MAE 5870 – Análise de Séries temporais
Lista #5
data de entrega: 20/06/2017

4.18 Consider two processes

$$x_t = w_t \quad \text{and} \quad y_t = \phi x_{t-D} + v_t$$

where w_t and v_t are independent white noise processes with common variance σ^2, ϕ is a constant, and D is a fixed integer delay.

(a) Compute the coherency between x_t and y_t.
(b) Simulate $n = 1024$ normal observations from x_t and y_t for $\phi = .9$, $\sigma^2 = 1$, and $D = 0$. Then estimate and plot the coherency between the simulated series for the following values of L and comment:
(i) $L = 1$, (ii) $L = 3$, (iii) $L = 41$, and (iv) $L = 101$.

4.19 For the processes in Problem 4.18:

(a) Compute the phase between x_t and y_t.
(b) Simulate $n = 1024$ observations from x_t and y_t for $\phi = .9$, $\sigma^2 = 1$, and $D = 1$. Then estimate and plot the phase between the simulated series for the following values of L and comment:
(i) $L = 1$, (ii) $L = 3$, (iii) $L = 41$, and (iv) $L = 101$.

4.20 Consider the bivariate time series records containing monthly U.S. production as measured by the Federal Reserve Board Production Index and monthly unemployment as given in Figure 3.21.

(a) Compute the spectrum and the log spectrum for each series, and identify statistically significant peaks. Explain what might be generating the peaks. Compute the coherence, and explain what is meant when a high coherence is observed at a particular frequency.

(b) What would be the effect of applying the filter

$$u_t = x_t - x_{t-1} \quad \text{followed by} \quad v_t = u_t - u_{t-12}$$

to the series given above? Plot the predicted frequency responses of the simple difference filter and of the seasonal difference of the first difference.

(c) Apply the filters successively to one of the two series and plot the output. Examine the output after taking a first difference and comment on whether stationarity is a reasonable assumption. Why or why not? Plot after taking the seasonal difference of the first difference. What can be noticed about the output that is consistent with what you have predicted from the frequency response? Verify by computing the spectrum of the output after filtering.

4.23 Suppose x_t is a stationary series, and we apply two filtering operations in succession, say,

$$y_t = \sum_r a_r x_{t-r} \quad \text{then} \quad z_t = \sum_s b_s y_{t-s}.$$

(a) Show the spectrum of the output is

$$f_z(\omega) = |A(\omega)|^2 |B(\omega)|^2 f_x(\omega),$$

where $A(\omega)$ and $B(\omega)$ are the Fourier transforms of the filter sequences a_t and b_t, respectively.

(b) What would be the effect of applying the filter

$$u_t = x_t - x_{t-1} \quad \text{followed by} \quad v_t = u_t - u_{t-12}$$

to a time series?

(c) Plot the predicted frequency responses of the simple difference filter and of the seasonal difference of the first difference. Filters like these are called seasonal adjustment filters in economics because they tend to attenuate frequencies at multiples of the monthly periods. The difference filter tends to attenuate low-frequency trends.

4.26 Fit an autoregressive spectral estimator to the Recruitment series and compare it to the results of Example 4.13.

4.32 Consider the problem of approximating the filter output

$$y_t = \sum_{k=-\infty}^{\infty} a_k x_{t-k}, \quad \sum_{-\infty}^{\infty} |a_k| < \infty,$$

by

$$y_t^M = \sum_{|k|<M/2} a_k^M x_{t-k}$$

for $t = M/2 - 1, M/2, \ldots, n - M/2$, where x_t is available for $t = 1, \ldots, n$ and

$$a_t^M = M^{-1} \sum_{k=0}^{M-1} A(\omega_k) \exp\{2\pi i \omega_k t\}$$

with $\omega_k = k/M$. Prove

$$E\{(y_t - y_t^M)^2\} \leq 4\gamma_x(0) \Bigg(\sum_{|k|\geq M/2} |a_k| \Bigg)^2.$$

4.34 The data set `climhyd`, contains 454 months of measured values for six climatic variables: (i) air temperature [`Temp`], (ii) dew point [`DewPt`], (iii) cloud cover [`CldCvr`], (iv) wind speed [`WndSpd`], (v) precipitation [`Precip`], and (vi) inflow [`Inflow`], at Lake Shasta in California; the data are displayed in Figure 7.3. We would like to look at possible relations among the weather factors and between the weather factors and the inflow to Lake Shasta.

(a) First transform the inflow and precipitation series as follows: $I_t = \log i_t$, where i_t is inflow, and $P_t = \sqrt{p_t}$, where p_t is precipitation. Then, compute the square coherencies between all the weather variables and transformed inflow and argue that the strongest determinant of the inflow series is (transformed) precipitation. [*Tip:* If `x` contains multiple time series, then the easiest way to display all the squared coherencies is to first make an object of class *spec*; e.g., `u = spectrum(x, span=c(7,7), plot=FALSE)` and then plot the coherencies suppressing the confidence intervals, `plot(u, ci=-1, plot.type="coh")`.]

(b) Fit a lagged regression model of the form

$$I_t = \beta_0 + \sum_{j=0}^{\infty} \beta_j P_{t-j} + w_t,$$

using thresholding, and then comment of the predictive ability of precipitation for inflow.

4.37 Consider the same model as in the preceding problem.

(a) Prove the optimal smoothed estimator of the form

$$\widehat{x}_t = \sum_{s=-\infty}^{\infty} a_s y_{t-s}$$

has

$$a_s = \frac{\sigma_w^2}{\sigma^2} \frac{\theta^{|s|}}{1-\theta^2}.$$

(b) Show the mean square error is given by

$$E\{(x_t - \widehat{x}_t)^2\} = \frac{\sigma_v^2 \sigma_w^2}{\sigma^2(1-\theta^2)}.$$

(c) Compare mean square error of the estimator in part (b) with that of the optimal finite estimator of the form

$$\widehat{x}_t = a_1 y_{t-1} + a_2 y_{t-2}$$

when $\sigma_v^2 = .053, \sigma_w^2 = .172$, and $\phi_1 = .9$.