MAE 5870 – Análise de Séries temporais
Lista #4
data de entrega: 06/06/2017

4.1 Repeat the simulations and analyses in Examples 4.1 and 4.2 with the following changes:

(a) Change the sample size to $n = 128$ and generate and plot the same series as in Example 4.1:

$$x_{t1} = 2\cos(2\pi\, .06\, t) + 3\sin(2\pi\, .06\, t),$$
$$x_{t2} = 4\cos(2\pi\, .10\, t) + 5\sin(2\pi\, .10\, t),$$
$$x_{t3} = 6\cos(2\pi\, .40\, t) + 7\sin(2\pi\, .40\, t),$$
$$x_t = x_{t1} + x_{t2} + x_{t3}.$$

What is the major difference between these series and the series generated in Example 4.1? (Hint: The answer is *fundamental*. But if your answer is the series are longer, you may be punished severely.)

(b) As in Example 4.2, compute and plot the periodogram of the series, x_t, generated in (a) and comment.

(c) Repeat the analyses of (a) and (b) but with $n = 100$ (as in Example 4.1), and adding noise to x_t; that is

$$x_t = x_{t1} + x_{t2} + x_{t3} + w_t$$

where $w_t \sim \text{iid N}(0, 25)$. That is, you should simulate and plot the data, and then plot the periodogram of x_t and comment.

4.4 A time series was generated by first drawing the white noise series w_t from a normal distribution with mean zero and variance one. The observed series x_t was generated from

$$x_t = w_t - \theta w_{t-1}, \quad t = 0, \pm 1, \pm 2, \ldots,$$

where θ is a parameter.

(a) Derive the theoretical mean value and autocovariance functions for the series x_t and w_t. Are the series x_t and w_t stationary? Give your reasons.

(b) Give a formula for the power spectrum of x_t, expressed in terms of θ and ω.

4.6 In applications, we will often observe series containing a signal that has been delayed by some unknown time D, i.e.,

$$x_t = s_t + As_{t-D} + n_t,$$

where s_t and n_t are stationary and independent with zero means and spectral densities $f_s(\omega)$ and $f_n(\omega)$, respectively. The delayed signal is multiplied by some unknown constant A.

(a) Prove
$$f_x(\omega) = [1 + A^2 + 2A\cos(2\pi\omega D)]f_s(\omega) + f_n(\omega).$$

(b) How could the periodicity expected in the spectrum derived in (a) be used to estimate the delay D? (Hint: Consider the case where $f_n(\omega) = 0$; i.e., there is no noise.)

4.8 Figure 4.31 shows the biyearly smoothed (12-month moving average) number of sunspots from June 1749 to December 1978 with $n = 459$ points that were taken twice per year; the data are contained in `sunspotz`. With Example 4.10 as a guide, perform a periodogram analysis identifying the predominant periods and obtaining confidence intervals for the identified periods. Interpret your findings.

4.9 The levels of salt concentration known to have occurred over rows, corresponding to the average temperature levels for the soil science data considered in Figures 1.15 and 1.16, are in `salt` and `saltemp`. Plot the series and then identify the dominant frequencies by performing separate spectral analyses on the two series. Include confidence intervals for the dominant frequencies and interpret your findings.

4.16 Consider two time series

$$x_t = w_t - w_{t-1},$$

$$y_t = \tfrac{1}{2}(w_t + w_{t-1}),$$

formed from the white noise series w_t with variance $\sigma_w^2 = 1$.

(a) Are x_t and y_t jointly stationary? Recall the cross-covariance function must also be a function only of the lag h and cannot depend on time.

(b) Compute the spectra $f_y(\omega)$ and $f_x(\omega)$, and comment on the difference between the two results.

(c) Suppose sample spectral estimators $\bar{f}_y(.10)$ are computed for the series using $L = 3$. Find a and b such that

$$P\left\{a \le \bar{f}_y(.10) \le b\right\} = .90.$$

This expression gives two points that will contain 90% of the sample spectral values. Put 5% of the area in each tail.