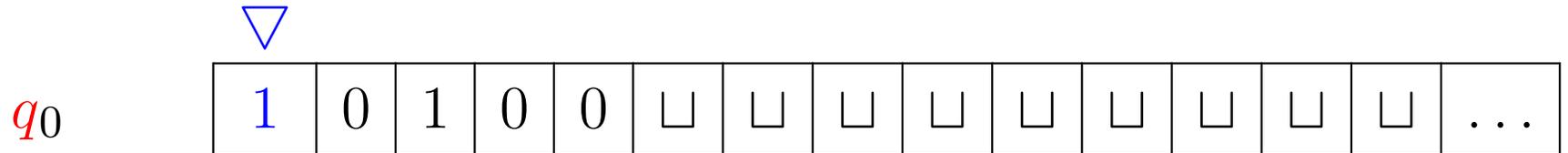
Simulação de M_0

estado



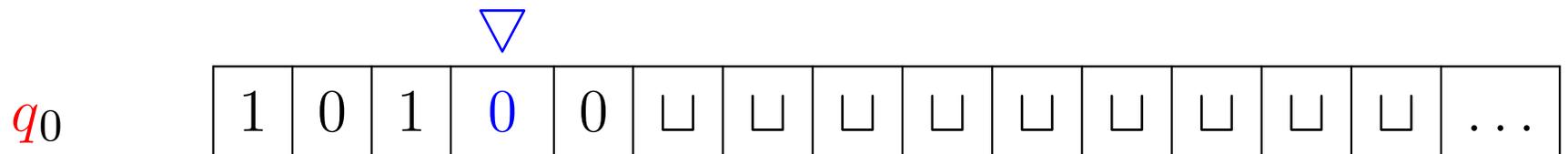
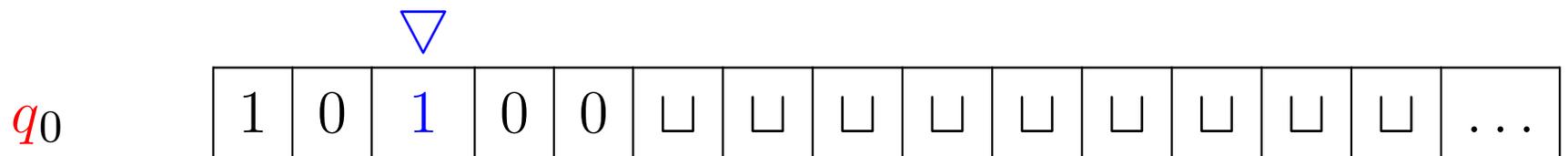
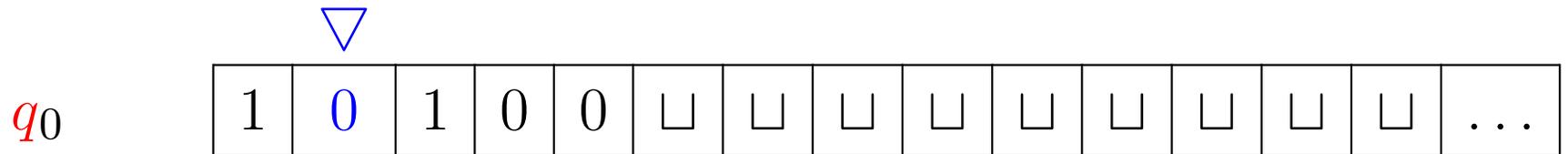
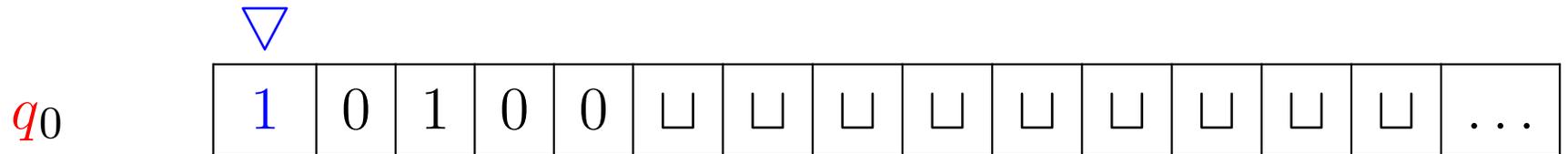
Simulação de M_0

estado



Simulação de M_0

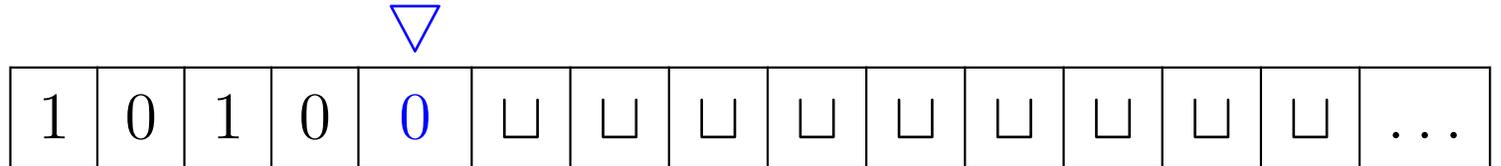
estado



Simulação de M_0

estado

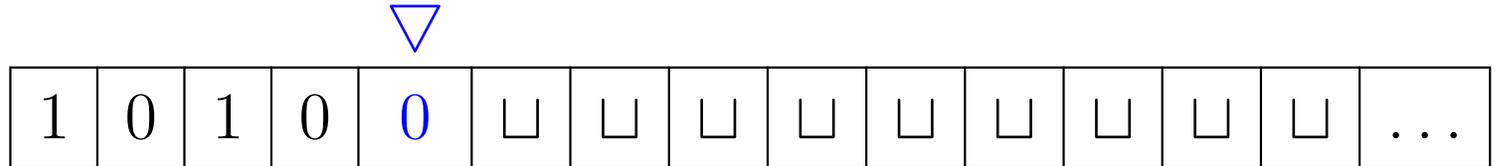
q_0



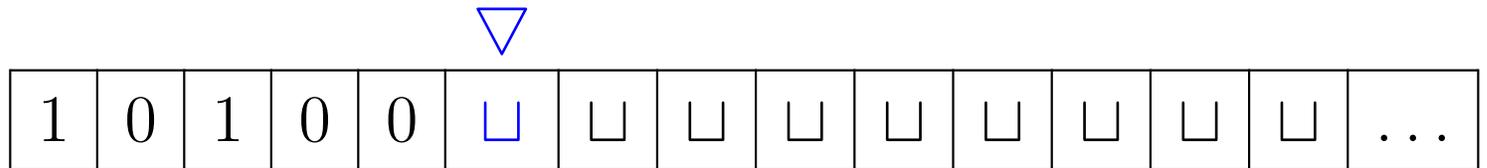
Simulação de M_0

estado

q_0



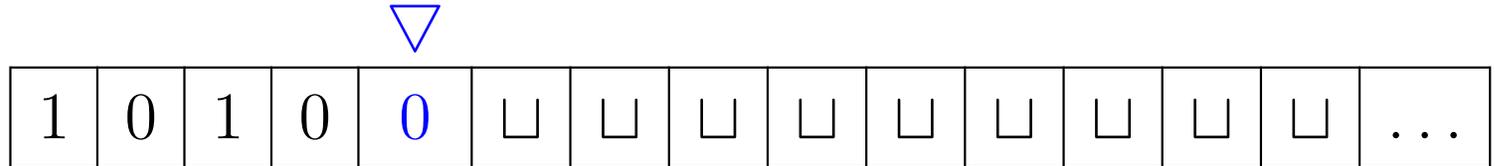
q_0



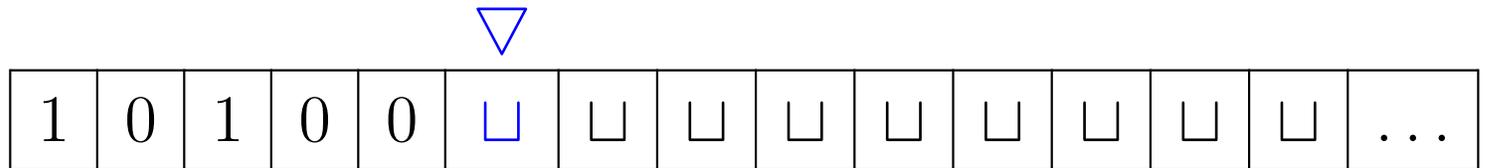
Simulação de M_0

estado

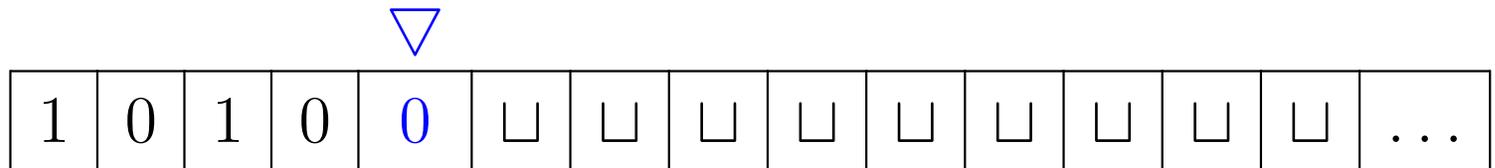
q_0



q_0



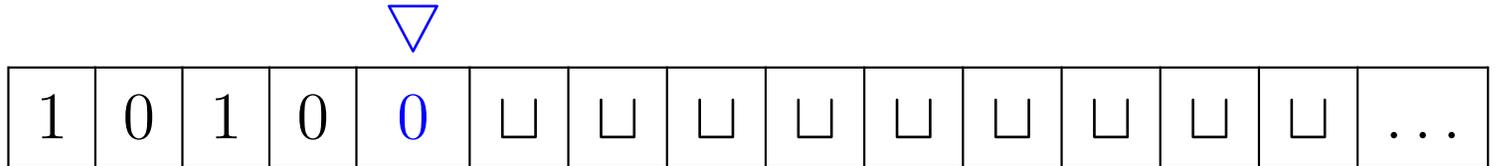
q_1



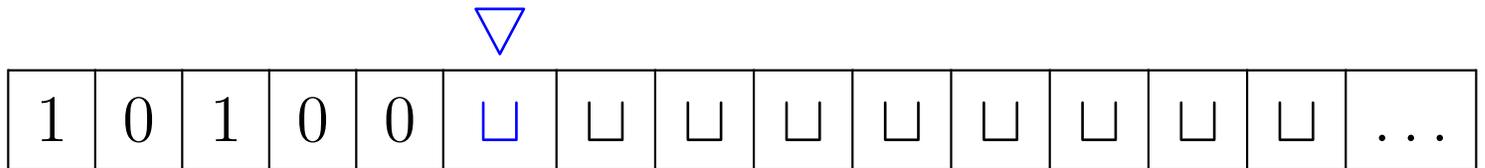
Simulação de M_0

estado

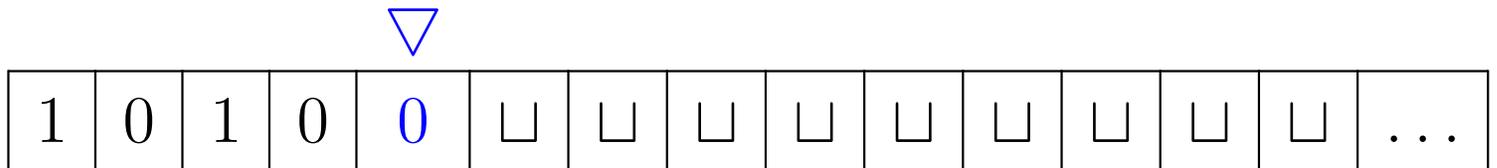
q_0



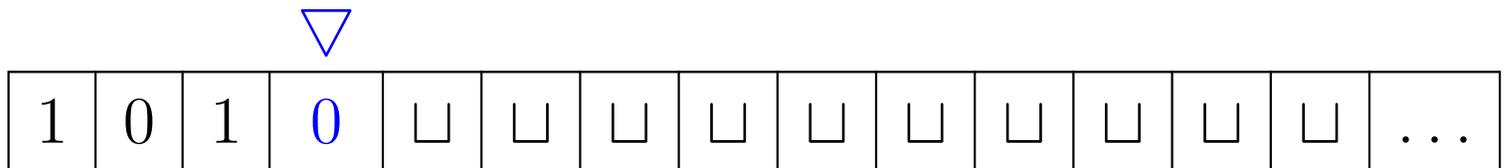
q_0



q_1



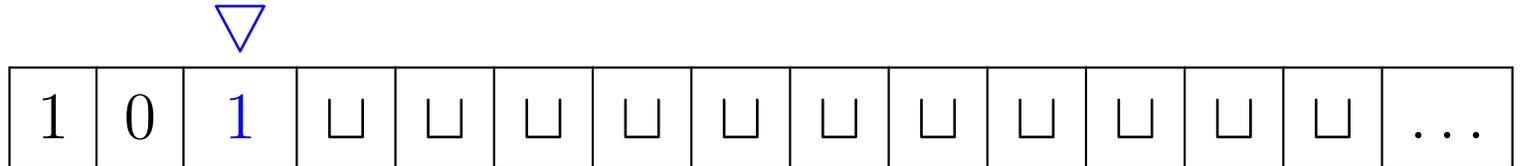
q_2



Simulação de M_0

estado

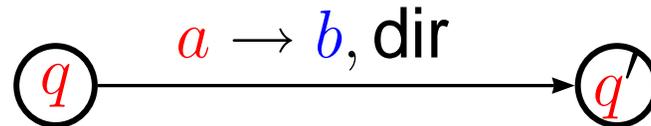
q_a



M_0 pára e **aceita** a cadeia 10100

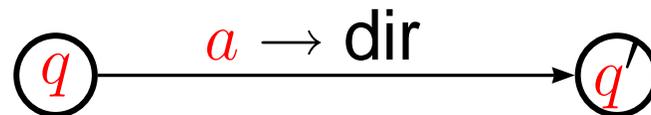
Diagrama de estados

Temos um **vértice** para cada q em Q e **arcos** com rótulos representando a **função de transição**.



Temos que $a, b \in \Sigma$ e o rótulo “ $a \rightarrow b, \text{dir}$ ” que aparece na transição de q para q' , representa que

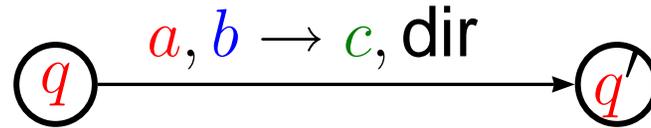
$$\delta(q, a) = (q', b, \text{dir}), \quad \text{dir} \in \{L, R\}.$$



Representa que

$$\delta(q, a) = (q', a, \text{dir}), \quad \text{dir} \in \{L, R\}.$$

Diagrama de estados

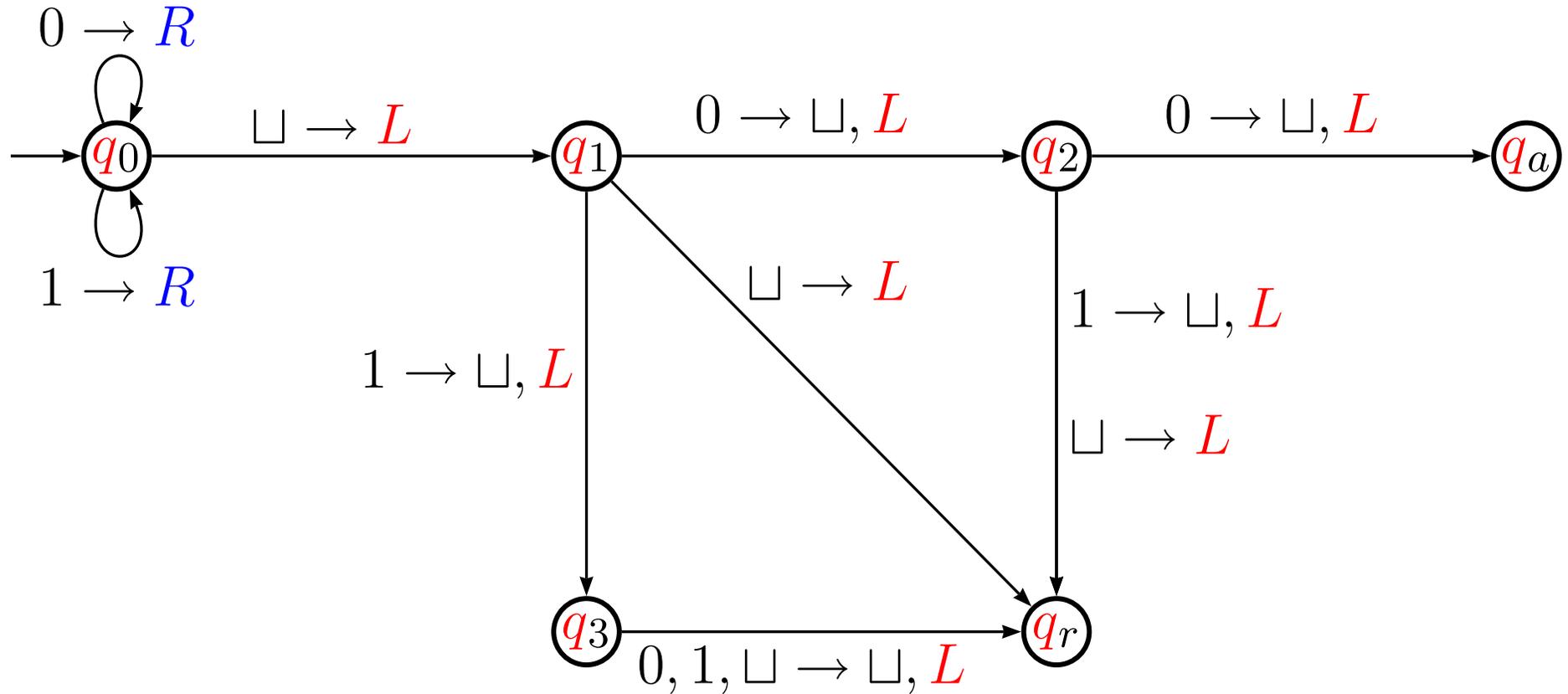


Temos que $a, b, c \in \Sigma$ e o rótulo “ $a, b \rightarrow c, \text{dir}$ ” que aparece na transição de q para q' , representa que

$$\delta(q, a) = (q', c, \text{dir}), \quad \text{dir} \in \{L, R\} \quad \text{e}$$

$$\delta(q, b) = (q', c, \text{dir}), \quad \text{dir} \in \{L, R\}.$$

Diagrama de estados para M_0



Pseudo código para M_0

q_0 :

```
se fita[ $c$ ] = 0
    então fita[ $c$ ]  $\leftarrow$  0
         $c \leftarrow c + 1$ 
    vá para  $q_0$ 
se fita[ $c$ ] = 1
    então fita[ $c$ ]  $\leftarrow$  1
         $c \leftarrow c + 1$ 
    vá para  $q_0$ 
se fita[ $c$ ] =  $\sqcup$ 
    então fita[ $c$ ]  $\leftarrow$   $\sqcup$ 
         $c \leftarrow c - 1$ 
    vá para  $q_1$ 
```

Pseudo código para M_0

Abreviadamente:

q_0 :

se $\text{fita}[c] = 0$ **ou** $\text{fita}[c] = 1$

então $c \leftarrow c + 1$

vá para q_0

se $\text{fita}[c] = \sqcup$

então $c \leftarrow c - 1$

vá para q_1

Pseudo código para M_0

q_1 :

```
se fita[ $c$ ] = 0
    então fita[ $c$ ]  $\leftarrow$   $\sqcup$ 
     $c \leftarrow c - 1$ 
    vá para  $q_2$ 
se fita[ $c$ ] = 1
    então fita[ $c$ ]  $\leftarrow$   $\sqcup$ 
     $c \leftarrow c - 1$ 
    vá para  $q_3$ 
se fita[ $c$ ] =  $\sqcup$ 
    então  $c \leftarrow c - 1$ 
    vá para  $q_{\text{rejeição}}$ 
```

Pseudo código para M_0

q_2 :

```
se fita[ $c$ ] = 0
    então fita[ $c$ ]  $\leftarrow$   $\sqcup$ 
     $c \leftarrow c - 1$ 
    vá para  $q_{\text{aceitação}}$ 
se fita[ $c$ ] = 1
    então fita[ $c$ ]  $\leftarrow$   $\sqcup$ 
     $c \leftarrow c - 1$ 
    vá para  $q_{\text{rejeição}}$ 
se fita[ $c$ ] =  $\sqcup$ 
    então  $c \leftarrow c - 1$ 
    vá para  $q_{\text{rejeição}}$ 
```

Pseudo código para M_0

q_3 :

```
se fita[ $c$ ] = 0 ou fita[ $c$ ] = 1 ou fita[ $c$ ] =  $\sqcup$   
então fita[ $c$ ]  $\leftarrow$   $\sqcup$   
 $c \leftarrow c - 1$   
vá para  $q_{\text{rejeição}}$ 
```

Pseudo código para M_0

$q_{\text{rejeição}}:$

rejeite e pare

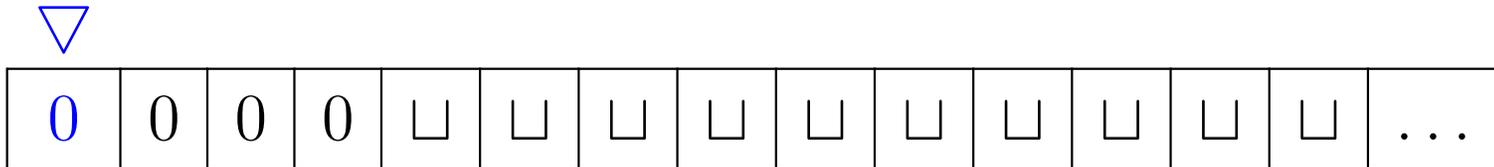
$q_{\text{aceitação}}:$

aceite e pare

Exemplo de MT

Máquina M_2 que decide $A = \{0^{2^n} : n \geq 0\}$.

$M_2 =$ “Com entrada w :



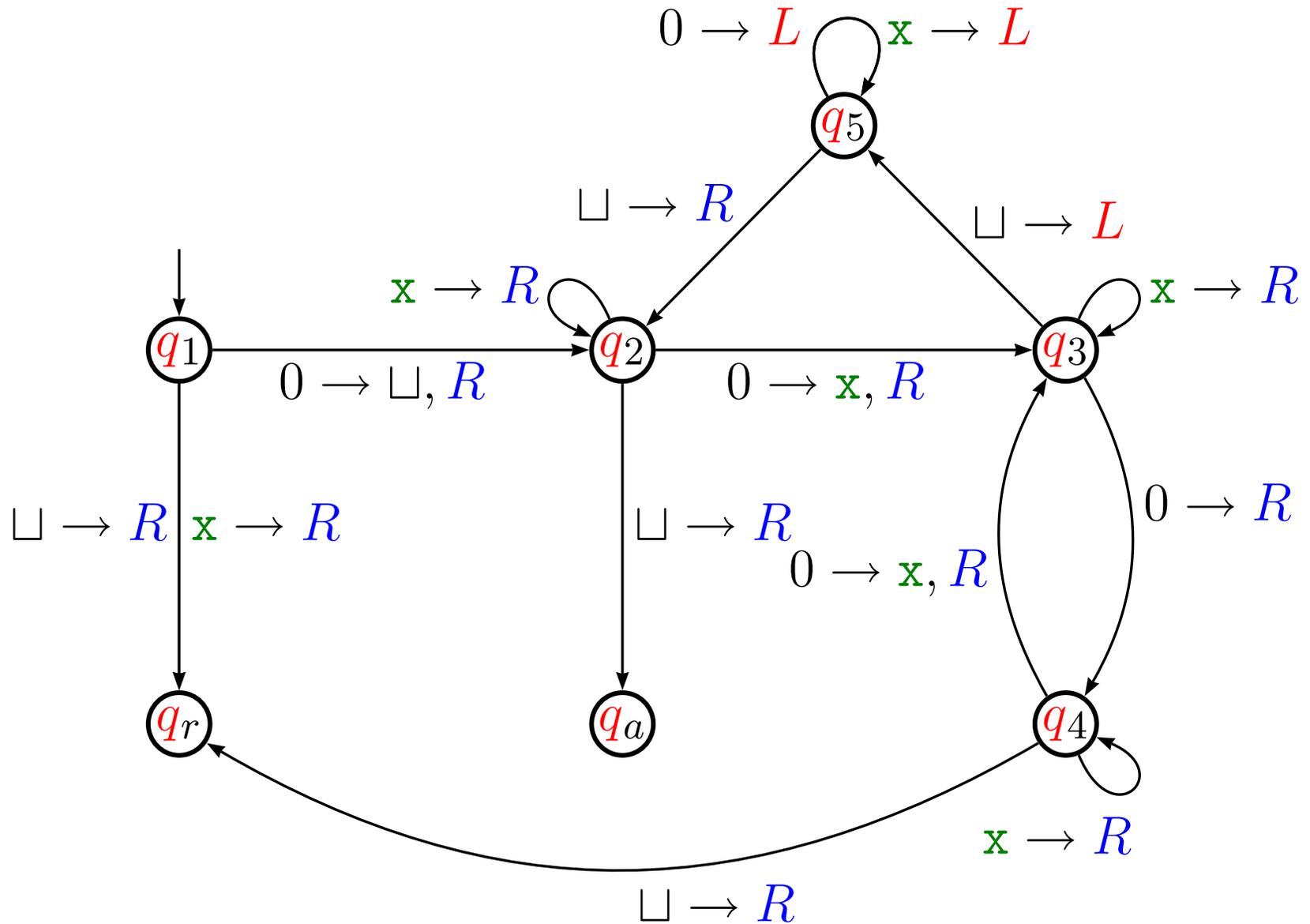
Exemplo de MT

Máquina M_2 que decide $A = \{0^{2^n} : n \geq 0\}$.

M_2 = “Com entrada w :

1. Ande na fita da esquerda para a direita riscando um 0 não e outro sim.
2. Se no passo 1 existe somente um 0, **aceite**.
3. Se no passo 1 a fita contém um número ímpar maior do que 1 de 0s, **rejeite**.
4. Mova a cabeça para a esquerda da fita e volte ao passo 1.”

Diagrama de estados para M_2



Descrição formal

$M_2 = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{aceitação}}, q_{\text{rejeição}})$:

- $Q = \{q_1, q_2, q_3, q_4, q_5, q_{\text{aceitação}}, q_{\text{rejeição}}\}$,
- $\Sigma = \{0\}$,
- $\Gamma = \{0, \mathbf{x}, \sqcup\}$ e
- δ é descrita no diagrama de estados.

Descrição formal

$M_2 = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{aceitação}}, q_{\text{rejeição}})$:

- $Q = \{q_1, q_2, q_3, q_4, q_5, q_{\text{aceitação}}, q_{\text{rejeição}}\}$,
- $\Sigma = \{0\}$,
- $\Gamma = \{0, \mathbf{x}, \sqcup\}$ e
- δ é descrita no diagrama de estados.

No diagrama de estados o rótulo “ $0 \rightarrow \sqcup, R$ ” que aparece na transição de q_1 para q_2 , representa que

$$\delta(q_1, 0) = (q_2, \sqcup, R).$$

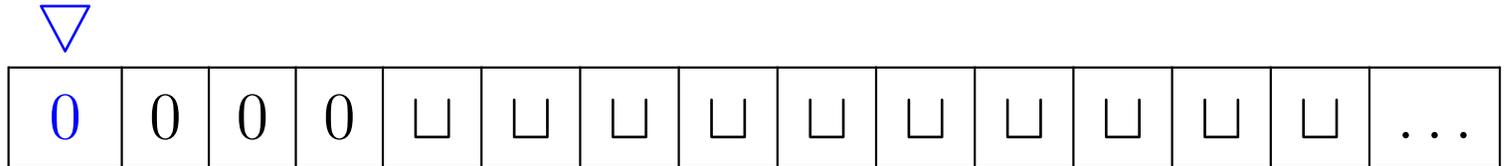
O rótulo “ $0 \rightarrow R$ ” na transição de q_3 para q_4 representa que

$$\delta(q_3, 0) = (q_4, 0, R).$$

Simulação de M_2

estado

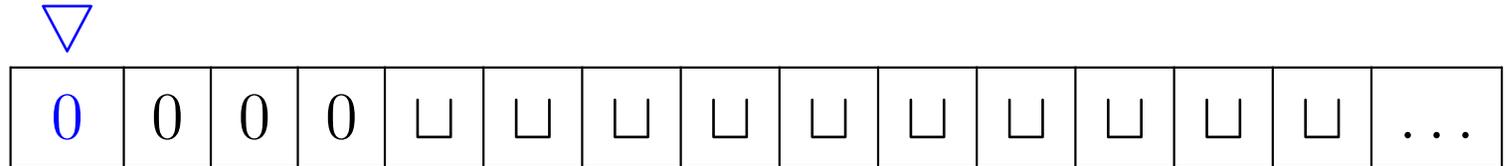
q_1



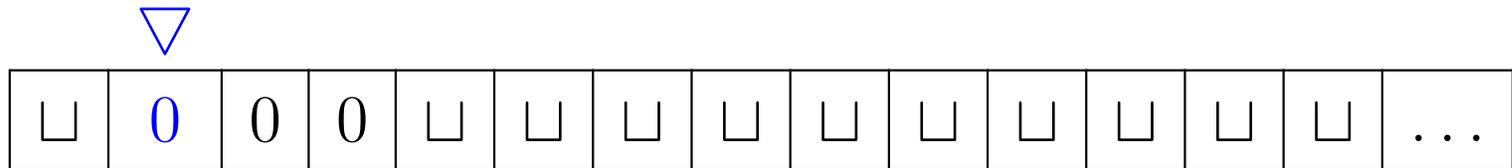
Simulação de M_2

estado

q_1

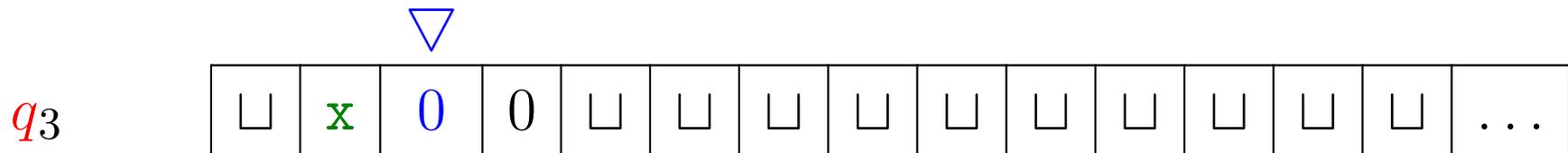
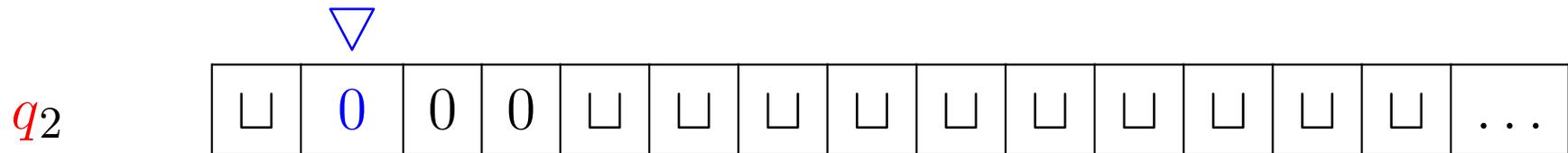
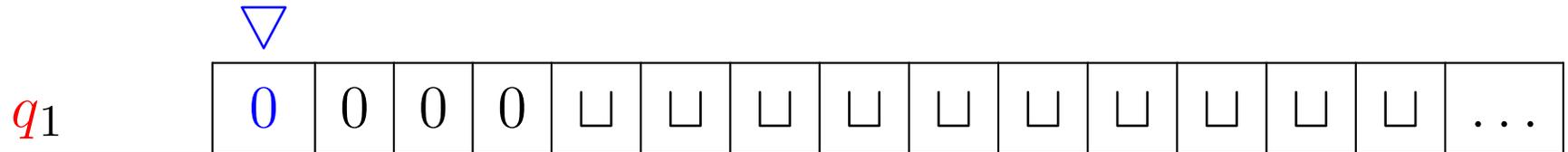


q_2



Simulação de M_2

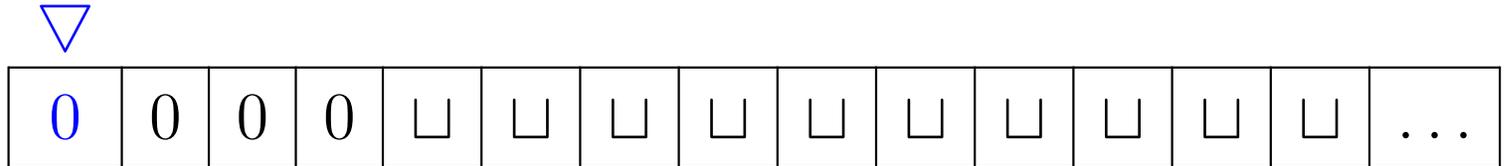
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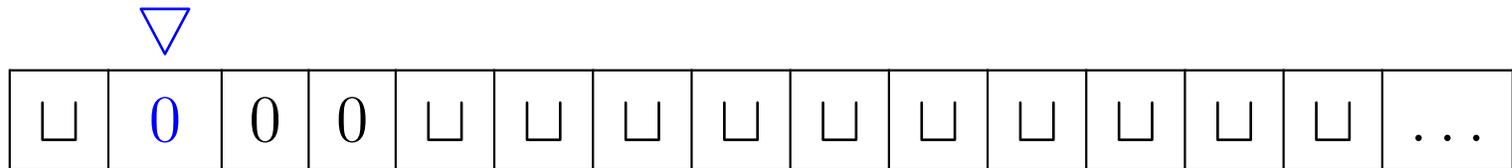
Simulação de M_2

estado

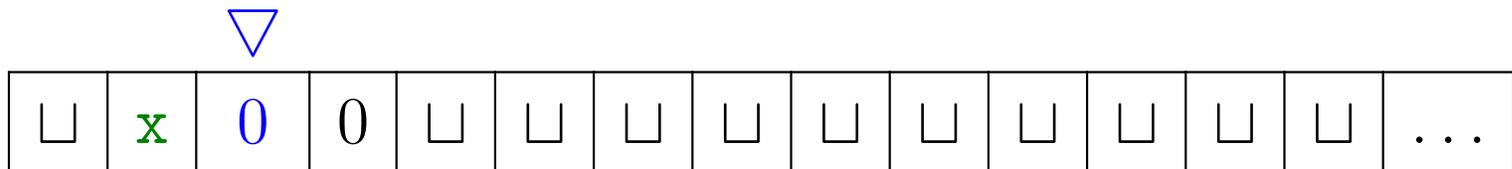
q_1



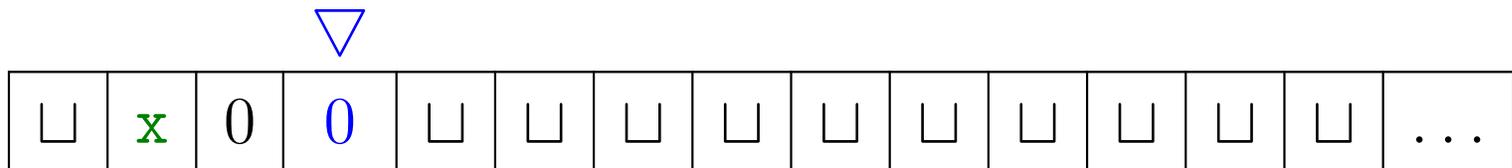
q_2



q_3



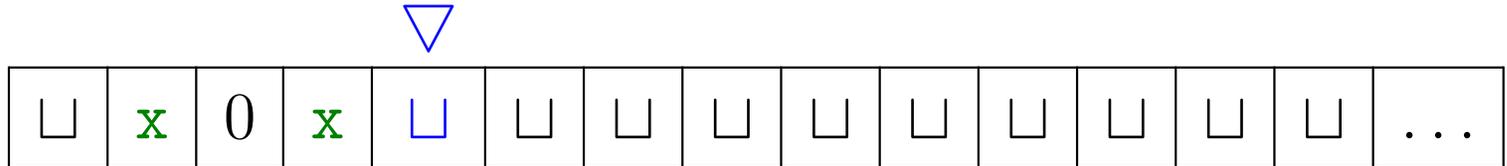
q_4



Simulação de M_2

estado

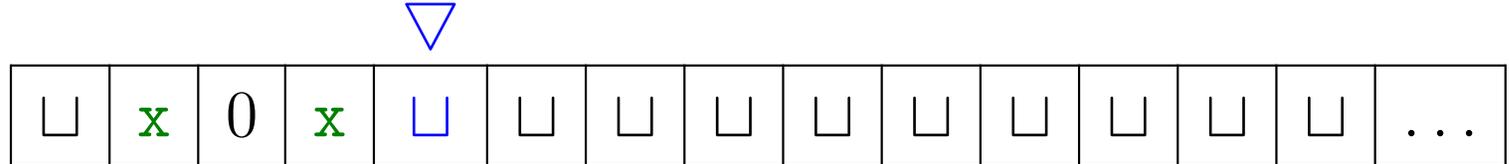
q_3



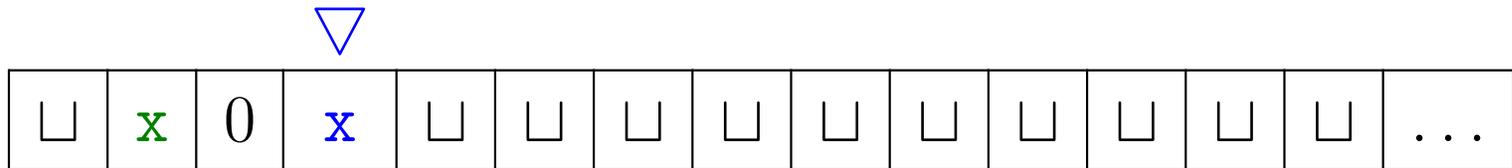
Simulação de M_2

estado

q_3



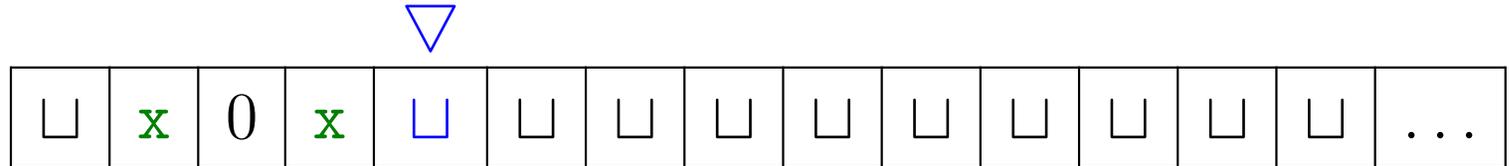
q_5



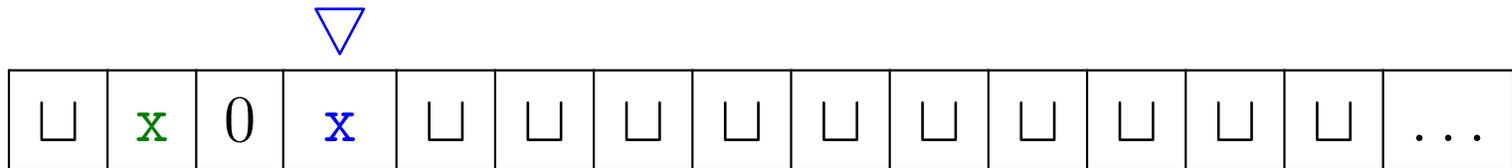
Simulação de M_2

estado

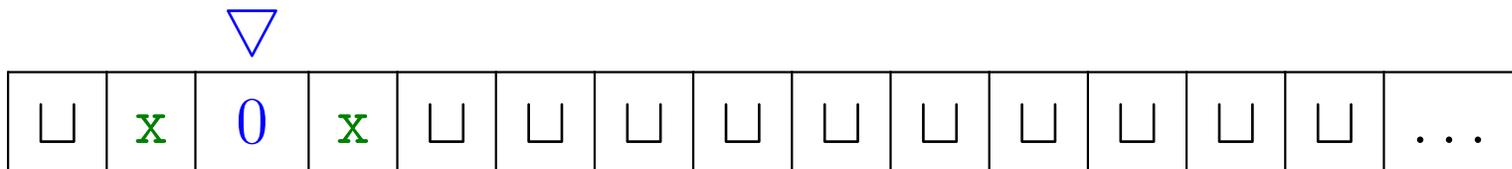
q_3



q_5



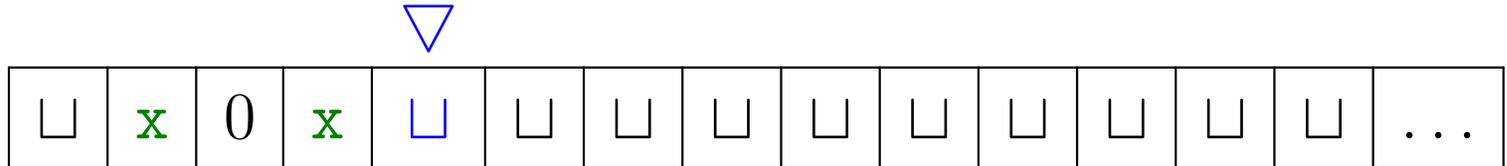
q_5



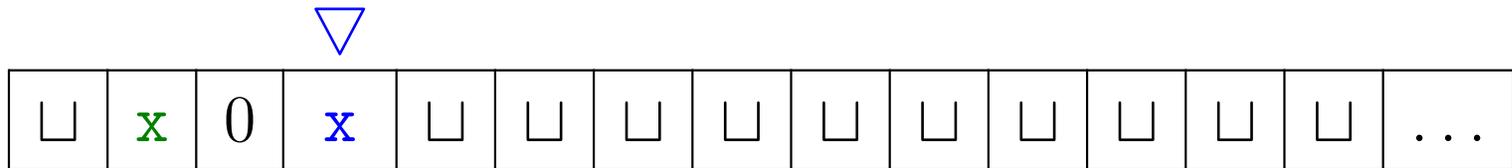
Simulação de M_2

estado

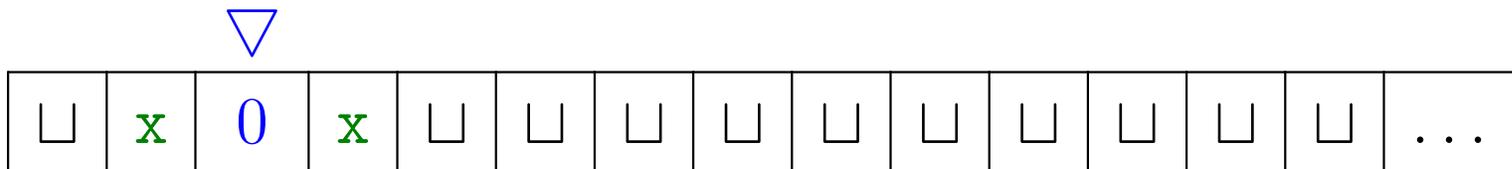
q_3



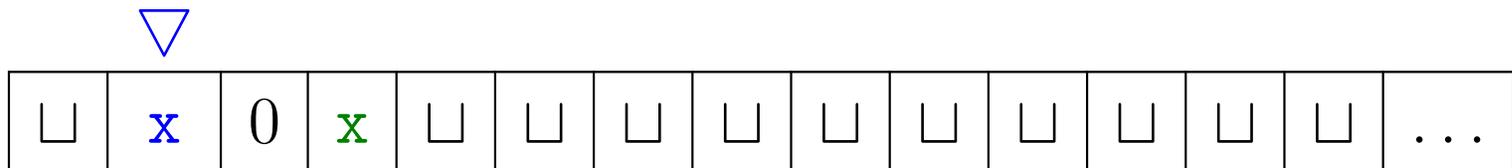
q_5



q_5



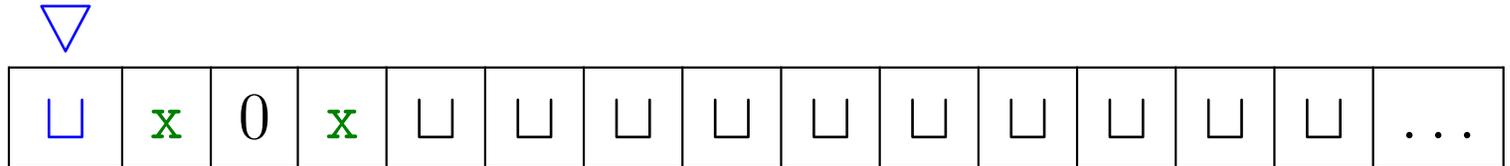
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Simulação de M_2

estado

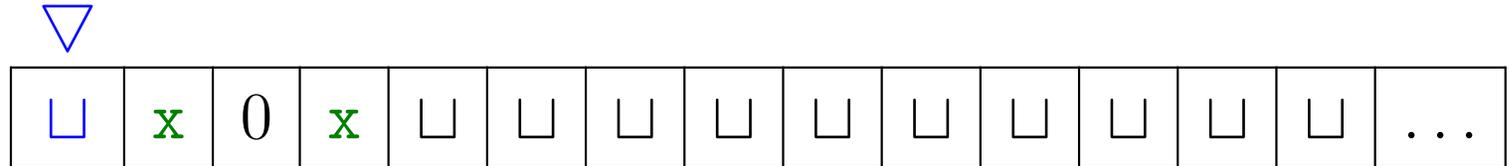
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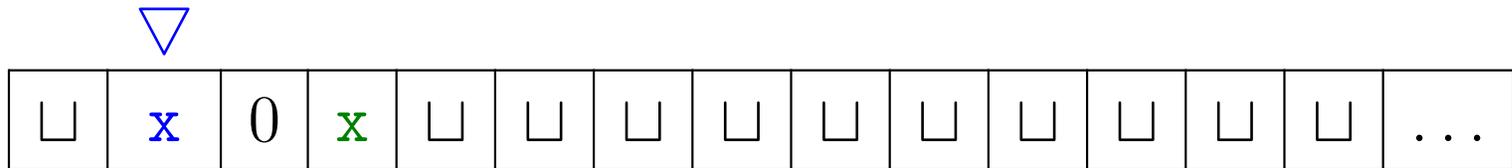
Simulação de M_2

estado

q_5

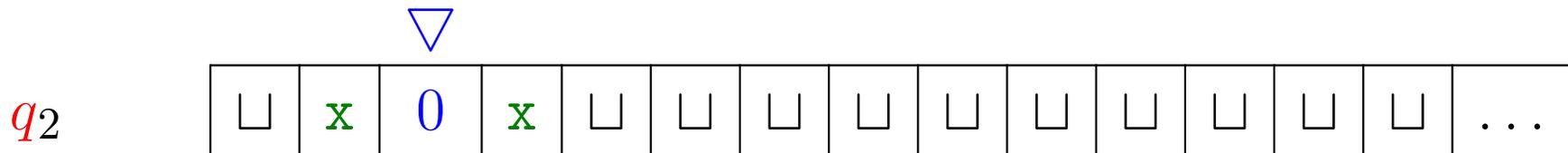
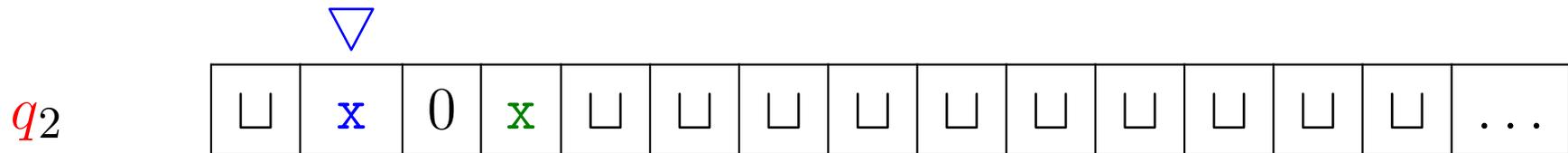
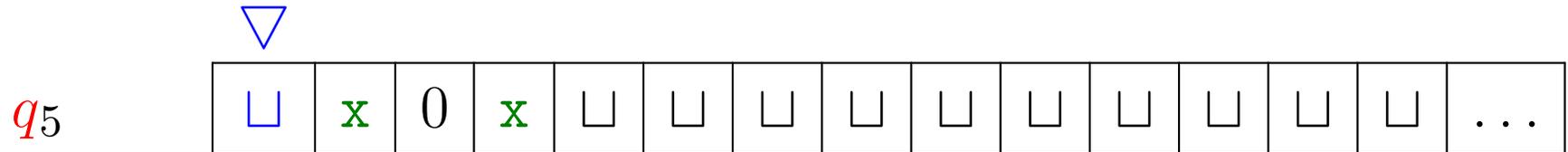


q_2



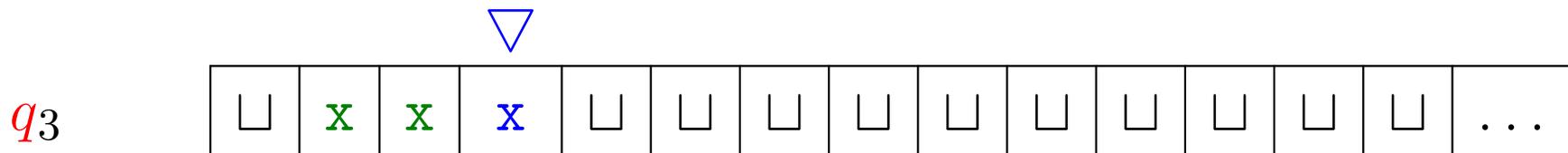
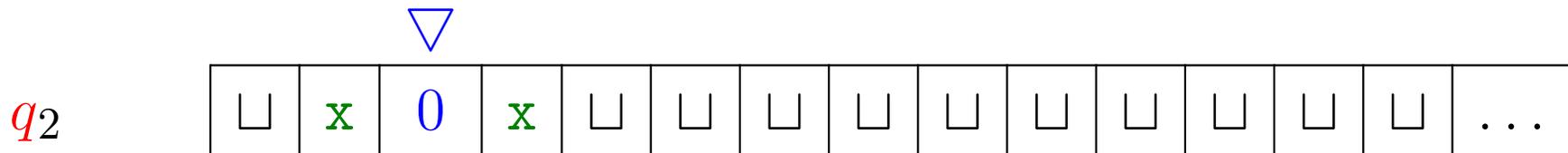
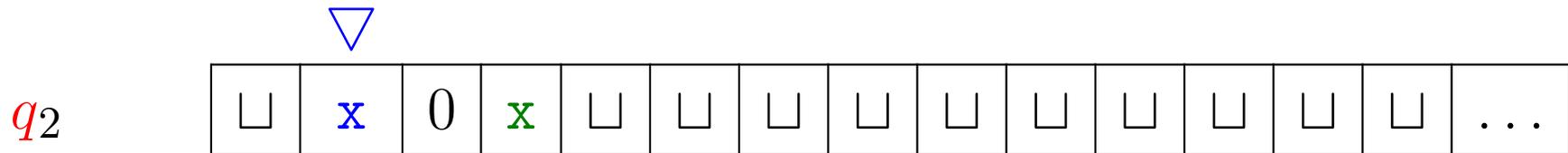
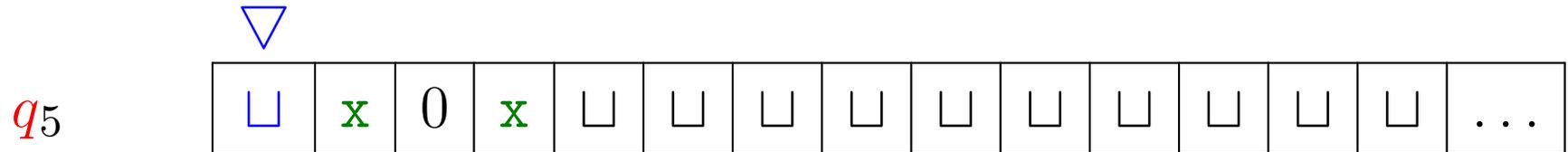
Simulação de M_2

estado



Simulação de M_2

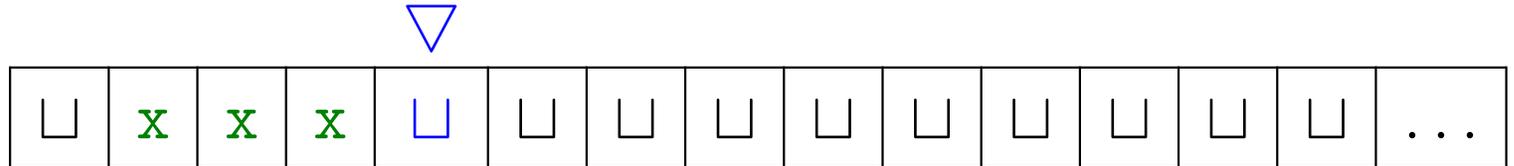
estado



Simulação de M_2

estado

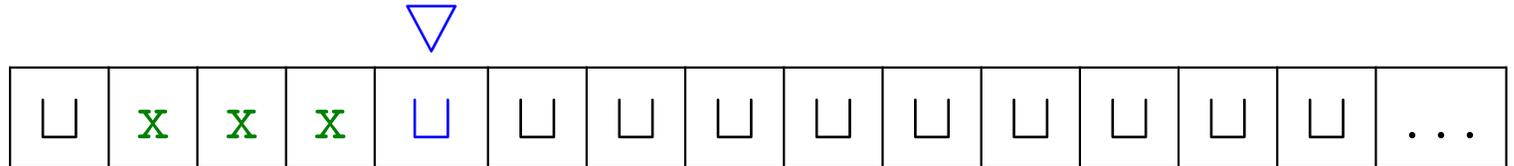
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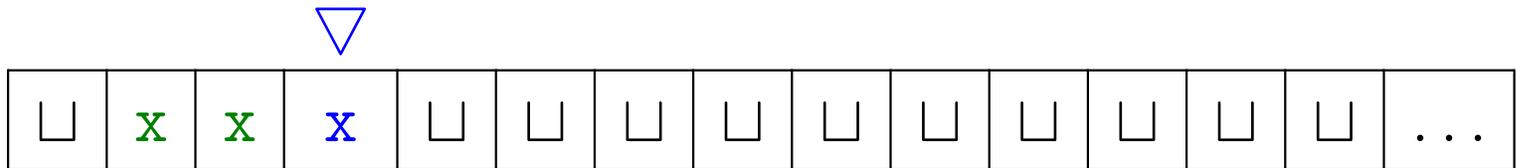
Simulação de M_2

estado

q_3



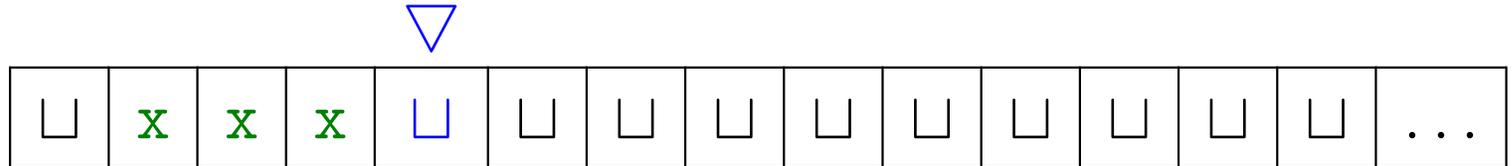
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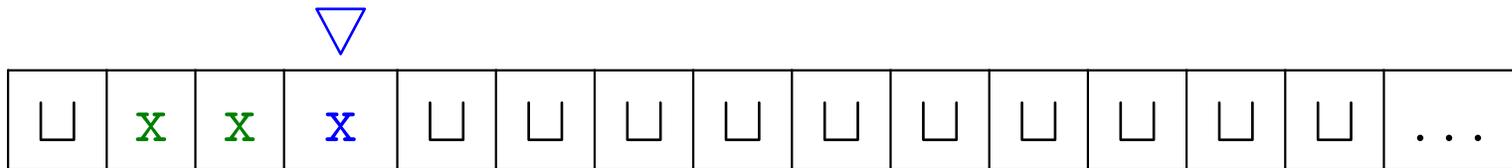
Simulação de M_2

estado

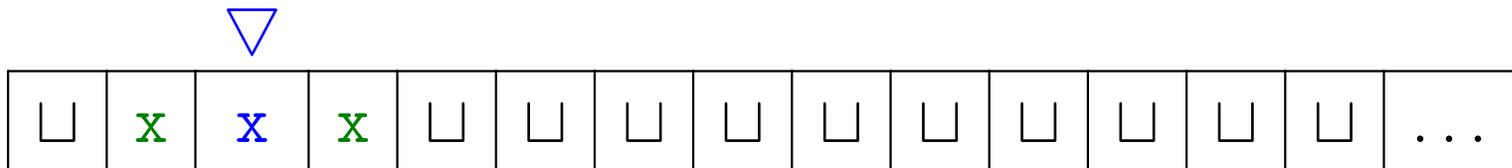
q_3



q_5



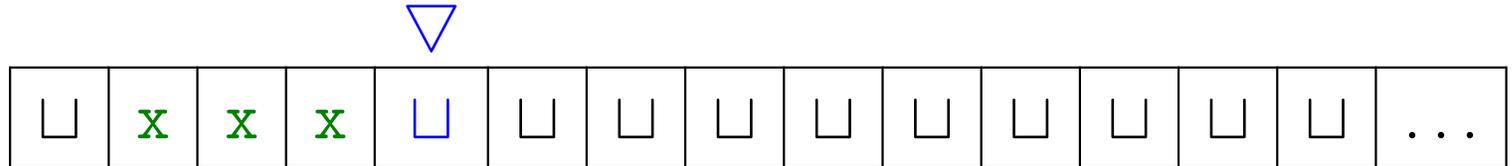
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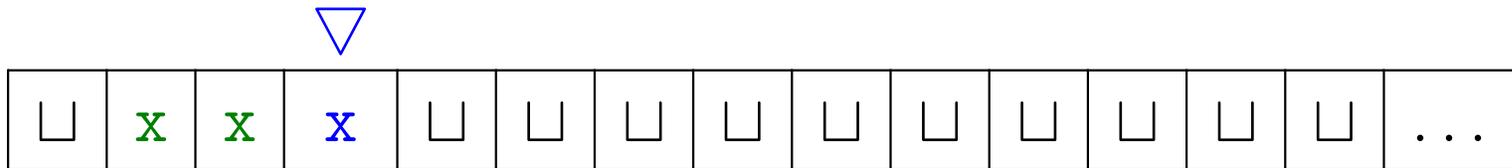
Simulação de M_2

estado

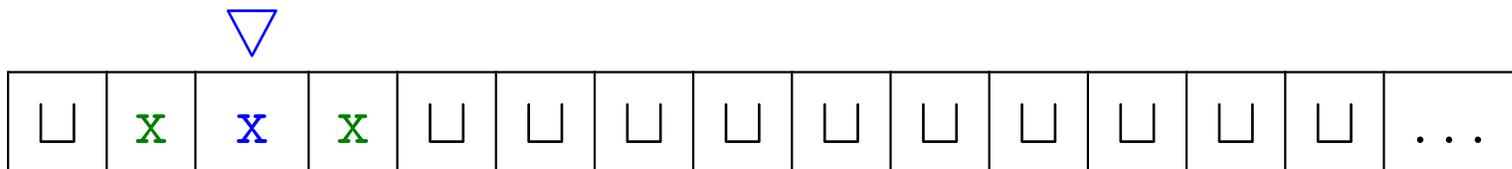
q_3



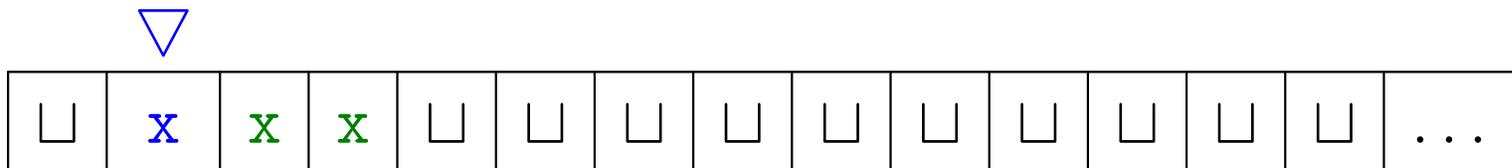
q_5



q_5



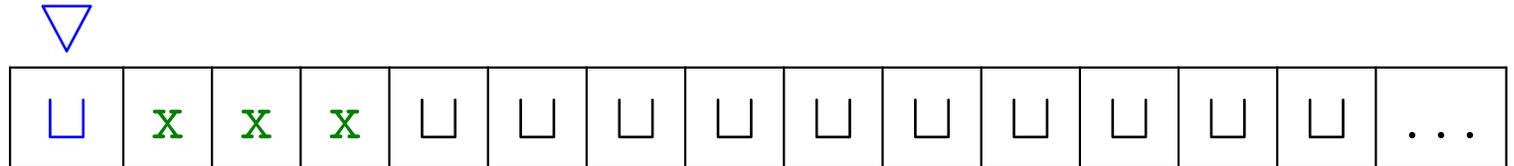
q_5



Simulação de M_2

estado

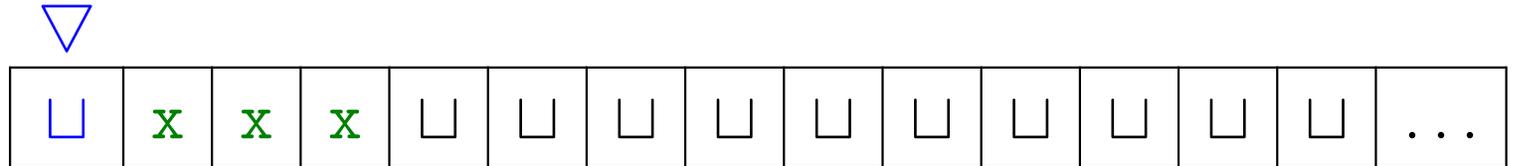
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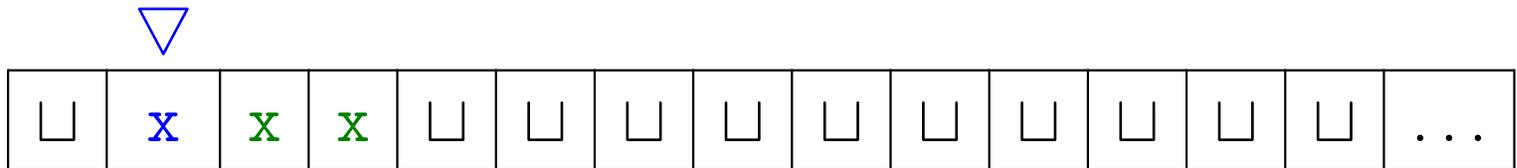
Simulação de M_2

estado

q_5



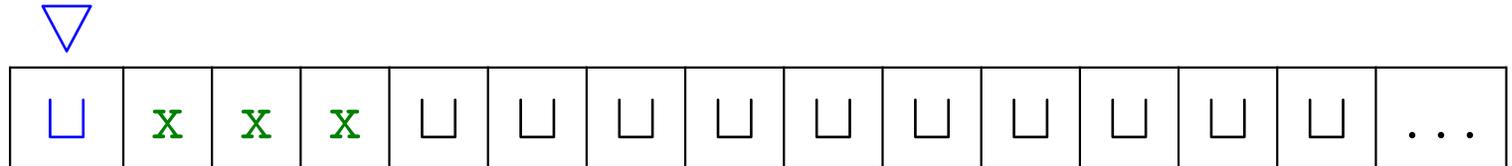
q_2



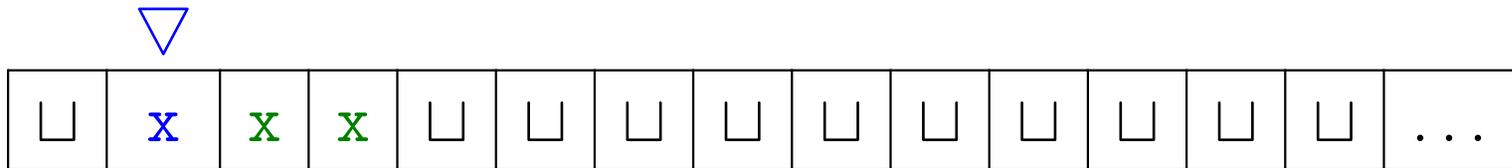
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estado

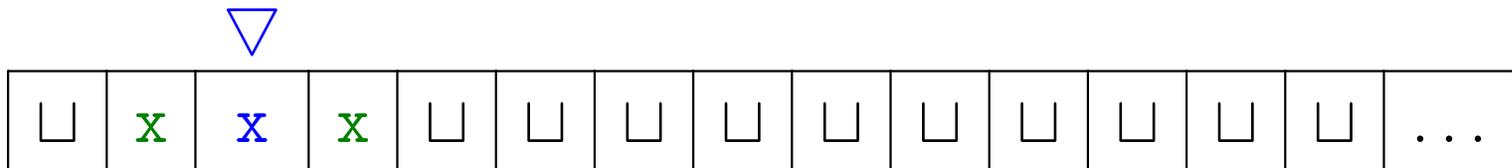
q_5



q_2



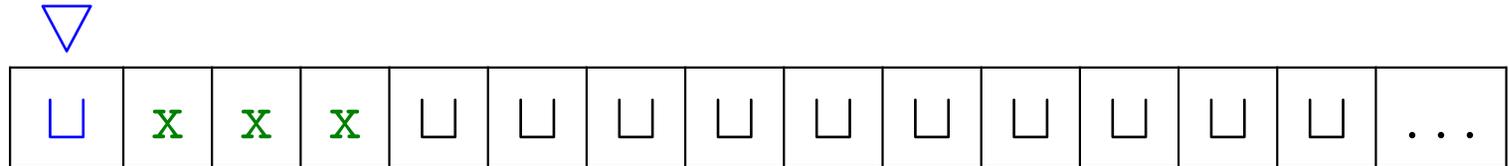
q_2



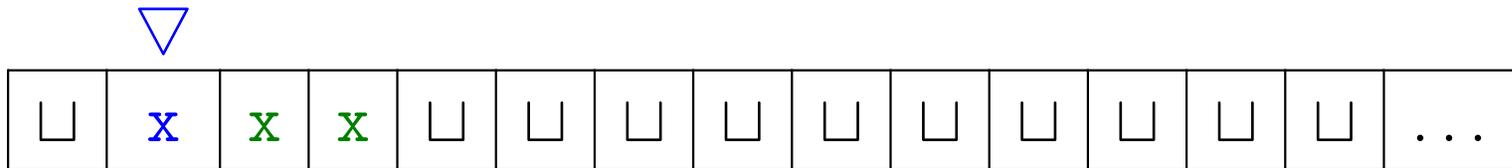
Simulação de M_2

estado

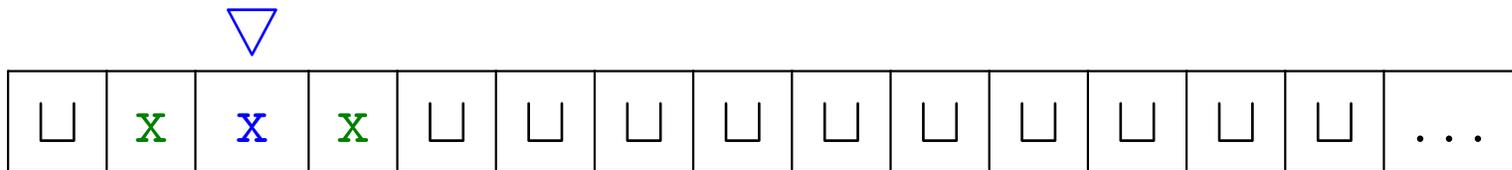
q_5



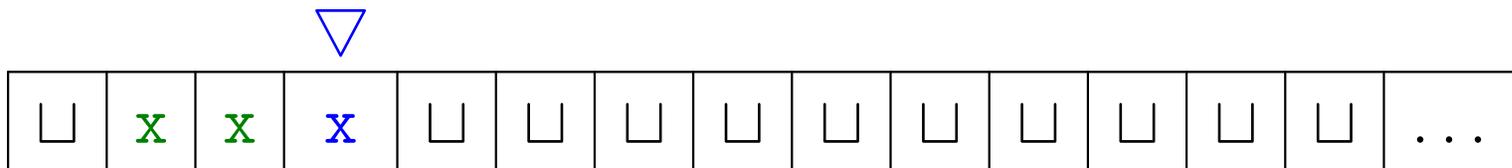
q_2



q_2



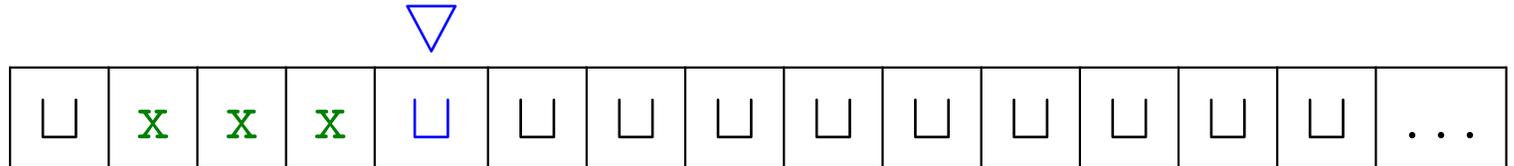
q_2



Simulação de M_2

estado

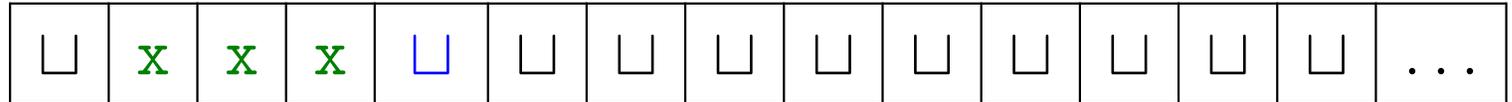
q_2



Simulação de M_2

estado

q_2



q_a

