Koszul in São Paulo: A Historical View

Claudio Gorodski University of São Paulo

2nd Workshop SPJM: Koszul in São Paulo, His Work and Legacy 13-14 November 2019

글 > : < 글 >

Education

• Jean-Louis André Stanislas Koszul was born in Strasbourg on 3 January 1921.

< 注) < 注)

Ξ.

Education

- Jean-Louis André Stanislas Koszul was born in Strasbourg on 3 January 1921.
- He entered ENS Ulm in 1940 and defended his thesis with H. Cartan^{1B}.

★ Ξ → < Ξ → </p>

Education

- Jean-Louis André Stanislas Koszul was born in Strasbourg on 3 January 1921.
- He entered ENS Ulm in 1940 and defended his thesis with H. Cartan^{1B}.



ENS ULM, CLASS OF 1940 -> (=> (=> (=>) (=>) (=>) (Claudio Gorodski Koszul in São Paulo: A Historical View

PhD thesis



 PhD thesis: "Homology and cohomology of Lie algebras", defended on 1949.

イロン イ団 とくほ とくほとう

3

Koszul's Thèse d'État

PhD thesis



Koszul's Thèse d'État

- PhD thesis: "Homology and cohomology of Lie algebras", defended on 1949.
- According to H. Cartan (1987): "Why did he turn to me as his 'guide' (so to speak)? Is it because he found inspiration in Élie Cartan's work on the topology of Lie groups? Perhaps he was surprised to note that mathematical knowledge is not necessarily transmitted by descent. In any case, he helped me to better know what my father had brought to the theory."

▲ ●
 ●

- (E) (E)

• As early as 1947, Koszul published three notes in CRAS.

- 4 注 > - 4 注 > - -

A ►

= 990

217

M. le Secrétraine perpérent signale, parmi les pièces imprimées de la Correspondance :

1º JOSEPH DE TONI. Diagnoses algarum novarum post sylloges editionem descriptarum. I. Myxophyceæ. Centuria IX,

2º BRUNO BÖTTCHER et ARTHUR LÜTTRINGHAUS. Über Trithione. I.

3º Institut de médecine vétérinaire exotique. Revue d'élevage et de médecine vétérinaire des pays tropicaux. Tome I (Nouvelle série), nº 1.

CORRESPONDANCE.

ALGRBRE. - Sur les opérateurs de dérivation dans un anneau. Note (1) de M. JEAN-LOUIS KOSZUL, présentée par M. Élie Cartan.

La structure algébrique de l'anneau d'homologie d'une représentation, défini et étudié par M. J. Leray (³), peut être rattachée à l'étude de l'homologie définie par certains opérateurs de dérivation dans un anneau. Nous donnons ici ce point de rue qui sera utilisé dans une Note ultérieure sur l'homologie des espaces homogénes.

Notations : par opérateur dans un anneau \mathfrak{A} , on entend un opérateur pour sa structure de groupe abélien; T.x désigne le transformé de $x \in \mathfrak{A}$ par l'opérateur T.

 Anneau à dérivation supérieure. — Définition : un anneau à dérivation supérieure est un anneau & où sont définis :

1* une suite d'idéaux bilatères & (p entier) tels que & ⊃ &++, & @ & ⊂ &++,

$$\bigcup \mathscr{B} = \mathfrak{A}, \bigcap \mathscr{B} = \{0\};$$

2° un automorphisme involutif T de l'anneau et tel, que T. $\mathcal{B}^{\mu} = \mathcal{B}^{\mu}$ (on pose T. $x = \overline{x}$);

3º Un opérateur de dérivation D tel que

(D, 1)	$D^{\eta} = O$,
(D, a)	$D.dP \in dP$ (pour tout p),
(D, 3)	$\mathbf{D}.xy = (\mathbf{D}.x)y + \overline{x}(\mathbf{D},y),$
(D, 4)	$DT + TD \equiv O^{-(3)}$.

En général, on disposera dans & d'une notion de degré (*), c'est-à-dire d'une

(1) Séance du 21 juillet 1947.

(*) Comptes rendus, 222, 1946, pp. 1419-1422.

(1) Lorsque & est une algèbre, on parlers d'algèbre à dérivation supérieure si les opérateurs T et D sont en outre linéaires.

(*) M. H. Cartan m'a indiqué les avantages qu'il y avait parfois à ne pas utiliser cette particularité.

- As early as 1947, Koszul published three notes in CRAS.
- One of them was: Sur les opérateurs de dérivation dans un anneau. (French) C. R. Acad. Sci. Paris 225 (1947), 217–219.

イロト イポト イヨト イヨト

217

M. le Secrétraine perpérent signale, parmi les pièces imprimées de la Correspondance :

1º JOSEPH DE TONI. Diagnoses algarum novarum post sylloges editionem descriptarum. I. Myxophycew. Centuria IX.

2º BRUNO BÖTTCHER et ARTHUR LÜTTRINGHAUS. Über Trithione. I.

3° Institut de médecine vétérinaire exotique. Revue d'élevage et de médecine vétérinaire des pays tropicaux. Tome I (Nouvelle série), n° 1.

CORRESPONDANCE.

ALGRBRE. - Sur les opérateurs de dérivation dans un anneau. Note (1) de M. JEAN-LOUIS KOSZUL, présentée par M. Élie Cartan.

La structure algébrique de l'anneau d'homologie d'une représentation, défini et tiudié par M. J. Leray (³), peut être rattachée à l'étude de l'homologie définie par certains opérateurs de dérivation dans un anneau. Nous donnons ici ce point de vue qui sera utilisé dans une Note ultérieure sur l'homologie des espaces homogénes.

Notations : par opérateur dans un anneau \mathfrak{A} , on entend un opérateur pour sa structure de groupe abélien; T.x désigne le transformé de $x \in \mathfrak{A}$ par l'opérateur T.

 Anneau à dérivation supérieure. — Définition : un anneau à dérivation supérieure est un anneau & où sont définis :

1* une suite d'idéaux bilatères & (p entier) tels que & ⊃ &++, & @ & ⊂ &++,

$$\bigcup \mathcal{O} \mathcal{O} = \mathfrak{A}, \bigcup \mathcal{O} \mathcal{O} = \{0\};$$

2° un automorphisme involutif T de l'anneau \mathfrak{C} tel, que T, $\mathfrak{G}^{\mu} = \mathfrak{G}^{\mu}$ (on pose T. $x = \overline{x}$);

3º Un opérateur de dérivation D tel que

(D, 1)	$D^{\eta} = O$,
(D, a)	$D.dP \in dP$ (pour tout p),
(D, 3) ·	$\mathbf{D}.xy = (\mathbf{D}.x)y + \overline{x}(\mathbf{D},y),$
(D, 4)	$DT + TD \equiv O$ (³).

En général, on disposera dans et d'une notion de degré (*), c'est-à-dire d'une

(1) Séance du 21 juillet 1947.

(*) Comptes rendus, 222, 1946, pp. 1419-1422.

(*) Lorsque & est une algèbre, on parlera d'algèbre à dérivation supérieure si les opérateurs T et D sont en outre linéaires.

(*) M. H. Cartan m'a indiqué les avantages qu'il y avait parfois à ne pas utiliser cette particularité.

- As early as 1947, Koszul published three notes in CRAS.
- One of them was: Sur les opérateurs de dérivation dans un anneau. (French) C. R. Acad. Sci. Paris 225 (1947), 217–219.
- According to H. Cartan (1987): "Koszul was the first to give a precise algebraic formalization of the situation studied by Leray in his 1946 publication, which became the theory of spectral sequences. It took a good deal of insight to unravel what lay behind Leray's study. In this respect, Koszul's Note in the July 1947 CRAS is of historical significance."

イロト イポト イヨト イヨト

• Married to Denise Reyss-Brion in 1948 (they had 3 children).

< 注入 < 注入

A 10



Koszul in Strasbourg

- Married to Denise Reyss-Brion in 1948 (they had 3 children).
- Associate Professor at U. Strasbourg in 1949. His colleagues included R. Thom, M. Berger, B. Malgrange.

< ∃⇒



Koszul in Strasbourg

- Married to Denise Reyss-Brion in 1948 (they had 3 children).
- Associate Professor at U. Strasbourg in 1949. His colleagues included R. Thom, M. Berger, B. Malgrange.

• Promoted to Professor in 1956.

 Became a member of second generation of Bourbaki, together with J. Dixmier, R. Godement, S. Eilenberg, P. Samuel, J.-P. Serre and L. Schwartz.

글 > : < 글 >

Koszul in Bourbaki

- Became a member of second generation of Bourbaki, together with J. Dixmier, R. Godement, S. Eilenberg, P. Samuel, J.-P. Serre and L. Schwartz.
- H. Cartan remarked in (1987): "In the vehement discussions within Bourbaki, Koszul was not one of those who spoke loudly; but we learned to listen to him because we knew that if he opened his mouth he had something to say."



Seminaire Bourbaki (1951).



IN A BOURBAKI CONGRESS.

◆□ → ◆□ → ◆三 → ◆三 → ○ へ ⊙

• Group of mathematicians (starting in 1934) that set out to reconstruct the whole edifice of Mathematics on a unified, general and thus very abstract way: new organization of Mathematics, where concepts of structure, axiomatics and unity are essential, in the detriment of analysis of special cases.

- Group of mathematicians (starting in 1934) that set out to reconstruct the whole edifice of Mathematics on a unified, general and thus very abstract way: new organization of Mathematics, where concepts of structure, axiomatics and unity are essential, in the detriment of analysis of special cases.
- 10 books from 1939 to 1960 and one book (Algebraic Topology) in 2016.

- Group of mathematicians (starting in 1934) that set out to reconstruct the whole edifice of Mathematics on a unified, general and thus very abstract way: new organization of Mathematics, where concepts of structure, axiomatics and unity are essential, in the detriment of analysis of special cases.
- 10 books from 1939 to 1960 and one book (Algebraic Topology) in 2016.
- Impact on active research mathematicians was important in post WWII.

글 > : < 글 >

- Group of mathematicians (starting in 1934) that set out to reconstruct the whole edifice of Mathematics on a unified, general and thus very abstract way: new organization of Mathematics, where concepts of structure, axiomatics and unity are essential, in the detriment of analysis of special cases.
- 10 books from 1939 to 1960 and one book (Algebraic Topology) in 2016.
- Impact on active research mathematicians was important in post WWII.
- Influence on Mathematics Education worldwide, supported by national governments, in 1960-1970.

∃ > < ∃ >

- Group of mathematicians (starting in 1934) that set out to reconstruct the whole edifice of Mathematics on a unified, general and thus very abstract way: new organization of Mathematics, where concepts of structure, axiomatics and unity are essential, in the detriment of analysis of special cases.
- 10 books from 1939 to 1960 and one book (Algebraic Topology) in 2016.
- Impact on active research mathematicians was important in post WWII.
- Influence on Mathematics Education worldwide, supported by national governments, in 1960-1970.
- Overemphasis on rigour and high abstraction provoked critics and hostility, and led to a gulf with the way theoretical physics was practiced.

A B > A B >

- Group of mathematicians (starting in 1934) that set out to reconstruct the whole edifice of Mathematics on a unified, general and thus very abstract way: new organization of Mathematics, where concepts of structure, axiomatics and unity are essential, in the detriment of analysis of special cases.
- 10 books from 1939 to 1960 and one book (Algebraic Topology) in 2016.
- Impact on active research mathematicians was important in post WWII.
- Influence on Mathematics Education worldwide, supported by national governments, in 1960-1970.
- Overemphasis on rigour and high abstraction provoked critics and hostility, and led to a gulf with the way theoretical physics was practiced.
- Direct influence has somewhat waned from 1970 onwards for different reasons.

A B > A B >

-

◆□ > ◆□ > ◆豆 > ◆豆 > □ = ○ ○ ○ ○

• In 1939 L. Fantappiè returned to Italy; in 1942 G. Albanese left (Albanese would return in 1946 but he would die in the following year).

< 注) < 注)

 In 1939 L. Fantappiè returned to Italy; in 1942 G. Albanese left (Albanese would return in 1946 but he would die in the following year). In 1944, the faculty of the subsection Mathematics of FFCL is the Cathedratic Omar Catunda and the alumni: Cândido Lima da Silva Dias, Fernando Furquim de Almeida e Benedito Castrucci.

글 > : < 글 >

-

- In 1939 L. Fantappiè returned to Italy; in 1942 G. Albanese left (Albanese would return in 1946 but he would die in the following year). In 1944, the faculty of the subsection Mathematics of FFCL is the Cathedratic Omar Catunda and the alumni: Cândido Lima da Silva Dias, Fernando Furquim de Almeida e Benedito Castrucci.
- Movement to hire faculty from abroad: in 1943 André Dreyfus becomes Director of FFCL, and in 1944 visits the U.S. to bring people.

프 () () () (

-

- In 1939 L. Fantappiè returned to Italy; in 1942 G. Albanese left (Albanese would return in 1946 but he would die in the following year). In 1944, the faculty of the subsection Mathematics of FFCL is the Cathedratic Omar Catunda and the alumni: Cândido Lima da Silva Dias, Fernando Furquim de Almeida e Benedito Castrucci.
- Movement to hire faculty from abroad: in 1943 André Dreyfus becomes Director of FFCL, and in 1944 visits the U.S. to bring people.
- A. Weil^{1B} is hired in January 1945 and stays till end of 1947.

(B) < B)</p>

- In 1939 L. Fantappiè returned to Italy; in 1942 G. Albanese left (Albanese would return in 1946 but he would die in the following year). In 1944, the faculty of the subsection Mathematics of FFCL is the Cathedratic Omar Catunda and the alumni: Cândido Lima da Silva Dias, Fernando Furquim de Almeida e Benedito Castrucci.
- Movement to hire faculty from abroad: in 1943 André Dreyfus becomes Director of FFCL, and in 1944 visits the U.S. to bring people.
- A. Weil^{1B} is hired in January 1945 and stays till end of 1947.
- O. Zariski spends 1945 in São Paulo.

- In 1939 L. Fantappiè returned to Italy; in 1942 G. Albanese left (Albanese would return in 1946 but he would die in the following year). In 1944, the faculty of the subsection Mathematics of FFCL is the Cathedratic Omar Catunda and the alumni: Cândido Lima da Silva Dias, Fernando Furquim de Almeida e Benedito Castrucci.
- Movement to hire faculty from abroad: in 1943 André Dreyfus becomes Director of FFCL, and in 1944 visits the U.S. to bring people.
- A. Weil^{1B} is hired in January 1945 and stays till end of 1947.
- O. Zariski spends 1945 in São Paulo.
- J. Dieudonné^{1B} (May 1946-November 1947) and J. Delsarte^{1B} (1948-1951; several stays) come.

- In 1939 L. Fantappiè returned to Italy; in 1942 G. Albanese left (Albanese would return in 1946 but he would die in the following year). In 1944, the faculty of the subsection Mathematics of FFCL is the Cathedratic Omar Catunda and the alumni: Cândido Lima da Silva Dias, Fernando Furquim de Almeida e Benedito Castrucci.
- Movement to hire faculty from abroad: in 1943 André Dreyfus becomes Director of FFCL, and in 1944 visits the U.S. to bring people.
- A. Weil^{1B} is hired in January 1945 and stays till end of 1947.
- O. Zariski spends 1945 in São Paulo.
- J. Dieudonné^{1B} (May 1946-November 1947) and J. Delsarte^{1B} (1948-1951; several stays) come.
- In 1952: S. Eilenberg^{2B}, L. Schwartz^{2B}, C. Ehresmann.^{1B}

- In 1939 L. Fantappiè returned to Italy; in 1942 G. Albanese left (Albanese would return in 1946 but he would die in the following year). In 1944, the faculty of the subsection Mathematics of FFCL is the Cathedratic Omar Catunda and the alumni: Cândido Lima da Silva Dias, Fernando Furquim de Almeida e Benedito Castrucci.
- Movement to hire faculty from abroad: in 1943 André Dreyfus becomes Director of FFCL, and in 1944 visits the U.S. to bring people.
- A. Weil^{1B} is hired in January 1945 and stays till end of 1947.
- O. Zariski spends 1945 in São Paulo.
- J. Dieudonné^{1B} (May 1946-November 1947) and J. Delsarte^{1B} (1948-1951; several stays) come.
- In 1952: S. Eilenberg^{2B}, L. Schwartz^{2B}, C. Ehresmann.^{1B}
- A. Grothendieck^{3B} (1953-1955).

· (문) · (문) · (문

- In 1939 L. Fantappiè returned to Italy; in 1942 G. Albanese left (Albanese would return in 1946 but he would die in the following year). In 1944, the faculty of the subsection Mathematics of FFCL is the Cathedratic Omar Catunda and the alumni: Cândido Lima da Silva Dias, Fernando Furquim de Almeida e Benedito Castrucci.
- Movement to hire faculty from abroad: in 1943 André Dreyfus becomes Director of FFCL, and in 1944 visits the U.S. to bring people.
- A. Weil^{1B} is hired in January 1945 and stays till end of 1947.
- O. Zariski spends 1945 in São Paulo.
- J. Dieudonné^{1B} (May 1946-November 1947) and J. Delsarte^{1B} (1948-1951; several stays) come.
- In 1952: S. Eilenberg^{2B}, L. Schwartz^{2B}, C. Ehresmann.^{1B}
- A. Grothendieck^{3B} (1953-1955).
- J.-L. Koszul^{2B} (1956, 1958).

Mathematics subsection of FFCL



ESCOLA NORMAL CAETANO DE CAMPOS (1954); 3RD FLOOR HOSTED THE MATHEMATICS SUBSECTION IN 1938-1942 (APPROX.)

(B) < B)</p>

- Rua Alfredo Ellis, 301 Paraíso (1942-1948)
- Avenida Brigadeiro Luís Antônio, 1277 (1948-1949)

() <) <)
 () <)
 () <)
</p>

Mathematics subsection of FFCL



USP Maria Antônia (antigo Liceu Nacional Rio Branco); hosted the Mathematics subsection in 1949-1964 (approx.) (=) = 00

Influence of Bourbaki on FFCL

Claudio Gorodski Koszul in São Paulo: A Historical View

5 990

・ロン ・四 と ・ ヨ と ・ ヨ と …

• Teaching courses, publishing in Brazilian journals.

< 注→ < 注→ -

э.

- Teaching courses, publishing in Brazilian journals.
- Members of Sociedade de Matemática de São Paulo; editorial work.

글 🖌 🔺 글 🕨

- Teaching courses, publishing in Brazilian journals.
- Members of Sociedade de Matemática de São Paulo; editorial work.
- Changes in contents and perspective of research and teaching:

프 () () () (

3

- Teaching courses, publishing in Brazilian journals.
- Members of Sociedade de Matemática de São Paulo; editorial work.
- Changes in contents and perspective of research and teaching:
 - Theses in support of candidature to Cathedratic Professor.
 - Theses in support of candidature to Doctoral degree.
 - Syllabi of courses.

글 🖌 🔺 글 🕨

3

- Teaching courses, publishing in Brazilian journals.
- Members of Sociedade de Matemática de São Paulo; editorial work.
- Changes in contents and perspective of research and teaching:
 - Theses in support of candidature to Cathedratic Professor.
 - Theses in support of candidature to Doctoral degree.
 - Syllabi of courses.
- PIRES, Rute da Cunha. A presença de Nicolas Bourbaki na Universidade de São Paulo (Tese). Doutorado em Educação Matemática, PUC/SP, São Paulo, 2006.

(B) < B)</p>

Koszul in São Paulo, first visit

Claudio Gorodski Koszul in São Paulo: A Historical View

(< E) < E)</p>

A ►

2

 Visits to Brazil seem to be co-organized with IMPA (created in 1952; CNPq and Capes were created in 1951).

글 🖌 🔺 글 🕨

Koszul in São Paulo, first visit



- Visits to Brazil seem to be co-organized with IMPA (created in 1952; CNPq and Capes were created in 1951).
- First visit to USP in 1956; course given in August and September.

A B > A B >

A >

Koszul in São Paulo, first visit



- Visits to Brazil seem to be co-organized with IMPA (created in 1952; CNPq and Capes were created in 1951).
- First visit to USP in 1956; course given in August and September.
- Lecture notes (taken by José de Barros Neto) *Faisceaux et cohomologie*, published in 1957; general treatise on Cech cohomology with coefficients on a sheaf.

• Course given in September and October of 1956.

< 注 → < 注 →

æ

Koszul in São Paulo, first visit



- Course given in September and October of 1956.
- Lecture notes (taken by Chaim Samuel Honig and Carlos Benjamin de Lyra) *Variétés Kahleriennes*, published in 1957.

(日) (同) (三) (三)

Koszul in São Paulo, first visit



- Course given in September and October of 1956.
- Lecture notes (taken by Chaim Samuel Honig and Carlos Benjamin de Lyra) Variétés Kahleriennes, published in 1957.
- Also course on Multilinear Algebra, notes by L. H. Jacy Monteiro, published in 1956.

- ∢ ∩ ¬ >

- (B) (B)

Claudio Gorodski Koszul in São Paulo: A Historical View

★ E ► < E ►</p>

A ►

2

Koszul in São Paulo, second visit



• Course on symmetric spaces given in September and October of 1958; lecture notes published in 1959.

э

Claudio Gorodski Koszul in São Paulo: A Historical View

Koszul in São Paulo, second visit



- Course on symmetric spaces given in September and October of 1958; lecture notes published in 1959.
- According to R. Bott in Math Reviews: "These are notes on symmetric spaces from a seminar conducted by the author in São Paulo during the fall of 1958. The notes are meant for readers who know the rudiments of geometry and the theory of Lie groups, and are very enjoyable. The pace is quick, and considerable material is covered elegantly. Apart from the more or less standard theorems on symmetric spaces, the author discusses the geometry of geodesics, the Bergmann metric, and finally investigates the bounded domains in considerable detail."

Brief chronology of symmetric spaces

• Introduced by É. Cartan in 1925-1930.

글 > : < 글 >

Brief chronology of symmetric spaces

- Introduced by É. Cartan in 1925-1930.
- Theory of Lie groups in book form: Pontrjagyn (1939) and Chevalley (1946).

글 > : < 글 >

Brief chronology of symmetric spaces

- Introduced by É. Cartan in 1925-1930.
- Theory of Lie groups in book form: Pontrjagyn (1939) and Chevalley (1946).
- In 1954-1955, A. Borel visited U. Chicago, where Chern was a professor.

글 > : < 글 >

3

- Introduced by É. Cartan in 1925-1930.
- Theory of Lie groups in book form: Pontrjagyn (1939) and Chevalley (1946).
- In 1954-1955, A. Borel visited U. Chicago, where Chern was a professor. In 1957-1958 R. Palais was an instructor in Chicago, S. Helgason was assistant professor and J. Wolf was a student of Chern. Under leadership of Chern, they decided to run a seminar on symmetric spaces.

- Introduced by É. Cartan in 1925-1930.
- Theory of Lie groups in book form: Pontrjagyn (1939) and Chevalley (1946).
- In 1954-1955, A. Borel visited U. Chicago, where Chern was a professor. In 1957-1958 R. Palais was an instructor in Chicago, S. Helgason was assistant professor and J. Wolf was a student of Chern. Under leadership of Chern, they decided to run a seminar on symmetric spaces.
- Helgason's, Wolf's and Borel's books on symmetric spaces came out of this.

프 () () () (

- Introduced by É. Cartan in 1925-1930.
- Theory of Lie groups in book form: Pontrjagyn (1939) and Chevalley (1946).
- In 1954-1955, A. Borel visited U. Chicago, where Chern was a professor. In 1957-1958 R. Palais was an instructor in Chicago, S. Helgason was assistant professor and J. Wolf was a student of Chern. Under leadership of Chern, they decided to run a seminar on symmetric spaces.
- Helgason's, Wolf's and Borel's books on symmetric spaces came out of this.
- According to P. Cartier (2018): "Depuis le début de l'entreprise Bourbaki, André Weil lui [Koszul] avait assigné comme l'une de ses tâches de récupérer l'héritage d'Élie Cartan sur les espaces riemanniens symétriques et les domaines bornés homogènes."

• Chapter 4 of Koszul's notes on symmetric spaces is about the concept of "groupes de isometries focalement complets" or "variationally complete actions" introduced by Bott-Samelson's seminal paper *Applications of the theory of Morse to symmetric spaces* from 1958.

프 () () () (

-

- Chapter 4 of Koszul's notes on symmetric spaces is about the concept of "groupes de isometries focalement complets" or "variationally complete actions" introduced by Bott-Samelson's seminal paper *Applications of the theory of Morse to symmetric spaces* from 1958.
- Bott-Samelson (1958): isotropy actions of symmetric spaces are variationally complete.

프 () () () (

-

- Chapter 4 of Koszul's notes on symmetric spaces is about the concept of "groupes de isometries focalement complets" or "variationally complete actions" introduced by Bott-Samelson's seminal paper *Applications of the theory of Morse to symmetric spaces* from 1958.
- Bott-Samelson (1958): isotropy actions of symmetric spaces are variationally complete.
- Conlon (1971): hyperpolar actions are variationally complete.

프 > > ㅋ ㅋ >

- Chapter 4 of Koszul's notes on symmetric spaces is about the concept of "groupes de isometries focalement complets" or "variationally complete actions" introduced by Bott-Samelson's seminal paper *Applications of the theory of Morse to symmetric spaces* from 1958.
- Bott-Samelson (1958): isotropy actions of symmetric spaces are variationally complete.
- Conlon (1971): hyperpolar actions are variationally complete.
- Gorodski-Thorbergsson (2002): Variationally complete actions on nonnegatively curved symmetric spaces are hyperpolar.

(B)

- Chapter 4 of Koszul's notes on symmetric spaces is about the concept of "groupes de isometries focalement complets" or "variationally complete actions" introduced by Bott-Samelson's seminal paper *Applications of the theory of Morse to symmetric spaces* from 1958.
- Bott-Samelson (1958): isotropy actions of symmetric spaces are variationally complete.
- Conlon (1971): hyperpolar actions are variationally complete.
- Gorodski-Thorbergsson (2002): Variationally complete actions on nonnegatively curved symmetric spaces are hyperpolar.
- Lytchak-Thorbergsson (2007): Variationally complete actions on nonnegatively curved manifolds are hyperpolar.

A B A A B A



• Koszul became Professor in Grenoble in 1963.

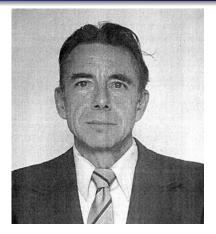
★@> ★ E> ★ E> = E



Alexandre A. M. Rodrigues (1930-2018) in 2016.

- Koszul became Professor in Grenoble in 1963.
- Hosted A. A. M. Rodrigues in Grenoble (1967-1970).

< E > < E >

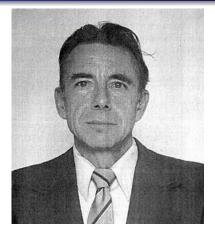


- Koszul became Professor in Grenoble in 1963.
- Hosted A. A. M. Rodrigues in Grenoble (1967-1970).

< 注 → < 注 →

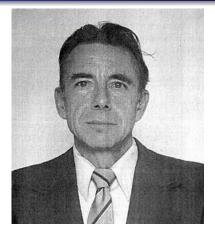
э

• Won Prix Jaffé in 1975.



- Koszul became Professor in Grenoble in 1963.
- Hosted A. A. M. Rodrigues in Grenoble (1967-1970).
- Won Prix Jaffé in 1975.
- President of SMF in 1978 (co-founder of CIRM, Luminy).

글 🖌 🔺 글 🕨



- Koszul became Professor in Grenoble in 1963.
- Hosted A. A. M. Rodrigues in Grenoble (1967-1970).
- Won Prix Jaffé in 1975.
- President of SMF in 1978 (co-founder of CIRM, Luminy).
- Elected to Academia de Ciências do Estado de São Paulo (1981).

- ∢ ≣ →

3 N

-



- Koszul became Professor in Grenoble in 1963.
- Hosted A. A. M. Rodrigues in Grenoble (1967-1970).
- Won Prix Jaffé in 1975.
- President of SMF in 1978 (co-founder of CIRM, Luminy).
- Elected to Academia de Ciências do Estado de São Paulo (1981).
- Visited USP again in 1986: one inaugural talk at Instituto de Estudos Avançados, hosted by A. A. M. Rodrigues: "The genesis of Bourbaki".

A 32 b



- Koszul became Professor in Grenoble in 1963.
- Hosted A. A. M. Rodrigues in Grenoble (1967-1970).
- Won Prix Jaffé in 1975.
- President of SMF in 1978 (co-founder of CIRM, Luminy).
- Elected to Academia de Ciências do Estado de São Paulo (1981).
- Visited USP again in 1986: one inaugural talk at Instituto de Estudos Avançados, hosted by A. A. M. Rodrigues: "The genesis of Bourbaki".

- E - A - E - A

Retirement in 1986.



- Koszul became Professor in Grenoble in 1963.
- Hosted A. A. M. Rodrigues in Grenoble (1967-1970).
- Won Prix Jaffé in 1975.
- President of SMF in 1978 (co-founder of CIRM, Luminy).
- Elected to Academia de Ciências do Estado de São Paulo (1981).
- Visited USP again in 1986: one inaugural talk at Instituto de Estudos Avançados, hosted by A. A. M. Rodrigues: "The genesis of Bourbaki".
- Retirement in 1986.
- Deceased on 12 January 2018.

< ∃ > < ∃ >