http://www.ime.usp.br/~banach/bwb2014/

First Brazilian Workshop in Geometry of Banach Spaces

August 25-29, 2014

Scientific committee

J. M. F. Castillo (U. Extremadura)
V. Ferenczi (U. São Paulo)
R. Haydon (U. Oxford)
W. B. Johnson (Texas A&M)
G. Pisier (Paris 6 & Texas A&M)
Th. Schlumprecht (Texas A&M)
S. Todorcevic (CNRS Paris & U. Toronto)

Organizing committee:

F. Baudier (Paris 6 & Texas A&M)
C. Brech (U. São Paulo)
V. Ferenczi (U. São Paulo), chair
E. M. Galego (U. São Paulo)
J. Lopez-Abad (ICMAT & U. São Paulo)

USP 80 1934 2014
IME-USP
FAPESP
CAPES
INCTMat
Proex-PPG-MAT IME-USP

Abstracts

Tutorial

Pandelis Dodos

Ramsey Theory and the geometry of Banach spaces

One of the most illuminating imports in Banach Space Theory is the field of Ramsey Theory. The mini-course will be devoted in the interaction between the two fields and will be concentrated on the following general schemes.

1. The Ramsey-theoretical background: first, we shall review some recent advances in Ramsey Theory focusing, in particular, on those which are relevant to Functional Analysis.

2. Applications: next, we shall present some applications of Ramsey Theory to the geometry of Banach spaces. There are several excellent expositions in the literature presenting most of the work, up to 2010, that has been done in this direction. Thus, we will focus on very recent results.

3. Open problems: last (but not least) we shall discuss some open problems in the geometry of Banach spaces which seem amenable to combinatorial and/or Ramsey-theoretic manipulations. The problems will be exposed together with the Ramsey-theoretic tools that seem most relevant to them.

Plenary talks

Spiros A. Argyros

On Banach spaces with rich $\mathcal{L}_\infty$-structure

In the 80's H. P. Rosenthal asked whether every $\mathcal{L}_\infty$-saturated Banach space contains c_0 isomorphically. We will discuss Rosenthal's problem in the opposite direction, namely the existence of a $\mathcal{L}_\infty$ space which is $\mathcal{L}_\infty$-saturated and does not contain c_0. We will present some partial results concerning this problem which are included in a recent joint work with P. Motakis.

Robert Deville

Construction of differentiable functions

We first give an overview about the possibility of constructing smooth functions between two Banach spaces with some properties and the geometry of these spaces. We then prove that for many pairs (X, Y) of classical Banach spaces, there exists a bounded, Lipschitz, Gâteaux-differentiable function from X to Y whose derivatives are all far apart.

Gilles Godefroy

Borel families of Banach spaces

Through a proper use of the Effros-Borel structure, a precise (and useful) meaning can be given to the notion of Borel collection of Banach spaces. It turns out that being Borel is actually a quite restrictive condition. We will survey in this talk some recent results on this topic, and several natural open questions which occur in this field. Technicalities will be avoided whenever possible.

Petr Hajek

Polynomial approximation in Banach spaces

The classical Stone-Weierstrass theorem claims that every continuous function on the unit ball of a finite dimensional Banach space can be arbitrarily well approximated by polynomials. In our talk we will address the situation in infinite dimensional Banach spaces. It is well-known that analogous theorem never holds, and our main focus will be the study of the approximable functions depending on the degree of the polynomials which are allowed to generate the approximating polynomial via the algebraic operations.

This talk is based on two recent papers by the author jointly with S. D Alessandro and M. Johanis.

Piotr Koszmider

The elusive geometry of the Banach space ℓ_∞/c_0

We will survey old and recent results as well as open problems concerning the geometry of the Banach space ℓ_∞/c_0. Many properties of ℓ_∞/c_0 (also as a Banach algebra) depend on higher combinatorics of subsets of the integers modulo finite sets. We will sketch the difficulties of the initial stage of the search for combinatorial principles which would yield a transparent structural theory of the Banach space ℓ_∞/c_0.

Vladimir Pestov

Around the problem of amenability of loop groups

We survey what is known (at least, to the speaker) about the open problem of amenability of groups of C^∞ loops with values in a simple compact Lie group, in particular we explain why this problem is closely linked to the topic of the conference.

Christian Rosendal

The intrinsic geometry of topological groups

Gromov's program of studying the geometry of finitely or compactly generated groups has had a broad impact on mathematics connecting many hitherto unrelated areas and techniques. However, beyond the category of locally compact groups, little progress has been made, since the absence of canonical generating sets indicated that general topological groups may not have any intrinsically defined geometry.

We shall remedy this situation by presenting a general framework for defining and identifying canonical coarse and quasi-isometry types of a large number of familiar topological groups, which incidentally may be seen as a common generalisation of the corresponding quasi-isometry types of compactly generated locally compact groups and of Banach spaces. We shall also discuss how this study requires the use of more general Banach spaces than the reflexive spaces which suffice in the classical setting.

Gideon Schechtman

An injective ℓ_1 preserving operator on $L_1(0,1)$ which is not an isomorphism

I shall discuss the construction of an injective operator $T : L_1(0,1) \to L_1(0,1)$ which is an isomorphism when restricted to any isomorph of ℓ_1 (and consequently, for some $\epsilon > 0$, an isomorphism on $L_1(A)$ for any $A \subset [0,1]$ of measure at most ϵ) but is not an isomorphism on all of $L_1[0,1]$. Some further consequences of the existence of such an operator will be discussed. Joint work with Johnson, Nasseri and Tkocz.

Thomas Schlumprecht

On Zippin's Embedding Theorem

In 1988 Zippin proved that every separable reflexive Banach space X embeds into a reflexive Banach space with a basis, and that every Banach space with separable dual embeds into a space with shrinking basis.

We will present a new proof of these results. Roughly speaking, instead of starting by embedding X into a *large* space Z with basis (for example $C[0,1]$) and then "prune" this space until it has the desired properties, we start with a Markushevich basis of X, and *augment* X until this basis becomes a Schauder basis. This proof leads to a space Y with basis containing X which, in many respects, is much closer to X than in previous constructions. For example we will solve a problem stated by Pelczynski and prove that X and its superspace Y with basis share the same Szlenk index.

Stevo Todorcevic

Sequences in Large Banach Spaces

This will be an overview of the recent work on determining the minimal density bounds on a given Banach space X that guarantee the existence of basic sequences in X with extra properties such as the unconditionality, subsymetricity, etc.

Contributed talks

Razvan Anisca

On hereditary approximation property

The talk stems from the recent investigation by Johnson and Szankowski of Banach spaces with the hereditary approximation property, that is, spaces whose all subspaces have the approximation property. Among several interesting results, their main theorem states that if X is a Banach space such that the sequence $\{d_n(X)\}$, which gives the isomorphism constant to ℓ_2^n from its n-dimensional subspaces, grows sufficiently slow as $n \to \infty$, then X must have the hereditary approximation property.

We identify a rather large class of Banach spaces X with the property that when the rate of growth of $\{d_n(X)\}$ is at least the same as $\log n^\beta$, for some $\beta > 1$, then X does not have the hereditary approximation property.

Antonio Aviles

Classification of separable Banach spaces under analytic determinacy

We show that, under the axiom of analytic determinacy, separable Banach spaces can be classified into a hierarchy of four categories, depending on the complexity of the lattice of weakly compact sets. This classification can be refined using Ramsey theory when the metric structure is taken into account.

This is joint work with G. Plebanek and J. Rodriguez.

Cleon Barroso

On Separable Quotients

In this talk I will present a new theorem on the existence of separable quotients, which generalizes some results of Plichko for spaces with a fundamental biorthogonal system.

Dana Bartosova

Generalizations of Gowers' Theorem

Inspired by a problem from topological dynamics, we generalize Gowers' fin_k Theorem, We show how this proves a dual Ramsey property for a certain class of trees, and its connection to the original problem. We raise the question how our generalizations can be applied in the Banach space theory. This is a joint work with Aleksandra Kwiatkowska.

Kevin Beanland

Tsirelson spaces with constraints

In this talk we discuss a method of building Tsirelson-like Banach spaces using constraints. This concept first appeared in work of Odell and Schlumprecht in the late 1990s. However, more recently, these spaces have been used (for example by Argyros and Motakis) to solve several problems in Banach space theory. We will briefly explain the method of constructing Tsirelson spaces with constraints and give some recent applications of these spaces to problems involving distortion, strictly singular operators, Krivine p's and spreading models.

Bruno Braga

On the complexity of some classes of Banach spaces

This work lies in the intersection of the geometry of Banach spaces and descriptive set theory. We find lower and/or upper bounds for the descriptive complexity of several natural classes of separable Banach spaces.

Jamilson Campos

Type of multilinear operators and polynomials

We introduce the classes of multilinear operators of type $(p_1, ..., p_n)$ and n-homogeneous polynomials of type p. Namely, a continuous n-linear operator $T \in \mathcal{L}(E_1, ..., E_n; F)$ has *type* $(p_1, ..., p_n)$, $\frac{1}{2} \leq \frac{1}{p_1} + \cdots + \frac{1}{p_n} \leq 1$, if there is a constant $C > 0$ such that, however we choose finitely many vectors $\left(x_j^{(1)}, ..., x_j^{(n)}\right)$ in $E_1 \times \cdots \times E_n$, $j \in \{1, ..., k\}$,

$$\left(\int_0^1 \left\|\sum_{j=1}^k r_j(t) T\left(x_j^{(1)}, ..., x_j^{(n)}\right)\right\|^2 dt\right)^{1/2} \leq C \cdot \prod_{i=1}^n \left(\sum_{j=1}^k \left\|x_j^{(i)}\right\|^{p_i}\right)^{1/p_i},$$

where $(r_j)_{j=1}^\infty$ are the Rademacher functions. A continuous n-homogeneous polynomial $P \in \mathcal{P}(^n E; F)$ has *type* p, $n \leq p \leq 2n$, if there is a constant $C > 0$ for which

$$\left(\int_0^1 \left\|\sum_{j=1}^k r_j(t) P(x_j)\right\|^2 dt\right)^{1/2} \leq C \cdot \left(\sum_{j=1}^k \|x_j\|^p\right)^{n/p},$$

regardless of the choice of finitely many vectors $x_1, ..., x_k$ in E.

One of our first results on these concepts asserts that the class of multilinear operators of type $(p_1, ..., p_n)$, denoted by $\tau^n_{p_1,...p_n}$, is quite "large" in the sense that if $\frac{1}{p} = \frac{1}{p_1} + \cdots + \frac{1}{p_n}$, then

$$\mathcal{L}(\tau_{p_1}, \mathcal{L}, ..., \mathcal{L}) \cup \cdots \cup \mathcal{L}(\mathcal{L}, ..., \mathcal{L}, \tau_{p_n}) \cup (\tau_p \circ \mathcal{L}) \subseteq \tau^n_{p_1,...p_n},$$

where τ_p and τ_{p_i}, $i \in \{1, ..., n\}$, are the linear ideals of operators of type p and p_i, respectively, $\mathcal{L}(\tau_{p_1}, \mathcal{L}, ..., \mathcal{L})$ is the multi-ideal constructed by factorization method from τ_{p_1} and $\mathcal{L}$, and $\tau_p \circ \mathcal{L}$ is the composition multi-ideal generated by τ_p.

This is a joint work with Geraldo Botelho.

Alejandro Chavez-Dominguez

Stability of low-rank matrix recovery and its connections to Banach space geometry

Compressed sensing deals with the problem of recovering a vector in a high-dimensional space from a lower-dimensional measurement, under the assumption that the vector is sparse (that is, it has relatively few non-zero coordinates). One of the best-known techniques to achieve such recovery is the ℓ_p-minimization, and its properties are related to the geometry of the Banach spaces involved: theorems by Kashin-Temlyakov and Foucart-Pajor-Rauhut-Ullrich relate the stability of sparse vector recovery via ℓ_p-minimization to the Gelfand numbers of identity maps between finite-dimensional ℓ_p-spaces. In many practical situations the space of unknown vectors has in fact a matrix structure, a good example being the famous matrix completion problem (also known as the Netflix problem) where the unknown is a matrix and we are given a subset of its entries. In this case sparsity gets replaced by the more natural condition of having low rank, and the last few years have witnessed an explosion of work in this area. In this talk we present matrix analogues of the aforementioned results, relating the stability of low-rank matrix recovery via Schatten p-minimization to the Gelfand numbers of identity maps between finite-dimensional Schatten p-spaces.

Joint work with Denka Kutzarova (UIUC).

Wilson Cuellar Carrera

Compatible complex structures on Kalton-Peck space

The Kalton-Peck space Z_2 is a twisted sum of ℓ_2 with ℓ_2 associated to a singular quasi-linear map Ω_2. It exhibits very interesting properties. For example, Z_2 has an unconditional FDD consisting of 2-dimensional spaces; however it has no unconditional basis and no local unconditional structure. An old conjecture (still open) about the Kalton-Peck space is that it's not isomorphic to its hyperplanes.

A proof to this conjecture may arise by proving that the hyperplanes of Z_2 do not admit complex structures. Recall that a real Banach space X is said to admit a complex structure if there exists an $\mathbb{R}$-linear operator I on X such that $I^2 = -id$. When such operators exist, we can give a $\mathbb{C}$-linear structure on X by setting a complex scalar multiplication as follows:

$$(\alpha + i\beta)x = \alpha x + \beta I(x) \ \ \forall \alpha, \beta \in \mathbb{R}, \ \ \forall x \in X.$$

Endowed with the equivalent norm $|||x||| = \sup_{0 \leq \theta \leq 2\pi} \|e^{i\theta} x\|$ we obtain a complex Banach space.

We say that a complex structure u on ℓ_2 is compatible with Z_2 if there exists a complex structure U on Z_2 such that the restriction of U to the copy of ℓ_2 in Z_2 is u. In this work we prove that there exists a complex structure on ℓ_2 which is not compatible with Z_2. We also study a notion of compatible complex structure on hyperplanes of Z_2 and prove that hyperplanes of Z_2 do not admit compatible complex structures.

This is a joint work with professors J. M. F. Castillo, V. Ferenczi and Y. Moreno.

References

[1] N.J. Kalton and N.T. Peck, Twisted sums of sequence spaces and the three space problem, Trans. Amer. Math. Soc. 255 (1979) 1–30.

[2] J. Lindenstrauss and L. Tzafriri, Classical Banach spaces I, sequence spaces, Ergeb. Math. 92, Springer-Verlag 1977.

Barnabas Farkas

Representations of ideals in Banach spaces

I shall talk about a joint work with P. Borodulin-Nadzieja and G. Plebanek (University of Wrocław). We investigated ideals on ω (or on arbitrary countable sets) of the following form: Given a Banach space, or in general, a Polish Abelian group G and a sequence $x = (x_n)_{n \in \omega}$ in G. Then the *generalized summable ideal* associated to G and x is defined as follows

$$\mathcal{I}_x^G = \Big\{ A \subseteq \omega : \sum x | A \ \text{ is unconditionally convergent in } G \Big\}.$$

In other words, $A \in \mathcal{I}_x^G$ iff $\sum x|A$ is unconditionally Cauchy iff for every $\varepsilon > 0$ there is an $N \in \omega$ such that $\|\sum\{x_n : n \in F\}\| < \varepsilon$ for every $F \in [A \setminus N]^{<\omega}$ (in the case of Polish Abelian groups, it can be formalized by using neighborhoods of 0).

Let $\mathcal{J}$ be an ideal on ω and G be as above. We say that $\mathcal{J}$ is *representable in G* if there is a sequence x in G such that $\mathcal{J} = \mathcal{I}_x^G$.

We found two general results: Given an ideal $\mathcal{J}$ on ω

Theorem I. $\mathcal{J}$ is representable in a Polish Abelian group iff $\mathcal{J}$ is an analytic P-ideal.

Theorem II. $\mathcal{J}$ is representable in a Banach space iff $\mathcal{J}$ is a non-pathological analytic P-ideal.

Explanations:

- $\mathcal{J}$ is *analytic* if it is an analytic subset of $\mathcal{P}(\omega) \simeq 2^\omega$ equipped with the usual product (Polish group) topology.
- $\mathcal{J}$ is a *P-ideal* if for every countable $\{A_n : n \in \omega\} \subseteq \mathcal{J}$ there is an $A \in \mathcal{J}$ such that $A_n \subseteq^* A$ (i.e. $|A_n \setminus A| < \infty$) for every n.
- $\mathcal{J}$ is *tall* if every infinite subset of ω contains an infinite element of $\mathcal{J}$ (we will need it below).

S. Solecki proved that analytic P-ideals are exactly those ideals which can be written of the form

$$\mathrm{Exh}(\varphi) = \left\{A \subseteq \omega : \lim_{n\to\infty} \varphi(A \setminus n) = 0\right\}$$

for some lower semicontinuous (lsc) submeasure φ on ω (in particular, every analytic P-ideal is $F_{\sigma\delta}$). An analytic P-ideal $\mathcal{J}$ is *non-pathological*, if it can be written on the form $\mathrm{Exh}(\varphi)$ for some *non-pathological* lsc submeasure φ on ω, that is, φ is equal to the pointwise supremum of measures dominated by φ.

Let me mention here the definitions of some classical examples of non-pathological analytic P-ideals:

- the *summable ideal*:

$$\mathcal{I}_{1/n} = \left\{A \subseteq \omega : \sum_{n\in A} \frac{1}{n+1} < \infty\right\};$$

- the *density zero ideal*:

$$\mathcal{Z} = \left\{A \subseteq \omega : \frac{|A \cap [2^n, 2^{n+1})|}{2^n} \to 0\right\};$$

- *Farah's ideal*:

$$\mathcal{J}_F = \left\{ A \subseteq \omega : \sum_{n \in A} \frac{\min(n, |A \cap [2^n, 2^{n+1})|)}{n^2} < \infty \right\};$$

- the *trace* of the null ideal:

$$\operatorname{tr}(\mathcal{N}) = \left\{ A \subseteq 2^{<\omega} : \{x \in 2^\omega : \exists^\infty\, n\ x|n \in A\} \text{ is a null set in } 2^\omega \right\}.$$

Theorem II motivated further investigations of interactions between geometry of Banach spaces and combinatorics of ideals:

Question A. (from the combinatorial point of view) Given a Banach space X. Can we characterize those ideals which are representable in X?

Question B. (form the geometric point of view) Given an ideal $\mathcal{J}$. Can we characterize those Banach spaces in which $\mathcal{J}$ is representable?

It has been turned out that these questions are highly non-trivial, even in the case of $X = c_0$ and $X = \ell_1$. At this moment we have the following results.

$X = c_0$: $\mathcal{I}_{1/n}$ and $\mathcal{Z}$ are easily representable in c_0 but neither $\operatorname{tr}(\mathcal{N})$ (because it is *totally bounded* and *summable-like*) nor $\mathcal{J}_F$ (because of the next theorem) are representable in c_0.

Theorem. A tall F_σ P-ideal $\mathcal{J}$ is representable in c_0 iff it is a classical summable ideal, that is, $\mathcal{J} = \left\{ A \subseteq \omega : \sum_{n \in A} r_n < \infty \right\}$ for some sequence $(r_n)_{n \in \omega}$ of positive reals.

$X = \ell_1$: This case seems to be even more difficult than c_0. The only thing we know is that there is a tall F_σ P-ideal (the *Rademacher-ideal*) which is not a classical summable ideal but representable in ℓ_1.

Vinícius Fávaro

Spaceability in Banach and quasi-Banach spaces of vector-valued sequences

We prove new results on the existence of infinite dimensional Banach (or quasi-Banach) spaces formed by vector-valued sequences with special properties.

Given a Banach space X, in [1] the authors introduce a large class of Banach or quasi-Banach spaces formed by X-valued sequences, called *invariant sequences spaces*, which encompasses several classical sequences spaces as particular cases (cf. [1, Example 1.2]). Roughly speaking, the main results of [1] prove that, for every invariant sequence space E of X-valued sequences and every subset Γ of $(0, \infty]$, there exists a closed infinite dimensional subspace of E formed, up to the null vector, by sequences not belonging to $\bigcup_{q\in\Gamma} \ell_q(X)$; as well as a closed infinite dimensional subspace of E formed, up to the null vector, by sequences not belonging to $c_0(X)$. In this work we consider the following much more general situation: given Banach spaces X and Y, a map $f\colon X \longrightarrow Y$, a set $\Gamma \subseteq (0,+\infty]$ and an invariant sequence space E of X-valued sequences, we investigate the existence of closed infinite dimensional subspaces of E formed, up to the origin, by sequences $(x_j)_{j=1}^\infty \in E$ such that either

$$(f(x_j))_{j=1}^\infty \notin \bigcup_{q\in\Gamma} \ell_q(Y) \text{ or } (f(x_j))_{j=1}^\infty \notin \bigcup_{q\in\Gamma} \ell_q^w(Y) \text{ or } (f(x_j))_{j=1}^\infty \notin c_0(Y).$$

For example, given an unbounded linear operator $u\colon X \longrightarrow Y$, we prove the existence of an infinite dimensional Banach space formed, up to the null vector, by null sequences (absolutely p-summable sequences, respectively) in X whose images under u are non-null (non-weakly p-summable, respectively) in Y.

This is a joint work with G. Botelho.

References

[1] G. Botelho, D. Diniz, V. V. Fávaro and D. Pellegrino, *Spaceability in Banach and quasi-Banach sequence spaces*, Linear Algebra Appl. **434** (2011), 1255–1260.

Manuel González

Twisted sums of Banach spaces generated by complex interpolation

We study some non-trivial twisted sums $X\oplus_\Omega X$ of a Banach space X obtained by complex interpolation, where Ω is the centralizer arising from the interpolation schema.

When $X = \ell_2$, we obtain new examples of *singular* (the quotient map from $X \oplus_\Omega X$ onto X is strictly singular) twisted Hilbert spaces, like the Kalton-Peck space Z_2.

When $X = \mathcal{F}_1$, Ferenczi's uniformly convex H.I. space, our methods produce the first example of an H.I. twisted sum $\mathcal{F}_2$ of an H.I. space. Moreover, using Rochberg's method of iterating twisted sums, we show the existence of a sequence $\mathcal{F}_n$ of H.I. spaces so that $\mathcal{F}_{m+n}$ is a singular twisted sum of $\mathcal{F}_m$ and $\mathcal{F}_n$, and for $n > l$ the space $\mathcal{F}_l \oplus \mathcal{F}_{m+n}$ is a nontrivial twisted sum of $\mathcal{F}_{l+m}$ and $\mathcal{F}_n$.

Joint work with Jesus M. F. Castillo and Valentin Ferenczi.

Pedro L. Kaufmann

Products of free spaces and applications

In recent years, much attention has been dedicated to the so-called *free spaces* over a metric space. These Banach spaces are natural isometric preduals to spaces of Lipschitz functions and encode important geometric properties of the original metric space, in particular concerning optimal transport. Despite of their simple definition, many basic questions on free spaces remain unanswered. In this exposition, we show that the free space over a Banach space X, denoted by $\mathcal{F}(X)$, is isomorphic to the ℓ_1-sum of countable copies of $\mathcal{F}(X)$. As applications, we deduce a non-linear version of Pełczyński's decomposition method for free spaces and identify the free space over any n-dimensional compact riemannian manifold with $\mathcal{F}(\mathbb{R}^n)$, up to isomorphism.

Niels Laustsen

Maximal left ideals of operators acting on a Banach space

I shall report on joint work with Garth Dales, Tomasz Kania, Tomasz Kochanek and Piotr Koszmider, in which we study the following two questions concerning the finitely-generated, maximal left ideals of the Banach algebra $\mathcal{B}(E)$ of all (bounded, linear) operators acting on an infinite-dimensional Banach space E:

(I) Does $\mathcal{B}(E)$ necessarily contain a maximal left ideal which is not finitely generated?

(II) Is every finitely-generated, maximal left ideal of $\mathcal{B}(E)$ necessarily of the form

$$\{T \in \mathcal{B}(E) : Tx = 0\}$$

for some $x \in E \setminus \{0\}$?

Since the two-sided ideal $\mathcal{F}(E)$ of finite-rank operators is not contained in any of the maximal left ideals considered in (II), a positive answer to Question (II) would imply a positive answer to Question (I). For this reason, it seems natural to also consider the following, formally more specific, variant of Question (II):

(III) Is $\mathcal{F}(E)$ ever contained in a finitely-generated, maximal left ideal of $\mathcal{B}(E)$?

However, Questions (II) and (III) turn out to be equivalent (in the sense that (II) has a positive answer if and only if (III) has a negative answer). We answer Question (I) in the positive for a large number of Banach spaces, and conjecture that the answer is always positive. In contrast, we show that Question (II) has a positive answer for some, but not all, Banach spaces.

Brice Mbombo

Test spaces for topologically amenable groups

An action of a countable discrete group G on a compact Haursdorff space X is called *topologically amenable* if there exist a weak*-continuous maps b^n from X to the space $\mathcal{P}(G)$, of probability measures on G such that $\lim_{n\longrightarrow\infty} \sup_{x\in X}\|gb_x^n - b_{gx}^n\|_1 = 0$ for all $g \in G$.A countable discrete group G is topologically amenable if it admits a topologically amenable action on a compact space.
Inspired by two characterizations of amenable groups due to Bogatyi and Ferdorchuk (A countable group G is amenable if and only if every continuous action of G on the Hilbert cube $I^{\aleph_0}$ admits an invariant probability measure) and Giordano and De la harpe (A countable group G is amenable if and only if every continuous action of G on the Cantor set $D^{\aleph_0}$ admits an invariant probability measure) we give two new characterizations of topologically amenable groups: A countable group G is topologically amenable if and only if it admits an amenable action on the Hilbert cube $I^{\aleph_0}$ or on the Cantor set $D^{\aleph_0}$.

This work is Joint work with Vladimir Pestov and Yousef Al-Gadid.

Pavlos Motakis

The stabilized set of p's in Krivine's theorem can be disconnected

J. L. Krivine's theorem states that for every Banach space X with a basis, there exists a $p \in [1, \infty]$ such that ℓ_p is finitely block represented in X. The set of all such p's is called the Krivine set of X. As it was proved by H.P. Rosenthal, this set is stabilized on some block subspace Y of X, i.e. the Krivine set of Y and the corresponding one of any of its further block subspaces coincide. The form of such a stabilized Krivine set has been a subject of study, since Rosenthal asked whether it always had to be a singleton. This question was answered negatively by E. Odell and Th. Schlumprecht by constructing a space having $[1, \infty]$ as its stabilized Krivine set. The question that followed was if such a stabilized Krivine set had to be an interval, which was asked by P. Habala and N. Tomczak-Jaegermann as well as by E. Odell. We answer this question in the negative direction by constructing, for every $F \subseteq [1, \infty]$ which is either finite or consists of an increasing sequence and its limit, a reflexive Banach space X with an unconditional basis such that for every infinite dimensional block subspace Y of X, the Krivine set of Y is precisely F. This construction also addresses some open problems concerning spreading models.

This is a joint work with K. Beanland and D. Freeman.

Przemysław Ohrysko

On spectra of measures

In this talk I would like to present the most important topics from the article 'On the relationships between Fourier - Stieltjes coefficients and spectra of measures' (joint work with Michal Wojciechowski) which will be published soon in Studia Mathematica. This work concentrates on problems related to Banach algebra $M(T)$ (convolution algebra of measures on circle group) and especially on spectra of measures. Despite from the fact that they can be very complicated in general, we have proved that it is possible to force measures to have 'natural' spectra (equal to the closure of the range of its Fourier - Stieltjes transform) in a way which uses only whole range of the Fourier - Stieltjes transform without any information on the distribution.

Daniel Pellegrino

On the real polynomial Bohnenblust–Hille inequality

It was recently proved by Bayart et al. that the complex polynomial Bohnenblust–Hille inequality is subexponential. We show that, for real scalars, this does no longer hold. Moreover, we show that, if $D_{\mathbb{R},m}$ stands for the real Bohnenblust–Hille constant for m-homogeneous polynomials, then $\limsup_m D_{\mathbb{R},m}^{1/m} = 2$.

Joint work with J. Campos, P. Jimenez, G. Munoz and J. Seoane.

Antonin Prochazka

Low distortion embeddings into Asplund Banach spaces

We give a simple example of a countable metric space that does not embed bi-Lipschitz with distortion strictly less than 2 into any Asplund space. We also show that for each ordinal α, the space $C([0, \omega^\alpha \cdot 4])$ does not embed to $C_0([0, \omega^\alpha])$ with distortion strictly less than 2. Joint work with Luis Sánchez González.

Luis Sanchez-Gonzalez

Hamilton-Jacobi equations on Finsler manifolds

We study the subdifferentiability on Banach-Finsler manifolds as well as the study of a certain class of Hamilton-Jacobi equations defined on this context. In particular, we prove that the Eikonal equation has a unique viscosity solution on Banach-Finsler manifolds.

Joint work with J. Jaramillo, M. Jimenez-Sevilla and J.L. Rodenas.

Bunyamin Sari

Uniform classification of classical Banach spaces

Many of the most obvious questions regarding the uniform classification of the classical Banach spaces are open. We will give a selective overview of those. Our focus will the scope and limitations of the techniques developed over the years and we will offer some new insights.

Richard Smith

Boundaries and isomorphic polyhedrality

We present a new series of equivalent conditions for a separable Banach space to be isomorphically polyhedral. We give a sufficient linear topological condition on the boundary of a general Banach space to be isomorphically polyhedral, and present a result which suggests that the same condition may also be necessary in WLD spaces.

Part of this work is joint with Vladimir Fonf and Stanimir Troyanski.

Jarno Talponen

Extracting bimonotone basic sequences from long weakly null sequences

This talk involves the following problem. Given a long weakly null normalized sequence of vectors in a Banach space, when can one find a long subsequence which is a bimonotone basic sequence? In fact, we will consider finding a 'block' sequence in the above question. Some geometric technical tools to address the above problem are also discussed. This in an on-going work.

Pedro Tradacete

Disjoint sequences in Banach lattices

We will discuss several problems concerning sequences of disjoint vectors in Banach lattices. In particular, we will consider disjointly homogeneous Banach lattices (those in which any two sequences of disjoint vectors share an equivalent subsequence) and the problem of complementability of the span of a disjoint sequence.

Despoina Ioanna Zisimopoulou

A hierarchy of separable commutative Calkin Algebras

For every $\mathcal{T}$ countably branching well founded tree with a unique root, we construct a $\mathcal{L}_\infty$ space $X_\mathcal{T}$. The construction uses a recursive iteration rooted to the Argyros-Haydon method for constructing Bourgain-Delbaen spaces.

For each tree $\mathcal{T}$ as above, the main property of the space $X_\mathcal{T}$ is that for every $s \in \mathcal{T}$ there exists a norm one projection I_s and each bounded operator T on $X_\mathcal{T}$ is approximated by a linear combination of $(I_s)_{s\in\mathcal{T}}$ plus a compact operator. We denote by $\mathcal{C}al(X_\mathcal{T})$ the Calkin Algebra of $X_\mathcal{T}$, i.e. the space $\mathcal{L}(X_\mathcal{T})/\mathcal{K}(X_\mathcal{T})$ and by $C(\mathcal{T})$ the algebra of continuous functions on the tree $\mathcal{T}$. It is shown that there exists a 1-1 linear homomorphism $\Phi_\mathcal{T} : \mathcal{C}al(X_\mathcal{T}) \to C(\mathcal{T})$ with dense range and $\|\Phi_\mathcal{T}\| = 1$. We conclude that the spaces $\mathcal{C}al(X_\mathcal{T})$ are separable, have the Dunford-Pettis property and their duals are separable with the Schur property.

As an application we obtain that for every countable compact metric space K with finite Cantor-Bendixson index, there exists a Banach space X such that its Calkin Algebra is homomorphic to the algebra $C(K)$.

This is a joint work with Pavlos Motakis and Daniele Puglisi.

Andras Zsak

Infinitely many closed ideals in the algebra of operators on $\ell_p \oplus \ell_q$ in the range $1 < p < q < \infty$.

We show that the algebra $\mathcal{L}(\ell_p \oplus \ell_q)$ of all bounded linear operators on $\ell_p \oplus \ell_q$ has infinitely many closed ideals. This solves a problem raised by A. Pietsch in the 70's. This is joint work with Th. Schlumprecht.

Posters

Victor Juan Hernandez Del Toro

Extrapolation in Banach space

In this work we present some results of Nigel Kalton in the Extrapolation Theory of Banach Spaces. We shall give ideas of the proof of the following theorem:

If X is a p-convex and q-concave Köthe space, where $1 < p < 2$ and $1/p + 1/q = 1$, then there are Köthe spaces X_0 and X_1 such that $X = (X_0, X_1)_{1/2}$, where $(X_0, X_1)_{1/2}$ is a Calderón Interpolation Space.

José Augusto Molina Garay

Towards an index theory for stratified manifolds

In this talk we concisely describe the role of tangent groupoids of compact manifolds in index theory. Our approach allows us to handle certain manifolds with boundary and may be extended to some stratified manifolds.

Joint work with A.R. da Silva

Heiki Niglas

Lipschitz functions and M-ideals

In [BW], D. Werner and H. Berninger tried to answer the following question.

Berninger–Werner problem. *Is the little Hölder space* $\mathrm{lip}([0,1]^\alpha)$ *an M-ideal in the Hölder space* $\mathrm{Lip}([0,1]^\alpha)$ *for every* $\alpha \in (0,1)$*?*

They showed that $\mathrm{lip}([0,1]^\alpha)$ is an M-ideal in a non-separable subspace of $\mathrm{Lip}([0,1]^\alpha)$, whilst they conjectured that the answer to the problem might be negative.

In [K2, Theorem 6.6], Nigel J. Kalton presented proof to the following theorem.

Theorem. *For a compact metric space M and every $\epsilon > 0$, the little Lipschitz space* $\mathrm{lip}(M)$ *is* $(1+\epsilon)$*-isomorphic to a subspace of* c_0.

After the proof Kalton remarked: "In particular this implies that $\mathrm{lip}(M)$ is an M-ideal in $\mathrm{Lip}(M)$ when M is compact and $\mathrm{lip}(M)$ is a predual of $\mathcal{F}(M)$. See [BW] for the case when $M = [0,1]$."

As it is well-known that $\mathrm{lip}(M^\alpha)$ is a canonical predual of $\mathcal{F}(M^\alpha)$ for a compact metric space M, Kalton solved Berninger–Werner problem in full generality, which means that the following holds: If M is a compact metric space, then the little Hölder space $\mathrm{lip}(M^\alpha)$ is an M-ideal in the Hölder space $\mathrm{Lip}(M^\alpha)$.

We show how to use Kalton's theorem to prove some results concerning properties of the spaces $\mathrm{lip}(M)$, $\mathcal{K}(\mathrm{lip}(M))$, and $\mathcal{L}(\mathrm{lip}(M))$ for a compact metric space M. We also present some further applications.

The research was partially supported by Estonian Science Foundation Grant 8976 and Estonian Targeted Financing Project SF0180039s08.

This is a joint work with Eve Oja and Indrek Zolk.

References

[BW] Heiko Berninger, Dirk Werner, *Lipschitz spaces and M-ideals*, Extracta Math. **18** (2003), 33–56.

[K2] Nigel J. Kalton, *Spaces of Lipschitz and Hölder functions and their applications*, Collect. Math. **55**, 2 (2004), 171–217.

Ksenia Niglas

On (a, B, c)-ideals in Banach Spaces

We say that a closed subspace Y of a Banach space X is an *ideal satisfying the* $M(a, B, c)$-*inequality* (in short, an $M(a, B, c)$-*ideal*) in X if there is a norm one projection P on X^* such that $\ker P = Y^\perp$ and

$$\|ax^* + bPx^*\| + c\|Px^*\| \le \|x^*\| \quad \forall b \in B, \forall x^* \in X^*.$$

This approach was first suggested by E. Oja and it allows us to handle well-known special cases of ideals, namely M-, h-, u- and $M(r, s)$-ideals (for definitions and references, see, e.g., [3]), in a more unified way.

We have developed easily verifiable equivalent conditions for a subspace of ℓ^2_∞ to be an $M(a, B, c)$-ideal.

Following what was done in [1] for $M(r, s)$-ideals, we obtain new results in a more general $M(a, B, c)$-setting. Our main results are as follows. Suppose X and Y are closed subspaces of a Banach space Z such that $X \subset Y \subset Z$. If X is an $M(a, B, c)$-ideal in Y and Y is an $M(d, E, f)$-ideal in Z, then X is an ideal satisfying a certain type of inequality in Z. Relying on this result, we show that if X is an $M(a, B, c)$-ideal in its second bidual, then X is an ideal satisfying a certain type of inequality in $X^{(2n)}$ for every $n \in \mathbb{N}$.

For illustration, we list here two corollaries of our results.

- *If X is an $M(a,B,c)$-ideal in Y and Y is an M-ideal in Z, then X is an $M(a,B,c)$-ideal in Z.*

- *If X is a u-ideal in X^{**}, then X is an $M\left(\frac{1}{2n-1},\left\{-\frac{2}{2n-1}\right\},0\right)$-ideal in $X^{(2n)}$ for every $n\in\mathbb{N}$.*

The talk is based on [2].

References

[1] R. Haller, *On transitivity of $M(r,s)$-inequalities and geometry of higher duals of Banach spaces*, Acta Comment. Univ. Tartu. Math. **6** (2002), 9–13.

[2] K. Niglas, I. Zolk, *On (a,B,c)-ideals in Banach spaces*, Acta Comment. Univ. Tartu. Math. **18** (2014), to appear.

[3] E. Oja, *Geometry of Banach spaces having shrinking approximations of the identity*, Trans. Amer. Math. Soc. **352** (2000), 2801–2823.

Daniel Nuñez Alarcón

On the coordinatewise multiple summing operators

We use standard notations and notions from multiple summing operators theory.

Definition 0.0.1. *Let $1\leq r<\infty$ and $D\subset\{1,\dots,m\}$ non-void and proper. We say that $U\in\mathcal{L}(X_1,\dots,X_m;Y)$ is* multiple $(r,1)$-summing in the coordinates of D *(or* multiple $(r,1)$-summing in D*) whenever U^D has its range in $\Pi^{|D|}_{(r,1)}\left(X^D;Y\right)$.*

Let us consider the functions $w_n:[1,q)^n\to[0,+\infty)$ and $f_n^k:=f_n^k(r_1,\dots,r_n):[1,q)^n\to[0,+\infty)^n$, defined by

$$\omega_n:=\omega_n(r_1,\dots,r_n)=\frac{qR}{1+R},$$

and

$$f_n^k:=f_n^k(r_1,\dots,r_n)=\frac{r_k}{R(q-r_k)},$$

for $k=1,\dots,n$, with $R=\sum_{k=1}^n\frac{r_k}{q-r_k}$.

The main result of [3, Theorem 5.1], is the following vector valued Bohnenblust–Hille type theorem:

Theorem 0.0.2. *Let $\{1,\ldots,m\}$ be the disjoint union of non-void proper coordinates subsets $C_1,\ldots,C_n$, Y be a Banach space with cotype q, and $1 \leq r_1,\ldots,r_n < q$. Assume that $U \in \mathcal{L}(X_1,\ldots,X_m;Y)$ is multiple $(r_k,1)$-summing in each coordinates subset C_k, $2 \leq k \leq n$. Then U is multiple $(\omega_n,1)$-summing, and*

$$\pi^m_{(\omega_n,1)}(U) \leq \sigma_n \prod_{k=1}^{n} \left\| U^{C_k} : X^{\widehat{C_k}} \to \Pi^m_{r_k,1}\left(X^{C_k};Y\right)\right\|^{f^k_n},$$

where σ_n only depends on n, $|C_1|,...,|C_n|$, $r_1,...,r_n$, q and $C_q(Y)$.

We use an interpolative technique from [1], our approach recover some recent results; for instance we recover the Theorem 0.0.2.

This work is contained in [2].

References

[1] N. Albuquerque, F. Bayart, D. Pellegrino and J. B. Seoane-Sepúlveda, *Sharp generalizations of the multilinear Bohnenblust–Hille inequality*, Journal of Functional Analysis, **266** (2014), no.6, p. 3726-3740.

[2] N. Albuquerque, D. Núñez-Alárcon, J. Santos and D. M. Serrano-Rodríguez *Absolutely summing multilinear operators via interpolation*, arXiv:1404.4949v1 [math.FA], 22Apr2014.

[3] A. Defant, D. Popa and U. Schwarting, *Coordinatewise multiple summing operators in Banach spaces*, Journal of Functional Analysis. **259** (2010), no.1, 220-242.

Diana Marcela Serrano Rodríguez

Multiple N-separately summing operators

Let $\mathbb{K}$ be the real or complex scalar field. The multilinear Bohnenblust–Hille inequality asserts that for every positive integer $n \geq 1$ there exists a sequence of positive scalars $(C_n)_{n=1}^{\infty}$ in $[1,\infty)$ such that

$$\left(\sum_{i_1,\ldots,i_n=1}^{N} \left|U(e_{i_1},\ldots,e_{i_n})\right|^{\frac{2n}{n+1}}\right)^{\frac{n+1}{2n}} \leq C_n \sup_{z_1,\ldots,z_n \in \mathbb{D}^N} |U(z_1,\ldots,z_n)|$$

for all n-linear forms $U : \mathbb{K}^N \times \cdots \times \mathbb{K}^N \to \mathbb{K}$ and every positive integer N, where $(e_i)_{i=1}^N$ denotes the canonical basis of $\mathbb{K}^N$ and $\mathbb{D}^N$ represents the open unit polydisc in $\mathbb{K}^N$.

The best known estimates of the multilinear Bohnenblust-Hille constants, was recently presented in [3, Proposition 3.1].

We use an interpolative technique from [1] to introduce the notion of *multiple N-separately summing operators*. Our approach extends and unifies some recent results; for instance we recover the best known estimates of the multilinear Bohnenblust-Hille constants from F. Bayart, D. Pellegrino and J. Seoane-Sepúlveda.

This work is contained in [2].

References

[1] N. Albuquerque, F. Bayart, D. Pellegrino and J. B. Seoane-Sepúlveda, *Sharp generalizations of the multilinear Bohnenblust–Hille inequality*, Journal of Functional Analysis, **266** (2014), no.6, p. 3726-3740.

[2] N. Albuquerque, D. Núñez-Alárcon, J. Santos and D. M. Serrano-Rodríguez *Absolutely summing multilinear operators via interpolation*, arXiv:1404.4949v1 [math.FA], 22Apr2014.

[3] F. Bayart, D. Pellegrino and J. B. Seoane-Sepúlveda, *The Bohr Radius of the n-dimensional polydisk is equivalent to $\sqrt{(\log n)/n}$*, arXiv:1310.2834v2 [math.FA], 15Oct2013.

List of Participants

Razvan Anisca
Lakehead University
ranisca@lakeheadu.ca

Spiros Argyros
National Technical University of Athens
sargyros@math.ntua.gr

Antonio Avilés
University of Murcia
avileslo@um.es

André Barbeiro
Universidade de São Paulo
andre.santoleri@gmail.com

Cleon Barroso
Universidade Federal do Ceará
cleonbar@mat.ufc.br

Dana Bartosova
Universidade de São Paulo
dana@ime.usp.br

Florent Baudier
IMJ-PRG and Texas A&M University
florent@math.tamu.edu

Kevin Beanland
Washington and Lee University
beanlandk@wlu.edu

Bruno Braga
University of Illinois at Chicago
demendoncabraga@gmail.com

Christina Brech
Universidade de São Paulo
brech@ime.usp.br

Jamilson Campos
Universidade Federal da Paraíba
jamilson@dce.ufpb.br

Alejandro Chavez-Dominguez
University of Texas at Austin
jachavezd@math.utexas.edu

William Corrêa
Universidade de São Paulo
will.hans.correa@gmail.com

Wilson Cuellar Carrera
Universidade de São Paulo
cuellar@ime.usp.br

Antonio Roberto da Silva
Universidade Federal do Rio de Janeiro
ardasilva@icloud.com

Robert Deville
Université de Bordeaux
Robert.Deville@math.u-bordeaux1.fr

Pandelis Dodos
University of Athens
pdodos@math.uoa.gr

Barnabas Farkas
Kurt Godel Research Center, University of Vienna
barnabasfarkas@gmail.com

Vinícius Fávaro
Universidade Federal de Uberlândia
vvfavaro@gmail.com

Valentin Ferenczi
Universidade de São Paulo
ferenczi@ime.usp.br

Michel Gaspar
Universidade de São Paulo
mgaspar@ime.usp.br

Gilles Godefroy
Centre National de la Recherche Scientifique
gilles.godefroy@imj-prg.fr

Manuel González
Universidad de Cantabria
manuel.gonzalez@unican.es

Petr Hajek
Czech Academy of Science and Czech Polytechnical University
hajek@math.cas.cz

Victor Juan Hernández Del Toro
Universidade de São Paulo
vjhernandez20@hotmail.com

Pedro Kaufmann
Universidade Federal de São Paulo
plkaufmann@gmail.com

Piotr Koszmider
Institute of Mathematics of the Polish Academy of Sciences
piotr.koszmider@impan.pl

Niels Laustsen
Lancaster University, UK
n.laustsen@lancaster.ac.uk

Jordi Lopez-Abad
ICMAT & Universidade de São Paulo
abad@icmat.es

Brice Mbombo
Universidade de São Paulo
bricero@yahoo.fr

José Augusto Molina Garay
Universidade Federal do Rio de Janeiro
molina@imca.edu.pe

Pavlos Motakis
National Technical University of Athens
pmotakis@central.ntua.gr

Heiki Niglas
University of Tartu
heiki.niglas@gmail.com

Ksenia Niglas
University of Tartu
ksenia.niglas@gmail.com

Daniel Núñez Alarcón
Universidade Federal de Pernambuco
danielnunezal@gmail.com

Przemysław Ohrysko
Institute of Mathematics of the Polish Academy of Sciences
p.ohrysko@gmail.com

Daniel Pellegrino
Universidade Federal da Paraíba
dmpellegrino@gmail.com

Vladimir Pestov
University of Ottawa and Universidade Federal de Santa Catarina
vpest283@uottawa.ca

Antonin Prochazka
Université de Franche-Comté
antonin.prochazka@univ-fcomte.fr

Michael Rincón
Universidade de São Paulo
michaelr@ime.usp.br

Christian Rosendal
University of Illinois at Chicago
rosendal.math@gmail.com

Luis Sanchez-Gonzalez
Universidad de Concepcion
lsanchez@ing-mat.udec.cl

Bunyamin Sari
University of North Texas
bunyamin@unt.edu

Guideon Schechtman
The Weizmann Institute
gideon@weizmann.ac.il

Thomas Schlumprecht
Texas A&M University
schlump@math.tamu.edu

Omar Selim
Universidade de São Paulo
oselim.mth@gmail.com

Diana Marcela Serrano Rodríguez
Universidade Federal de Pernambuco
dmserrano0@gmail.com

Richard Smith
University College Dublin
richard.smith@maths.ucd.ie

Clayton Suguio Hida
Universidade de São Paulo
clayton.hida@gmail.com

Jarno Talponen
University of Eastern Finland
talponen@iki.fi

Stevo Todorcevic
CNRS Paris and University of Toronto
stevo@math.toronto.edu

Pedro Tradacete
Universidad Carlos III Madrid
ptradace@math.uc3m.es

Despoina Ioanna Zisimopoulou
National Technical University of Athens
dzisimopoulou@hotmail.com

Andras Zsak
University of Cambridge
a.zsak@dpmms.cam.ac.uk