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Anosov families, renormalization and nonstationary subshifts.

ABSTRACT.

Abstract. We introduce the notion of an Anosov family, a generalization of an Anosov map of a manifold. This is a sequence of diffeomorphisms along compact Riemannian manifolds such that the tangent bundles split into expanding and contracting subspaces. We develop the general theory, studying sequences of maps up to a notion of isomorphism and with respect to an equivalence relation generated by two natural operations, gathering and dispersal. Then we concentrate on linear Anosov families on the two-torus. We study in detail a basic class of examples, the multiplicative families, and a canonical dispersal of these, the additive families. These form a natural completion to the collection of all linear Anosov maps. A renormalization procedure constructs a sequence of Markov partitions consisting of two rectangles for a given additive family. This codes the family by the nonstationary subshift of finite type determined by exactly the same sequence of matrices. Any linear positive Anosov family on the torus has a dispersal which is an additive family. The additive coding then yields a combinatorial model for the linear family, by telescoping the additive Bratteli diagram. The resulting combinatorial space is then determined by the same sequence of nonnegative matrices, as a nonstationary edge shift. This generalizes and provides a new proof for theorems of Adler and Manning.