UMA EXTENSÃO DE OPERADORES COMPACTOS POR OPERADORES COMPACTOS SEM MULTIPLICADORES NÃO TRIVIAIS

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Abstract. Uncountable almost disjoint families of infinite subsets of $\mathbb{N}$ (every two distinct members of the family have finite intersection) appeared in the work of Hausdorff, Sierpiński and Luzin at the begining of the 20th century. When used in the constructions of mathematical structures they induce interesting phenomena. Already Alexandroff and Urysohn ([1]) used such families $\mathcal{A}$ to construct what we now call a $\Psi_{\mathcal{A}}$-space, i.e., a locally compact topological space where points of $\mathbb{N}$ are isolated and form a dense subspace but $\overline{\mathbb{N}} \setminus \mathbb{N}$ is uncountable discrete. It turned out that combinatorial properties of almost disjoint families $\mathcal{A}$ correspond to topological properties of $\psi_{\mathcal{A}}$. Many topological examples or counterexamples where obtained in the form of $\Psi_{\mathcal{A}}$-spaces ([3]). One of them obtained in 1977 by Mrówka ([5]) had the property that its Stone-Cech compactification $\beta\Psi_{\mathcal{A}}$ was equal to its one-point compactification $\alpha\Psi_{\mathcal{A}}$, which is striking for a separable Frechet-Urysohn space.

Johnson and Lindenstrauss started looking in [4] at $C_0(\Psi_{\mathcal{A}})$ or its modifications as a Banach space in the context of WCG spaces. This class of spaces also induce interesting examples and counterexamples in Banach space theory. Using continuous functions on $\Psi_{\mathcal{A}}$-space one can express the definition of $\Psi_{\mathcal{A}}$-space in an algebraic way, that is $\Psi_{\mathcal{A}}$-space satisfies the following short exact sequence

$$0 \to c_0 \xrightarrow{\iota} \mathcal{C}_0(\Psi_{\mathcal{A}}) \to c_0(\kappa) \to 0,$$

where $\iota[c_0]$ is an essential ideal $C_0(\Psi_{\mathcal{A}})$, or in other words, $\mathbb{N}$ is a dense subset of $\Psi_{\mathcal{A}}$. Here κ is the cardinality of $\mathcal{A}$. This formulation explains why these spaces may be useful when considering "three space properties".

As $\mathbb{N}$ is dense in $\Psi_{\mathcal{A}}$ we can consider $C_0(\Psi_{\mathcal{A}})$ as a subspace of $\ell_\infty(\mathbb{N})$ which can be considered as the algebra of bounded diagonal operators on $\ell_2(\mathbb{N})$.

In my talk I would like to look at noncommutative operator algebras on $\ell_2(\mathbb{N})$, whose constructions mimick the constructions of the above mentioned commutative objects, where the algebra of compact operators $\mathcal{K}(\ell_2(\kappa))$ replaces $c_0(\kappa)$. In particular I would like to mention a construction obtained with Saeed Ghasemi in [2] of a C^*-subalgebra of $\mathcal{B}(\ell_2)$ satisfying the following short exact sequence

$$0 \to \mathcal{K}(\ell_2) \xrightarrow{\iota} \mathcal{A} \to \mathcal{K}(\ell_2(\mathfrak{c})) \to 0,$$

where $\iota[\mathcal{K}(\ell_2)]$ is an essential ideal of $\mathcal{A}$ with the additional Mrówka type property that the multiplier algebra $\mathcal{M}(\mathcal{A})$ of $\mathcal{A}$ is *-isomorphic to the unitization of $\mathcal{A}$.

As $K(\ell_2)$ and $K(\ell_2(\mathfrak{c}))$ are "very nice" and $\mathcal{A}$ is "very strange" due to its Mrówka type property, this example sheds light on "three space properties" among nonseparable C^*-algebras, in particular in the context of their stability ([6]).

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