Finite and infinite invariant measures for adic transformations

Alby Fisher, University of Sao Paulo
Seminar Ernst, Universite Aix-Marseilles, Luminy
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We will describe a new paper with Marina Talet of Aix-Marseille University, in which we classify the invariant Borel measures for adic transformations.

We restrict to finite rank (bounded alphabet size) and we assume the measure is finite on the path space of some sub-Bratteli diagram defined by erasing vertices or edges.

Vershik's adic transformations generalize the classical Kakutani-von Neumann odometer map, but go far beyond that example as they can be used to model for example irrational circle rotations, interval exchange transformations, substitution dynamical systems, and cutting-and-stacking constructions.

The first generalization beyond the odometer is to a primitive nonnegative integer matrix M which defines a subshift of finite type, with the adic transformationacting transversally to the shift map. In contrast to the shift map, these are uniquely ergodic, minimal and of zero entropy.

Things get much more interesting when one allows for a matrix sequence (a nonstationary adic transformation), for the nonprimitive case, and for infinite measures.

To classify these maps we extend the approach of Bezuglyi, Kwiatkowski, Medynets and Solomyak for the nonprimitive stationary case to the nonprimitive nonstationary case, allowing   for measures which may be locally infinite. We give a necessary and sufficient condition for the measure to be infinite. To carry this out we prove a nonstationary version of the Frobenius-Victory Theorem.

As an application of our general result, we introduce nested irrational rotations, giving a checkable condition for the tower measure to be infinite.