USP - UNIVERSIDADE DE SÃO PAULO
30º Simpósio Internacional de Iniciação Científica

Inscrição No.: 339

Título em Português:	Teoria dos Modelos e suas Aplicações
Título em Inglês:	Model Theory and its Applications
Área de Pesquisa:	Geometria e Topologia
Palavras Chave:	ultraproduto - Löweinheim-Skolem - Stone
Ag. Financiadora do Projeto:	CNPq - PIBIC
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	
Validado em:	**05/09/2022**

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Resumo do Trabalho em inglês:

MODEL THEORY AND ITS APPLICATIONS

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Objectives

This work had as its initial goal the study of pre-requisites of First Order Logic and Set Theory in order to start the study of Model Theory. After that, the student studied Model Theory techniques and its applications in Analysis, Algebra and Topology.

Materials and Methods

The reading of books and articles was used as methodology, as well as resolution of exercises and a detailed study of the proofs. There were also one hour weekly meetings to discuss the studied topics and to clarify questions that emerged.

Results

Definition 1. Let $(\mathcal{M}_i)_{i \in I}$ be a family of models for a first order language $\mathcal{L}$ and $\mathcal{U}$ an ultrafilter over I. The **ultraproduct** of $\{\mathcal{M}_i : i \in I\}$ by $\mathcal{U}$, denoted by $\Pi^{\mathcal{U}}_{i \in I}\mathcal{M}_i$, is the model $\mathcal{M}$ such that

- Its universe is $\mathcal{D} = \Pi_{i \in I}\mathcal{D}_i/\sim$, where $\sim$ is the equivalence relation in $\Pi_{i \in I}\mathcal{D}_i$ given by

$$(x_i)_{i \in I} \sim (y_i)_{i \in I} \text{ iff } \{i \in I : x_i = y_i\} \in \mathcal{U}$$

- The n-placed relations R in $\mathcal{M}$ are sets with elements $([(x^1)_i]_{i \in I},\ldots,[(x^n)_i]_{i \in I})$ such that

$$\{i \in I : R_{\mathcal{M}i}(x^1{}_i,\ldots,x^n{}_i)\} \in \mathcal{U}$$

where $R_{\mathcal{M}i}$ are relations of the models $\mathcal{M}_i$;

- The m-placed functions F in $\mathcal{M}$ are defined by

$$F([(x^1)_{i \in I}],\ldots,[(x^m)_{i \in I}]) = [(F_{\mathcal{M}i}(x^1{}_i,\ldots,x^m{}_i))_{i \in I}]$$

where $F_{\mathcal{M}i}$ are functions of the models $\mathcal{M}_i$;

- The constants in $\mathcal{M}$ are $c = [(c_{\mathcal{M}i})_{i \in I}]$, where $c_{\mathcal{M}i}$ are constants of the models $\mathcal{M}_i$.

Definition 2. $\mathcal{A}$ is an **elementary submodel** of $\mathcal{B}$ if $\mathcal{A} \subset \mathcal{B}$ and for all formulas φ of $\mathcal{L}$ in the variables $x_1,\ldots,x_n$, and all $a_1,\ldots,a_n \in \mathcal{A}$, we have

$$\mathcal{A} \vDash \varphi[a_1\ldots a_n] \text{ iff } \mathcal{B} \vDash \varphi[a_1\ldots a_n]$$

In this case we say that $\mathcal{B}$ is an **elementary extension** of $\mathcal{A}$.

One of the main applications of ultraproducts is the construction of the hyperreals, which is an example of elementary extension of the real numbers.

Theorem 3. (Łoś) Let V_i be valuations for $\mathcal{M}_i$ and V a valuation for $\Pi^{\mathcal{U}}_{i \in I}\mathcal{M}_i$ given by $V(x) = [(V_i(x))_{i \in I}]$. Then:
(i) For all formulas φ of $\mathcal{L}$ we have

$$(\mathcal{M},V) \vDash \varphi \text{ iff } \{i \in I : (\mathcal{M}_i,V_i) \vDash \varphi\} \in \mathcal{U}$$

(ii) For all sentences φ of $\mathcal{L}$ we have

$$\mathcal{M} \vDash \varphi \text{ iff } \{i \in I : \mathcal{M}_i \vDash \varphi\} \in \mathcal{U}$$

The classic version of the Löweinheim-Skolem Theorem (also known as Downward Löweinheim-Skolem Theorem) states that every

infinite model has a countable elementary submodel (as long as the language is countable). Using the Łoś Theorem one can prove the Upward Löweinheim-Skolem Theorem that says that every infinite model has arbitrarily large extensions.

Definition 4. A **chain of models** is an increasing sequence of models

$$\mathcal{A}_0 \subset \mathcal{A}_1 \subset \ldots \subset \mathcal{A}_\beta \subset \ldots, \beta < \alpha$$

whose length is an ordinal α. We say that a theory T is **preserved under unions of chains** if the union of any chain of models of T is a model of T.

Theorem 5. A theory T is preserved under unions of chains if, and only if, T has a set of universal-existential axioms.

One application in Algebra is the construction of a countable ordered field K such that, for a fixed natural number n, $x^n - a$ has a root in K, for all a positive rational number.

We take $K_0 = Q$ and a fixed $n \in N$. Now, let $\{a_m : m \in N\}$ be an enumeration of the positive rationals. For each $m \in N$, $m \geq 1$, we define $K_m = K_{m-1}[\sqrt[n]{a_{m-1}}]$. We have then a chain

$$K_0 \subset K_1 \subset \ldots \subset K_n \subset \ldots \subset$$

Defining $K = \cup K_n$ we have that K is countable. Since the theory of ordered fields has a set of universal-existential axioms, we have that K is an ordered field and, by construction, it has the roots of $x^n - a$ for all a positive rational.

Conclusions

Model Theory combines techniques from Logic and Set Theory, resulting in applications in several areas and unifying common arguments in Algebra.

References

[1] Chang, C. C.; Keisler, H. J. *Model Theory*. 3rd. ed. Amsterdam: North-Holland, 1990. (Studies in Logic and the Foundations of Mathematics, 73)

[2] Fajardo, R. *Lógica Matemática.* São Paulo, Edusp, 2017.

[3] Fajardo, R. *Teoria dos Conjuntos*. Material available at http://www.ime.usp.br/fajardo/Conjuntos.pdf.

[1]Scholarship student of PIBIC, a program promoted by CNPq.

Inscrição No.: 358

Título em Português:	A.D.A. - Assistente Distribuída Avançada
Título em Inglês:	A.D.A - Advanced Distributed Assistant
Área de Pesquisa:	Sistemas de Computação
Palavras Chave:	IoT - Aprendizado de Máqui - Assistente Inteligen
Ag. Financiadora do Projeto:	CNPq - PIBITI
Projeto:	Iniciação Tecnológica
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Resumo do Trabalho em inglês:

A.D.A. – Advanced Distributed Assistant

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Objectives

To develop a communication and control layer for intelligent devices to the Assistente Distribuída Avançada (A.D.A.) personal assistant, including everything from the software for embedded devices to the API for accessing the resources. Specific objectives include using low-cost hardware, highly adopted and open-source technologies, as well as testing and simulating the solution automatedly.

Materials and Methods

Espressif ESP32-WROOM-32E microcontrollers, with wireless connections based on the IEEE 802.11 standard, were used as the hardware platform. The *Constrained Application Protocol* (CoAP) was chosen as the communication protocol, with *Concise Binary Object Representation* (CBOR) as the data format. An implementation was built using the star network topology, with a hub unit responsible for receiving the requests, analyzing the validity of the payload, sending them to the respective devices, and reporting the result to the personal assistant.

Concerning the network access point, we have implemented two solutions. The first integrates with an already existing network, suited for home use, and on the other, the hub also becomes an access point, which is appropriate for use in corporate environments. Figure 1 presents a diagram with both solutions.

The *Constrained RESTful Environments* (CoRE) Link Format was adopted to facilitate the discovery of connected devices and their resources. An initial set of predefined commands were proposed and designed. These commands realize canonical functions such as activating and deactivating actuators connected to General Purpose Input/Output (GPIO) pins of the microcontroller, reading the state of those pins, and doing pulse-width modulation (PWM) on the output signal. Each request should contain the unique identifier of the target node device and one of the predefined commands, making the packet data size up to 32 bytes. Packages with custom commands, and eventually larger payload sizes, are also supported.

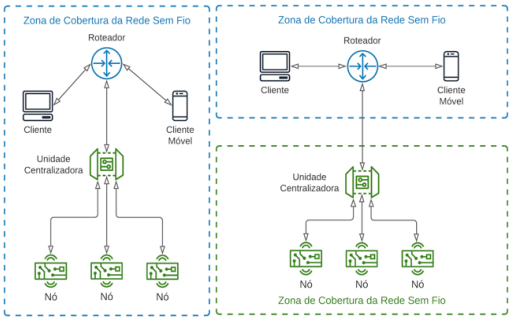

Figure 1: Diagram of network configurations

In addition to the predefined commands aimed at essential functions, an interface for defining new commands also was implemented, enabling the control of peripherals, including through I²C communication, Serial Peripheral Interface (SPI), and UART. Thereby, it's possible to integrate a higher range of

peripherals into the system compared to other solutions.

In addition to the compilation for the microcontroller, we have adapted the application code to compile natively in Linux-based systems on the x86 architecture, with support to virtual network interfaces and adding a module that mocks the inputs and outputs. Therefore, it is possible to test and simulate the application without building a controlled environment.

Results

We have conducted unit tests, automated tests in a simulated environment, and functionality tests in the target use environment. Among the activities performed are the activation and deactivation of an LED connected to the microcontroller, the reading of the LED status, and reading the output of a light sensor. To determine the average round-trip time (RTT) of the proposed solution, including the processing time of the requests, with intervals varying between one second and five minutes, were triggered, and 200 samples were collected. Figure 2 shows the average value for different packet sizes, excluding outlier values.

It is possible to notice that for the predefined commands, which cover a good amount of uses, the average time was below 41 milliseconds, with no noticeable delay between the execution of the request and the activation of the LED, for example.

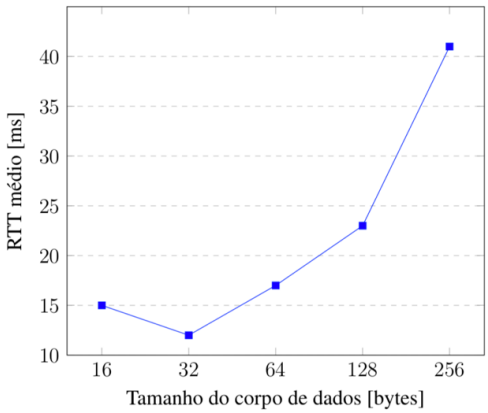

Figure 2: Average round-trip time on the target environment

Conclusions

The studies conducted in this research demonstrated the feasibility and efficiency of a control system for intelligent devices based on technologies that are already widespread, with low cost, and highly customizable. In this regard, the strategy of using a simple and with lower processing capacity hub, compared to that adopted by other solutions, proved to be adequate for managing smaller amounts of smart devices. Furthermore, the tests showed that it is possible to apply this infrastructure in a virtual assistant, where low latency is required.

References

Al-Fuqaha A., Guizani M., Mohammadi M., Aledhari M., and Ayyash M. (2015). Internet of things: A survey on enabling technologies, protocols, and applications. *IEEE Communications Surveys Tutorials,* 17(4):2347–2376.

E. Baccelli, O. Hahm, M. Günes, M. Wählisch and T. C. Schmidt, "RIOT OS: Towards an OS for the Internet of Things," In *IEEE Conference on Computer Communications Workshops (INFOCOM WKSHPS)*, 2013, pp. 79-80.

El-Shweky B. E., El-Kholy K., Abdelghany M., Salah M., Wael M., Alsherbini O.,Ismail Y., Salah K., and AbdelSalam, M. (2018). Internet of things: A comparative study. In *2018 IEEE 8th Annual Computing and Communication Workshop and Conference (CCWC)*, pp. 622–631.

Freire F., Rosa T., Feulo G., Elmadjian C., Cordeiro R., Moura S., Andrade A., de Omena L. A., Vicente A., Marques F., Sheffer A., Hideki O., Nascimento P.,Cordeiro D., and Goldman A. (2020). Toward Development of A.D.A. – Advanced Distributed Assistant. *In Anais do XXI Simpósio em Sistemas Computacionais de Alto Desempenho*, pp. 203–214, Porto Alegre, RS, Brasil. SBC.

Inscrição No.: 395

Título em Português:	Estabilidade Estrutural em Sistemas Dinâmicos
Título em Inglês:	Structural Stability in Dynamic Systems
Área de Pesquisa:	Matemática Aplicada
Palavras Chave:	Sistemas dinâmicos - Caos - Dinâmica de aplicaçõ
Ag. Financiadora do Projeto:	CNPq - PIBIC
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USP - UNIVERSIDADE DE SÃO PAULO
30º Simpósio Internacional de Iniciação Científica

Inscrição No.: 395

Resumo do Trabalho em inglês:

STRUCTURAL STABILITY IN DYNAMIC SYSTEMS

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Objectives

Structural stability is an extremely interesting subject within the studies of Dynamical Systems. Its application can greatly facilitate the analysis of functions with complicated dynamics, in addition to showing us some of the “order” behind chaotic systems.
This topic is, however, very dense, and for that reason it is valid to study examples that show us the process of proving the stability of a system. Thus, the concepts involved become less abstract, and their application in more difficult cases will be facilitated.
Here, we analyze a relatively simple case, but one that well illustrates the process behind this type of analysis.

Materials and Methods

Weekly, we held group meetings with the advisor, where one or a few students presented a topic previously studied by all. I mainly exposed topics from the book An Introduction to Chaotic Dynamical Systems [1], and occasionally from other sources on Dynamical Systems. In the meetings, we developed the topics discussed in more detail, forming a more solid knowledge on the basis necessary for future more complex analyses.
For the presentation in this edition of SIICUSP, an example seen in the meetings and elaborated in one of the topics of the book [1] was chosen.

Results

The idea behind structural stability is to be able to study the dynamics of a complicated system through other functions sufficiently close to it. In addition to the theoretical importance of the topic, there are practical implications of enormous relevance, such as the use of Numerical Analysis to obtain information about solutions of a dynamic process described by a function (in the discrete case) or a field (in the continuous case) that is structurally stable, or when we are interested in knowing whether small perturbations in the parameters of a system can abruptly interfere with its dynamic behavior.
To understand this theme and its applications, we first need to define some concepts. In what follows, A and B are metric spaces:

Definition 1: Two functions $f: A \to A$ and $g: B \to B$ são topologically conjugated if there is a homeomorphism $h: A \to B$ such that $h \circ f = g \circ h$.

Definition 2: Let $f: A \to A$ and $g: A \to A$ be two maps of class C^r, where A is an interval. The C^r ”distance” between f and g is given by $d_r(f, g) = sup_{x \in A}\{|f(x)-g(x)|,..., |f^{(r)}(x)-g^{(r)}(x)|\}$

Definition 3: Let J be an interval. A function $f: J \to J$ of class C^r is said to be C^r-structurally stable on J if there exists $\varepsilon > 0$ such that whenever $d_r(f, g) < \varepsilon$ for $g: J \to J$, it follows that f is topologically conjugated to g.
Proving that a dynamical system is structurally stable is, as we can see, very useful for the analysis of its perturbations. Here we develop

the proof that the function $L(x) = \frac{x}{2}$ is C^1-structurally stable in $\mathbb{R}$. It is simple and well-behaved, but it can still show us the idea of the process and illustrate the usual difficulties of these proofs.

We want an $\varepsilon > 0$ such that, if $d_1(L, g) < \varepsilon$, then L and g are topologically conjugated. We check that any $\varepsilon < \frac{1}{2}$ works. With $d_1(L, g) < \varepsilon$, we have $0 < g'(x) < 1 \ \forall x \in \mathbb{R}$, and therefore g is always increasing and has a single attractor fixed point (because it is so close to L, it crosses the line $y = x$).

Now, we introduce the use of fundamental domain. It is the smallest set of intervals that contain exactly one point from each orbit (other than the fixed point) of the function. The pair of intervals $5 < |x| \leq 10$, for instance, it is a fundamental domain of L. As $|g(10) - L(10)| < \varepsilon$ and $|g(-10) - L(-10)| < \varepsilon$, it follows that the fixed point of g is in the interval (-10,10), then to g we take the intervals $g(10) < x \leq 10$ and $-10 \leq x < g(-10)$.

We now want to build a homeomorphism $h: \mathbb{R} \to \mathbb{R}$ such that $h \circ L = g \circ h$. As we only need to prove that *there is* a h, there is no need to do it generically.

Thus, we define $h: [5, 10] \to [g(10), 10]$ and $h: [-10, -5] \to [-10, g(-10)]$ linearly, and $h(0)$ the fixed point of g. We can also take h increasing in such a way that $h(\pm 10) = \pm 10$. These values are arbitrary and were chosen for simplicity. The only criteria for them is to correctly create the fundamental domain.

Following the definitions made, we see that there is an $n \in \mathbb{Z}$ such that $L^n(x)$, if $x \neq 0$, belongs to the fundamental domain of L, and therefore $h \circ L^n(x)$ is well defined. Furthermore, we know that $L(x)$ is a homeomorphism. Since $g(x)$ is continuous and strictly increasing, we can say that it has a continuously increasing inverse, and therefore is also a homeomorphism.

With this, we know that $g^{-n}(x)$ is well defined, we can then take $h(x) = g^{-n} \circ h \circ L^n(x)$. Applying $g^n(x)$ on both sides, we get at last $g^n \circ h(x) = h \circ L^n(x)$. With the same artifice for L, we have $g \circ h(x) = h \circ L(x)$.

We can conclude that fixed n, inside the image by L^n of the fundamental domain of L, h is continuous, as the number of iterations is a fixed value. It remains now to show that at the edges of these intervals this is still valid, as well as at the origin.

Let's take x as one of the ends of one of the iterations of the fundamental domain. We can write, for small $\delta > 0$ and $\eta > 0$, $h(x + \delta) = g \circ h \circ L^{-1}(x + \delta)$ and $h(x - \eta) = g^2 \circ h \circ L^{-2}(x - \eta)$, so that we have points in two distinct intervals. We want to verify that the limit of both cases goes to $h(x)$ when δ and η tend to 0. To illustrate we choose, for instance, $x = 2.5$, so that when $\eta \to 0$, we have $h(2.5 - \eta) \to g^2(10)$; on the other hand, when $\delta \to 0$, $L^{-1}(x + \delta) \to 5$ and $h \circ L^{-1}(x + \delta) \to g(10)$, which implies that e $g \circ h \circ L^{-1}(x + \delta) \to g^2(10)$.

We can apply this to any nth iteration, and with that we see that the function h we constructed is continuous in $\mathbb{R}^*$, and indeed a homeomorphism, if we remove the fixed points of L and g of its domains. Finally, remembering that $h(0)$ is the fixed point of g, we need to show the continuity of h in 0.

Conclusions

The study of structural stability of dynamical systems is very useful, providing great tools for analyzing more complex cases. It is notable, however, that demonstrating such a property can be challenging. Understanding simpler cases, even without the presence of very interesting dynamics, allows us to create a familiarity with the concepts and prepare us for more laborious examples, in addition to strengthening the knowledge base on the subject.

References

[1] Devaney, R. L. An Introduction to Chaotic Dynamical Systems. Westview Press, 1986.

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Inscrição No.: 793

Título em Português:	Cardinais $\mathfrak{b}$, $\mathfrak{d}$, a hipótese do contínuo e o axioma de Martin
Título em Inglês:	Cardinals $\mathfrak{b}$ and $\mathfrak{d}$, the continuum hypothesis, and Martin axiom
Área de Pesquisa:	Álgebra
Palavras Chave:	cardinalidades - contínuo - axioma de Martin
Ag. Financiadora do Projeto:	CNPq - PIBIC
Projeto:	Iniciação Científica
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Resumo do Trabalho em inglês:

Cardinals $\mathfrak{b}$ and $\mathfrak{d}$, the continuum hypothesis and Martin's axiom

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Introduction

The aim of this project is to study some independence results through the impact of the continuum hypothesis and Martin's axiom on certain cardinals. In the 19th century, mathematician Georg Cantor, considered by many to be the father of set theory, conjectured the so-called continuum hypothesis (CH) which states the following:

Conjecture 1

There is no non-enumerable cardinal that is strictly smaller than $2^{\aleph_0} = |\mathbb{R}|$. That is, there is no cardinal κ such that:

$$\aleph_0 < \kappa < 2^{\aleph_0}$$

This conjecture, however, was proved to be independent of the Zermelo-Fraenkel axioms, which implies the existence of cardinality sets between $\aleph_0$ and $2^{\aleph}_0$ which, in ZF, without the addition of any other axiom, cannot have its cardinality described through a specific aleph.

Definition 1

We say that an ordinal κ is a **cardinal characteristic of the continuum** if

$$\aleph_0 < \kappa \leq 2^{\aleph_0}$$

The study of the cardinal characteristics of the continuum delimits an area of interest in modern set theory. Note that if CH is assumed to be true, all cardinal characteristics of the continuum would be the size of the continuum. However, this is not a fact in ZFC.

The cardinal characteristics $\mathfrak{b}$ and $\mathfrak{d}$

We will focus in this work to present the cardinal characteristics $\mathfrak{b}$ and $\mathfrak{d}$ relative, respectively, to the unlimited and dominant families of functions among the natural numbers. By its nature, we call the first the "bounding number" and the second the "dominant number". The notions of unlimited family and dominant family are described in the following definition.

Definition 2

Let $f, g \in \mathbb{N}^{\mathbb{N}}$. We say that $f <^* g$ if $\{n \in \mathbb{N} : g(n) < f(n)\}$ is finite.

A family $\mathscr{B} \subseteq \mathbb{N}^{\mathbb{N}}$ is said to be **unlimited** if $\nexists f \in \mathbb{N}^{\mathbb{N}}$ such that f dominates all functions of $\mathscr{B}$, i.e., $\forall f \in \mathbb{N}^{\mathbb{N}}\ \exists g \in \mathscr{B}$ such that $g \not<^* f$.

On the other hand, a family $\mathscr{D} \subseteq \mathbb{N}^{\mathbb{N}}$ is **dominant** if, for each $f \in \mathbb{N}^{\mathbb{N}}$, $\exists g \in \mathscr{D}$ such that $f <^* g$

Now look at the following sets:

$$\{|\mathscr{B}| : \mathscr{B} \text{ is unlimited family}\}$$

$$\{|\mathscr{D}| : \mathscr{D} \text{ is dominant family}\}$$

Since both are nonempty, they both have a minimum. We then define:

$$\mathfrak{b} := \min\{|\mathscr{B}| \ : \ \mathscr{B} \text{ is unlimited family}\}$$

$$\mathfrak{d} := \min\{|\mathscr{D}| \ : \ \mathscr{D} \text{ is dominant family}\}$$

We can prove that both are cardinal characteristics of the continuum and that, ensuring that every dominant family is unlimited, $\mathfrak{b} \leq \mathfrak{d}$. The study of comparisons between cardinal characteristics of the continuum is still relevant today. In some cases, proving this kind of claim requires very fine-grained arguments.

Martin's Axiom

Martin's axiom is an independent statement from the ZFC that says the following:

Axiom 1

If $\mathbb{P} = (P, \leq)$ is a partially ordered set that satisfies the countable chain condition (ccc) and D is a collection of dense subsets of P such that $|D| < 2^{\aleph_0}$ then $\exists$ filter $D-$generic on P.

This statement originates as an attempt to generalize the Rasiowa-Sikorski lemma, a lemma of extreme importance for the construction of the forcing theory. The strength and intuitive origin of this statement can also be seen by observing its various equivalences.

Consequences

Martin's axiom (MA) has several topological consequences. Many of them are generalizations of known theorems for enumerable cardinalities, being extended to non-enumerable cardinalities smaller than the continuum. One of these is the following generalization of Baire's theorem:

Theorem 1

Assuming true Martin's axiom (MA), then the union of less than $2^{\aleph_0}$ meager subsets of $\mathbb{R}^n$ is also meager in $\mathbb{R}^n$, i.e.:

$$\bigcup \mathcal{F} \in \mathcal{M} \, \forall \, \mathcal{F} \in [\mathcal{M}]^{2^{\aleph_0}}$$

Effect on cardinal characteristics $\mathfrak{b}$ and $\mathfrak{d}$

Note that the MA is a weaker version of the continuum hypothesis. However, Martin's axiom is strong enough to collapse the cardinals $\mathfrak{b}$, the $\mathfrak{d}$, among many others, into the continuum.

About references

HALBEISEN (2011) was used for the study of small cardinals. For the study of Martin's axiom, the same book was used, at first, and later, for a more in-depth study regarding consequences and equivalences, FREMLIN (1984) and CIESIELSKI (1997) were used.

References

CIESIELSKI, K. Set theory for the working mathematician. Cambridge University Press, 1997.

FREMLIN, D. Consequences of Martin's axiom. Cambridge University Press, 1984.

HALBEISEN, L. J. Combinatorial Set Theory With a Gentle Introduction to Forcing. London: Springer, 2011.

Inscrição No.: 794

Título em Português:	Conexidade Dinâmica
Título em Inglês:	Dynamic Connectivity
Área de Pesquisa:	Teoria da Computação
Palavras Chave:	estruturas de dados - grafos dinâmicos - conexidade
Ag. Financiadora do Projeto:	CNPq - PIBIC
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USP - UNIVERSIDADE DE SÃO PAULO
30º Simpósio Internacional de Iniciação Científica

Inscrição No.: 794

Resumo do Trabalho em inglês:

Dynamic Connectivity

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Objectives

Fully dynamic connectivity in graphs [1, 8] is the problem in which a graph can undergo insertion and removal of edges and one wants to efficiently perform queries of the type: decide if two vertices are in the same component of the graph. It is easy to solve this problem, keeping the graph with adjacency lists, and using breadth first search (BFS), for instance, to answer the queries. This however takes linear time per query. This work considers the particular case of the problem where the graph is a forest and describes a non-trivial solution that answers queries in logarithmic amortized time. The solution is the basis of a similarly efficient implementation for dynamic graphs.

Materials and Methods

The study began with the reading of section 1.1 of the Dynamic Graphics book [1] about the basic concepts of dynamic graphs and the possible data structures used to solve the problem. After this reading, link-cut trees [2] were chosen as the data structure to be studied and implemented. Lecture notes [4, 5] and some articles [2, 6] were used to understand them. Link-cut trees are based on a simpler data structure, the binary search trees (BSTs). The BSTs adopted were the splay trees [3], and the Algorithms book [7] was used to understand how splay trees work.

The first part of the work was the implementation of the data structure for the fully dynamic connectivity problem in forests, that is, in an acyclic graph, in order to make the problem simpler. The second part would be the implementation of the more general case, that is, the data structure for the fully dynamic connectivity problem in graphs. After the study, the implementation of the data structures in C language began. The splay trees were the first to be implemented, then the link-cut trees and finally the dynamic forest. The GitHub [9] platform was used for version control. In addition, Bash scripts were implemented for automated tests.

Results

The number of vertices of the underlying graph or forest will be denoted by n. The implemented library that handles dynamic forests is: the **dynamicForest(***n***)** creates a forest with n vertices without edges; the **addEdge(***u***, *v***)** adds an edge between vertices u and v; the **deleteEdge(***u***, *v***)** removes the edge between u and v; and the **connected(***u***, *v***)** returns true if u and v are in the same forest component and false otherwise. The **dynamicForest** routine has time cost $O(n)$ and the other operations have $O(\lg n)$ amortized cost.

The link-cut tree is a data structure used to represent forests, and maintains the dynamic forest, its library is: the **maketree()** creates a tree with a single node; the **link(***u***, *v***)** adds an edge between u and v, assuming that u is the root of its link-cut tree; the **cut(***v***)** removes the edge between v and its parent in the link-cut tree, assuming that v is not the root of its link-cut tree; and the **findroot(***u***)** returns the root of the link-cut tree containing u. The **maketree** routine runs in $O(1)$ time and the other operations have an amortized cost of $O(\lg n)$.

The implementation of the dynamic forest library from the link-cut trees library is straightforward. The **connected(***u***, *v***)** operation uses **findroot** on u and v and checks if they return the same root. The **addEdge(***u***, *v***)** operation executes **link(***u***, *v***)**, while the **deleteEdge(***u***, *v***)** operation chooses the

deepest between *u* and *v* in the link-cut tree to perform the **cut** operation.

The splay tree library is: the **makeSplay()** creates a splay tree with a single node; the **join(***u, v***)** unites the splay trees with roots in *u* and *v*, assuming *u* has maximum key in its splay tree and its key is smaller than all keys in the *v's* splay tree; the **split(***u***)** breaks *u*'s splay tree in two: one with all nodes with a key less than or equal to *u* and another with nodes with a key greater than *u*; and **splay(***u***)** makes node *u* root of its splay tree. The **splay** routine has an amortized time cost of O(lg *n*) and the other operations have cost of O(1).

Table 1 summarizes the efficiency of each operation of the dynamic forest for the traditional and the studied implementation. Note the trade off: to improve the efficiency of the queries, the link-cut tree implementation spends more time to add and remove edges.

Table 1: Time complexity of a traditional implementation and a link-cut tree implementation.

Operation	Adjacency lists + BFS	Link-cut trees (amortized time)
addEdge	O(1)	O(lg *n*)
deleteEdge	O(1)	O(lg *n*)
connected	O(*n*)	O(lg *n*)

Conclusions

This work is designed to solve the problem of dynamic connectivity in a context where the manipulated graph is huge and there will be many more queries than changes in the graph. Therefore, queries with linear cost are very undesirable in this situation. So it is essential to make queries more efficient, even if it makes the cost of inserting and removing edges in the graph a little more expensive. The result is a non-trivial but extremely efficient implementation for dynamic forests, the first step towards the more general case of dynamic graphs.

References

[1] C. Demetrescu, I. Finocchi, and G. F. Italiano. Handbook of Data Structures and Applications, chapter Dynamic Graphs. Chapman and Hall/CRC, 2004.

[2] D. D. Sleator and R. E. Tarjan. A data structure for dynamic trees. In Proc. of the 13th Annual ACM Symp. on Theory of Computing (STOC), 362–391, 1983.

[3] D. D. Sleator and R. E. Tarjan. Self adjusting binary search trees. Journal of the ACM, 32(3):652–686, 1985.

[4] E. Demaine, J. Holmgren (scriber), J. Jian (scriber), M. Stepanenko (scriber), and M. Ishaque (scriber). Link-cut trees problems. Lecture Notes in Advanced Data Structures, 2012.

[5] E. Demaine and K. Lai (scriber). Dynamic graph problems. Lecture Notes in Advanced Data Structures, 2007.

[6] M. R. Henzinger and V. King. Randomized dynamic graph algorithms with polylogarithmic time per operation. In Proc. of the 27th Annual ACM Symp. on Theory of Computing (STOC), 502–516, 1995.

[7] R. Sedgewick and K. Wayne. Algorithms. Addison-Wesley, 4 edition, 2011.

[8] J. Holm, K. de Lichtenberg, and M. Thorup. Poly-logarithmic deterministic fully-dynamic algorithms for connectivity, minimum spanning tree, 2-edge, and biconnectivity. Journal of the ACM, 48(4):723, 2001.

[9] D. Lawand. Dynamic connectivity implementation. GitHub, November 2022, https://github.com/danlawand/conexidade-dinamica.

Inscrição No.: 795

Título em Português:	Segmentação de tumores em imagens de ressonância magnética 3D
Título em Inglês:	Tumor segmentation in 3D magnetic resonance images
Área de Pesquisa:	Metodologia e Técnicas da Computação
Palavras Chave:	medical imaging - computer vision - AI
Ag. Financiadora do Projeto:	CNPq - PIBIC
Projeto:	Iniciação Científica
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Departamento:	
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Resumo do Trabalho em inglês:

Tumor segmentation of MRI scans

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Objectives

The goal of this project is to meet the clinical demand for annotations in Magnetic Resonance Images (MRIs). Firstly, we intend to analyze the datasets that are going to be utilized: the 2019 BraTS (*Multimodal Brain Tumor Segmentation*)[1] challenge dataset [1, 2, 3] and the 2021 FeTA[2] [4] challenge. Then, we should develop and execute neural network architectures on the MRIs, and also use the specialist networks approach on the data. Finally, it is necessary to measure and analyze the accuracy of the developed models, comparing the results with already known literature methods.

Methodology

The methodology used in this project consists of submitting the data from BraTS and then from FeTA to two pipelines. The first pipeline uses only the structure of five convolutional neural networks (CNNs) for the segmentation of the desired regions on the MRIs, without preprocessing. The networks that are developed and utilized are the Encoder-Decoder architectures: the 3D U-Nets [5] and the V-Nets [6]; and also the Fully Convolutional Networks (FCNs) [7]: HighRes3DNet (HR3N) [8], SkipDenseNet (SDN)[9] and Med3D [10].

The second pipeline consists of two steps. First, the MRI is submitted to a neural network called generalist ($\mathcal{G}$), that makes a rough binary prediction of the region of interest. From that binary prediction, it is possible to delimit a bounding box using the extremes of the predicted region. The second step utilizes a specialist network ($\mathcal{S}$) to refine the segmentations, submitting the bounding boxes to the neural networks seeking a multi-class segmentation. For the BraTS dataset, this pipeline should segment a whole tumor, while in the FeTA, it should segment a region from the fetal encefalus that contains three chosen structures: the cerebellum, the deep gray mass and the brain stem. The segmentation then undergoes a set of validations, intending to test the efficiency of the prediction. To measure the learning efficacy, we used three objective metrics: the Dice coefficient (DSC), the Jaccard index (IoU) and confusion matrices. The segmentation predictions are also saved as the images label, in such way that it is possible to visualize after the execution.

For the whole project, we used the programming language Python [3]. The neural network implementations uses the Pytorch library [4], that contains classes and methods to perform convolutions and other operations used in each architecture. For the images pre-processing, we used the Scikit-Image[5] library. Finally, to compute the metrics when testing the model, we used methods and classes from the Scikit-Learn[6] library.

Metric	V-Net	U-Net	HR3N	SDN	Med3D
DSC	0.5444	0.5828	0.49	0.6618	0.556
IoU	0.4583	0.4991	0.4091	0.5525	0.4491

Table 1: DSC and IoU metrics from the first *pipeline* executed on the BraTS dataset.

Results

The results of the execution of both pipelines on the BraTS dataset are shown in the Tables 1 and 2.

The Tables 3 and 4 show the metrics obtained from the execution of the experiments on the FeTA dataset.

[1] https://www.med.upenn.edu/cbica/brats2019/tasks.html
[2] https://feta-2021.grand-challenge.org
[3] https://python.org
[4] https://pytorch.org
[5] https://sckit-image.org
[6] https://sckit-learn.org

Metric	Strategy	V-Net	U-Net	HR3N	SDN	Med3D
DSC	$\mathcal{G}$	0.8132	0.5775	0.7498	0.8287	0.7159
	$\mathcal{S}$	0.6699	0.3103	0.3046	0.6404	0.3821
IoU	$\mathcal{G}$	0.6852	0.406	0.5998	0.7075	0.5575
	$\mathcal{S}$	0.5593	0.2681	0.266	0.5236	0.3061

Table 2: DSC and IoU metrics of the second pipeline executed on the BraTS dataset, comparing the execution between $\mathcal{G}$ and $\mathcal{S}$.

Metric	V-Net	U-Net	HR3N	SDN	Med3D
DSC	0.861	0.8355	0.8072	0.8749	0.7848
IoU	0.7635	0.7267	0.6884	0.7844	0.6586

Table 3: DSC and IoU metrics from the first *pipeline* executed on the FeTA dataset.

Conclusions

The application of convolutional neural networks to the problem of segmentation of medical images has obtained highly accurate results, similar to the doctors' annotations. In this project, the use of the U-Net 3D, V-Net, HighRes3DNet, SkipDenseNet, and Med3D architectures on two distinct datasets reached predictions with variable accuracies according to segmentation method and dataset complexity.

The tumor segmentation on the BraTS dataset obtained satisfactory accuracy (compared to doctors' annotations), mostly without the use of specialist networks, which have shown to be inadequate for the problem of glioma detection. In the FeTA dataset experiments, the results have shown to be more accurate than the previous one, mostly with the use of a pipeline with generalist and specialist networks.

For future researchs, it would be interesting to study the application of specialist networks on other encephalic structures and volumetry analysis on the obtained segmentations, generating clinical interpretations for the results. Another interesting challenge is utilizing fetal MRI segmentations to pathology detections, extracting objective metric for the comparison between healthy and pathological volumes.

Metric	Strategy	V-Net	U-Net	HR3N	SDN	Med3D
DSC	$\mathcal{G}$	0.8864	0.8094	0.7465	0.7711	0.8119
	$\mathcal{S}$	0.8996	0.8363	0.5905	0.9011	0.9313
IoU	$\mathcal{G}$	0.7959	0.6798	0.5955	0.6275	0.6833
	$\mathcal{S}$	0.8233	0.7296	0.4866	0.8263	0.8741

Table 4: DSC and IoU metrics of the second pipeline executed on the FeTA dataset, comparing the execution between $\mathcal{G}$ and $\mathcal{S}$.

References

[1] MENZE, B. H. et al. The multimodal brain tumor image segmentation benchmark (brats). *IEEE TMI*, IEEE, v. 34, n. 10, p. 1993–2024, 2014.

[2] BAKAS, S. et al. Identifying the best machine learning algorithms for brain tumor segmentation, progression assessment, and overall survival prediction in the brats challenge. *arXiv preprint arXiv:1811.02629*, 2018.

[3] BAKAS, S. et al. Advancing the cancer genome atlas glioma mri collections with expert segmentation labels and radiomic features. *Scientific Data*, Nature Publishing Group, v. 4, n. 1, p. 1–13, 2017.

[4] PAYETTE, K. et al. An automatic multi-tissue human fetal brain segmentation benchmark using the fetal tissue annotation dataset. *Scientific Data*, Nature Publishing Group, v. 8, n. 1, p. 1–14, 2021.

[5] ÇIÇEK, Ö. et al. 3d u-net: learning dense volumetric segmentation from sparse annotation. In: SPRINGER. *MICCAI*. [S.l.], 2016. p. 424–432.

[6] MILLETARI, F.; NAVAB, N.; AHMADI, S.-A. V-net: Fully convolutional neural networks for volumetric medical image segmentation. In: IEEE. *International Conference on 3D Vision (3DV)*. [S.l.], 2016. p. 565–571.

[7] LONG, J.; SHELHAMER, E.; DARRELL, T. Fully convolutional networks for semantic segmentation. In: *CVPR*. [S.l.: s.n.], 2015. p. 3431–3440.

[8] LI, W. et al. On the compactness, efficiency, and representation of 3d convolutional networks: brain parcellation as a pretext task. In: SPRINGER. *International Conference on Information Processing in Medical Imaging*. [S.l.], 2017. p. 348–360.

[9] BUI, T. D.; SHIN, J.; MOON, T. 3d densely convolutional networks for volumetric segmentation. *arXiv preprint arXiv:1709.03199*, 2017.

[10] CHEN, S.; MA, K.; ZHENG, Y. Med3d: Transfer learning for 3d medical image analysis. *arXiv preprint arXiv:1904.00625*, 2019.

Inscrição No.: 987

Título em Português:	Representações de grupos e álgebras de Lie
Título em Inglês:	Lie groups and Lie algebras representations
Área de Pesquisa:	Álgebra
Palavras Chave:	representação - grupo de Lie - álgebra de Lie
Ag. Financiadora do Projeto:	CNPq - PIBIC
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	
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Resumo do Trabalho em inglês:

1. REPRESENTATIONS OF LIE GROUPS & ALGEBRAS

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Objectives

The theory of representations of Lie groups is concerned with studying the smooth homomorphisms of a Lie group onto the general linear group — that is, the group $GL(V)$ of invertible endomorphisms $V \to V$ endowed with the Euclidean topology. Thus, given a Lie group G, a representation of G is a vector space V provided with a smooth homomorphism $\rho : G \to GL(V)$.

However, one can show that it is possible to study the representations of a Lie group G through the representations of the Lie algebra $\mathfrak{g}$ associated with G: a Lie algebra $\mathfrak{g}$ is a vector space provided with a bilinear operator $[,] : \mathfrak{g} \times \mathfrak{g} \to \mathfrak{g}$ which satisfies Jacobi's identity — commonly called a Lie bracket; in turn, the Lie algebra $\mathfrak{g}$ algebra associated to G is the algebra of smooth vector fields over G invariant by left translations, equipped with the usual Lie bracket ($[X, Y] = XY - YX$).

Analogously to the case of Lie groups, a representation of a Lie algebra $\mathfrak{g}$ algebra is a vector space V provided with a linear transformation $\rho : \mathfrak{g} \to \mathfrak{gl}(V)$ invariant under the Lie bracket — where $\mathfrak{gl}(V)$ is the Lie algebra associated to $GL(V)$, which coincides with the algebra of endomorphisms $V \to V$ endowed with the Lie brackts given by the comutator $[X, Y] = XY - YX$.

Materials and Methods

This is a project in the area of Pure Mathematics. Thus, no specific materials were required. The analysis of the results was done by means of half-yearly reports and the presentation of seminars — which were given online because of the then present epidemic of COVID-19. A dissertation about the topics studied was also written.

Results

In the last two semesters we studied:

- The basics of representations of Lie group and algebras
  - Definitions of Lie groups and algebras
  - Definitions of representations of Lie groups and algebras
  - The fundamental Lie theorems
  - The equivalence between representations of a Lie group and representations of the Lie algebra associated with that group
  - The equivalence between representations of a Lie algebra and representations of its universal enveloping algebra
- Representations of Compact Groups
  - The Haar measure
  - Unitary representations
  - The generalization of Maschke's theorem

- The Peter-Weyl theorem

- Representations of $\mathfrak{sl}_2(\mathbb{C})$
- Representations of $\mathfrak{sl}_3(\mathbb{C})$
- Representations of finite-dimensional complex semisimple Lie algebras
- Cartan's classification of finite-dimensional simple complex Lie algebras and Dynkin diagrams

Conclusions

The major conclusion of the project was the explicit construction of all finite-dimensional irreducible representations of a finite-dimensional semisimple complex Lie algebra.
For future projects we intend to study the theory of representations of Lie algebras in more general contexts, such as irreducible, infinite-dimensional weight representations of reductive finite-dimensional Lie algebras over an algebraically closed field of characteristic zero.

References

[1] Vera Serganova Caroline Gruson. A Journey Through Representation Theory: From Finite Groups to Quivers via Algebras. 2018.
[2] Luiz A. B. San Martin. Grupos de Lie. Unicamp, 2016.
[3] Luiz Antonio Barrera San Martin. Álgebras de Lie. 1ª ed. Unicamp, 1999.
[4] Claudio Procesi. Lie Groups: An Approach through Invariants and Representations. 1ª ed. Universitext. Springer, 2006.
[5] Joe Harris William Fulton. Representation theory. A first course. Corrected. Graduate Texts in Mathematics / Readings in Mathematics. Springer, 1991.

Inscrição No.: 988

Título em Português:	Órbitas e conjuntos alcançáveis de familias de campos vetores
Título em Inglês:	Orbits and attainable sets of vector fields
Área de Pesquisa:	Geometria e Topologia
Palavras Chave:	Geometria - órbitas - conjuntos alcançávei
Ag. Financiadora do Projeto:	CNPq - PIBIC
Projeto:	Iniciação Científica
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USP - UNIVERSIDADE DE SÃO PAULO
30º Simpósio Internacional de Iniciação Científica

Inscrição No.: 988

Resumo do Trabalho em inglês:

ORBITS AND ATTAINABLE SETS OF FAMILIES OF VECTOR FIELDS

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Objectives

Stablishing a good basic knowledge in differential geometry and manifold theory

Investigate and comprehend the some classic theorems of geometric control theory: the Stefan-Sussmann theorem about orbits of geometric control systems and Kremmer's theorem about attainable sets. Get a deeper understanding of the corolaries and consequences of these theorems, both for geometric control theory and other areas of mathematics.

Stablish a good basis for the study of optimal control theory and Pontryagin's maximality principle.

Materials and Methods

Weekly seminars during the period of the scientic initiation, where the topics to be studied where discussed and exposed.

Results

Results

Theorem 0.1 (Orbit Theorem). *Let $\mathcal{F} \subseteq \mathfrak{X}(M)$ and $p_0 \in M$. Then:*

- *$\mathcal{O}_{p_0}$ is an immersed and connected submanifold of M*
- *$T_p\mathcal{O}_{p_0} = span\{p \circ (Ad\ P)X : P \in \mathcal{P}, X \in \mathcal{F}\}$*

Where:

$$\mathcal{P} = \{e^{t_1 X_1} \circ \ldots \circ e^{t_k X_k} : t_1, \ldots, t_k \in \mathbb{R}; X_1, \ldots X_k \in \mathcal{F}\}$$

Theorem 0.2. *Let $\mathcal{F} \subseteq \mathfrak{X}(M)$ be a family such that Lie$\mathcal{F}$ is locally finitelly generated. Then:*

$$T_p\mathcal{O}_p = Lie_p\mathcal{F}$$

Theorem 0.3 (Teorema de Stefan–Sussmann). *Let Δ be a singular distribution. Then, the following are equivalent:*

- *Δ is invariant wrt. $X \in \mathfrak{X}(\Delta)$.*
- *Δ is integrable.*

Conclusions

Geometric control theory comes from an intersection between an area of pure maths (differential geometry) and an area of applied maths (control theory). Its study allows an analysis and an understanding in both direction, with results (such as the orbit theory) that offer insights into many different sub-areas of both areas.

References

[1] A. Agrachev Y. Sachkov
Control theory from the Geometric viewpoint
Encyclopedia of Mathematical

Science, Control Theory and Optimization.

[2] M. M. Alexandrino, R. G. Bettiol,
Lie Groups and Geometric Aspects of Isometric Actions
. Springer

Verlag (2015)

[3] D.D. Holm, T. Schamah, C. Stoica
Geometric Mechanics and Symmetry
, Oxford Texts in Applied and

Engineering Mathematics.

[4] J. M. Lee,
Introduction to smooth manifolds
, Springer

[5] M. Spivak,
A comprehensive introduction to differential geometry
, Perish or publish.

Inscrição No.: 1148

Título em Português:	Dois Teoremas Clássicos em Análise Matemática
Título em Inglês:	Two Classical Theorems in Mathematical Analysis
Área de Pesquisa:	Análise
Palavras Chave:	Stone Weierstrass - Conjuntos Compactos - Arzela Ascoli
Ag. Financiadora do Projeto:	CNPq - PIBIC
Projeto:	Iniciação Científica
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Resumo do Trabalho em inglês:

Two Classic Theorems of Mathematical Analysis

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Objectives

The Ascoli-Arzelà and the Stone-Weierstrass theorems are results of extreme importance in analysis. They both are statements about the space of continuous, real functions, defined in some compact domain. The former provides a tool for showing that certain subsets of this space are compact, while the latter gives us a sufficient condition for subalgebras being dense. Our goal was to study these theorems in three settings: the real line, metric spaces, and topological spaces. In this manner, we were able to better understand all the mentioned settings, and to get a feel for the way mathematics evolves, with results being progressively generalized.

Materials and Methods

Beginning in April of 2021, virtual seminars were made every week, in which the student presented the studied results, and the advisor answered questions and helped with comprehension. In September, the project was awarded a PIBIC scholarship by CNPq. In the beginning of 2022, the seminars transitioned from being virtual to being in person.

Results

We were able to achieve our objectives, with both theorems having been studied in all three settings. In some cases, more than one proof was seen. We were also able to study a stronger version of the Ascoli-Arzelà Theorem that only applies for separable and locally compact metric spaces. The Picard Theorem was also studied. In the case of the Stone-Weierstrass Theorem for topological spaces, the student managed to make a slight modification in the demonstration presented in the book Approximation & Optimization (Prolla, 1993), making the proof easier to understand. During our studies, the book Espaços Métricos (Lima, 2011) was read in its entirety. Many other books were also seen. They are all listed in the references section.

Now we shall state the most general version of each theorem and discuss some of their applications. We begin with some definitions and conventions. If X and Y are topological spaces, $C(X,Y)$ represents the set of all continuous functions from X to Y. If X is compact, we can treat $C(X,\mathbb{R})$ as a metric space with the uniform convergence metric. A subset $A \subset C(X,\mathbb{R})$ is said to be a subalgebra of continuous functions if A is closed under addition, scalar multiplication, and function multiplication. We also say that A separates points of X if, for any pair of distinct points $x, y \in X$, there exists $f \in A$ such that $f(x) \neq f(y)$. If the closure of a subspace of X is compact, the subspace is said to be relatively compact. Finally, we say that $S \subset C(X,\mathbb{R})$ is equicontinuous at $x \in X$ if, for each $\varepsilon > 0$ there is a neighborhood V of x such that $\sup_{f\in S}\sup_{t\in V}|f(x)-f(t)| < \varepsilon$. If S is equicontinuous at every point of X, it is said to be equicontinuous.

The Stone-Weierstrass theorem for topological spaces states that if X is a compact Hausdorff topological space, and if $A \subset C(X,\mathbb{R})$ is a subalgebra that contains the constant functions and separates points of X, then A is dense in X.

[1] This student was awarded a PIBIC/CNPq Scholarship.

Therefore, the theorem provides us with a large number of families of functions that can be used to approximate uniformly the continuous functions from X to $\mathbb{R}$. In particular, it says that continuous real functions defined in compact intervals can be approximated by polynomials. This last result is referred to as the Weierstrass Approximation Theorem. The Stone-Weierstrass Theorem also has consequences that are important to the theory of Fourrier Series. Indeed, it can be used to show that real continuous functions of period 2π can be uniformly approximated by trigonometric polynomials (linear combinations of functions of the type $\sin(n \cdot x)$ and $\cos(n \cdot x)$, where $n \in \mathbb{N}$).

On the other hand, the Ascoli-Arzelà theorem for topological spaces states that if X is a compact Hausdorff topological space, then $S \subset C(X, \mathbb{R})$ is relatively compact if, and only if, S is bounded and equicontinuous. In Analysis, it is frequently useful to know if a sequence of continuous functions defined in a compact interval has a convergent subsequence. From the Ascoli-Arzelà Theorem, we see that if the sequence is bounded and equicontinuous, then that does happen.

Conclusions

Throughout the project, many results and techniques from all three settings were learned. A few of the important ideas seen were convolutions, partitions of unity and nets. Because we began by studying the theorems in simpler settings, we were able to develop an intuition that would have been impossible had we begun by the most general cases. In this way we were also able to learn a lot about how mathematics develops: a result appears in a familiar case and, as time passes, mathematicians realize which properties the theorem really relies on, allowing them to extend the result to more general cases.

References

1. N. Dunford, J. Schwartz, Linear Operators, I, Interscience Publishers Inc., New York, 1988.
2. R. R. Goldberg, Methods of Real Analysis, Blaisdell Publishing Company, 1964.
3. E. L. Lima, Espaços Métricos, SBM-Projeto Euclides, 2011.
4. J. B. Prolla, Weierstrass-Stone, the Theorem, Approximation & Optimization, Verlag Peter Lang, 1993.
5. T. Tao, Compactness and compactification, The Princeton Companion to Mathematics (Ed. T. Gowers, J. Barrow-Green and I. Leader), Princeton University Press, 2008, pp 167-169.

Inscrição No.: 1150

Título em Português:	Grupos livres em anéis com divisão
Título em Inglês:	Free groups in division rings
Área de Pesquisa:	Álgebra
Palavras Chave:	division rings - free groups - group algebras
Ag. Financiadora do Projeto:	CNPq - PIBIC
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	
Validado em:	**15/09/2022**

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Resumo do Trabalho em inglês:

Free Groups in Division Rings

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Objectives

In 1972, J. Tits proved the following result: "A finitely generated subgroup of Gl(n, K), where K is a field, either contains a non-cyclic free group, or is solvable-by-locally finite". It is natural to wonder whether the same remains valid when the field K is replaced by a noncommutative division ring. In 1977, A. I. Lichtman gave a negative answer to that question and proposed the following conjecture: "The multiplicative group D* = D - {0} of a division ring contains a non-cyclic free group". The goal of the present work is to study some instances where the above conjecture is valid, and in which we can exhibit the free groups explicitly.

Materials and Methods

Many types of division ring constructions were studied in detail, such as: fields of fractions of Ore domains, Malcev-Neumann series rings of ordered group rings and quaternion algebras, both over fields of characteristic 2 and over fields of characteristic not 2.

Methods of identifying free groups were also studied and were applied in the examples constructed beforehand. In order to allow the analysis of more complex cases, such as that of the field of fractions of the first Weyl algebra, it was necessary to study specializations, as these provide a way to reduce problems about intricate division rings to those of simpler ones, such as the aforementioned quaternion algebras. Some of these concepts are detailed in what follows:

Def: A (not necessarily commutative) domain D is called **right Ore** if, given any two non-zero elements x, y in D, the principal right ideals xD, yD have a non-empty intersection.

It is known that these rings allow a construction akin to that of the field of fractions over a (commutative) integral domain, usually called a total right field of fractions.

Def: The **first Weyl algebra** is the algebra defined by **Q**<s, t; ts - st - 1 = 0>.

It is possible to prove that this algebra is right Ore and thus, admits a total right field of fractions.

Def: If G is a group and H is a normal subgroup, then a pair {x, y} is called **semi-free modulo H** if xH has order at least 3, yH has order 2 and <xH, yH> = <xH> $*$ <yH>, where the star denotes the free product.

It is a known fact that groups of the above form contain free groups of rank 2 (which can be explicitly determined). Since free groups can be lifted via homomorphisms, semi-free pairs modulo H are indicative of the presence of free pairs (which can also be explicitly determined).

Def: If K is a field of characteristic not 2 and a, b are non-zero elements of K, then the **quaternion algebra (a, b / K)** is the algebra given (by generators and relations) as K<i, j; $i^2 = a$, $j^2 = b$, ji = -ij>

Even though not all quaternion algebras are division rings, those that are such have the smallest possible dimension for a division ring over its center. This makes them particularly

simple to analyse and makes obtaining results easier.

Results

Of all the many results that were proven throughout the work, two particular theorems deserve a special recognition, as they are very illustrative of the techniques used:

1. The pair {1 + ts, 1 + t} is semifree modulo a subgroup of the multiplicative group of the field of fractions of the first Weyl algebra, $A_1(\mathbf{Q})$
2. If K is a field of characteristic not 2, G is a residually torsion-free nilpotent group, c and d are any non-zero elements of K and x, y are two non-commuting elements of G, then the pair {1 + cx, 1 + dy} generates a free group of rank 2 in the multiplicative group of the Malcev-Neumann series ring of KG.

Both of these results make use of so-called **specializations.** Simply put, a specialization from a division ring D to a division ring F is a homomorphism $\psi: S \longrightarrow F$, where S is a local subring of D, such that $\ker(\psi)$ is the maximal ideal of S.

This technique allows us to dodge the fact that division rings don't have ideals, by switching the domain to a local subring. As such, in both of the aforementioned results, the division ring in question can be specialized to another convenient one, in which better properties are known.

In particular, the first result cited above is obtained by constructing a specialization onto a quaternion algebra over the field of rational functions in two indeterminates, x and y. On these kinds of algebras, the following can be stated:

3. If K is a field of characteristic not 2, then the pair {1 + i, 1 + j} is free in the algebra (x, y / K(x, y))

Conclusions

Even though a direct proof of Lichtman's conjecture still seems out of reach, since we don't yet have a complete classification of division rings, we are able to determine its validity in many cases which naturally appear, such as when the division ring is of finite dimension over its center (as was shown in [5]).

There is still much to be done in this direction. In particular, the second result still hasn't been established when K is a field of characteristic 2. Another way in which this type of study can be conducted is to look for a free pair with aditional nice properties. For instance, it is interesting to see whether a division ring with involution admits a free symmetric or free unitary pair. These types of results will be studied in future works.

References

1. Goncalves, J. Z., Mandel, A. e Shirvani, M. "Free products of units in algebras I. Quaternion algebras." J. Algebra 214 (1999), pp. 301-316
2. Lichtman, A. I. "On the subgroups of the multiplicative group of skewfields." Proceedings of AMS 63 (1977), pp. 15-16
3. Tits, J. "Free groups in linear groups." J. Algebra, 20 (1972), pp. 250-270.
4. A. I. Lichtman. “Valuation methods in division rings”. J. Algebra 177 (1995), pp. 870– 898.
5. Gonçalves, J. Z. "Free Groups in Subnormal Subgroups and the Residual Nilpotence of the Group of Units of Group Rings" Canad. Math. Bull. 27 (1984), pp. 365-370.

Inscrição No.: 1305

Título em Português:	Utilização de Animações para Comparação de Hashes em Votação Eletrônica via Internet
Título em Inglês:	Using Animations to Compare Hashes in Electronic Voting via Internet
Área de Pesquisa:	Sistemas de Computação
Palavras Chave:	Hashes criptográfico - Votação Eletrônica - Internet
Ag. Financiadora do Projeto:	CNPq - PIBIC
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	
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USP - UNIVERSIDADE DE SÃO PAULO
30º Simpósio Internacional de Iniciação Científica

Inscrição No.: 1305

Resumo do Trabalho em inglês:

Using Animations to Compare Hashes in Electronic Voting via Internet

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Objectives

Secure electronic web voting systems require data comparison at various stages of the voting process. Some of this data are hashes, usually represented by strings, for example:

```
o4FqetQ2Ax1NV/I0LDhudrykb3jXeCE6NuLiPqNcRTi
```

Comparing these strings by humans tends to be tedious, which means that it is not always performed correctly [1]. In this scientific initiation project, we aimed to produce and report preliminary results of a software system that advances state-of-the-art electronic web voting by comparing hashes represented by animations. The basis of the system is the free software `Hashify` [2], responsible for animations, and the free software `Helios` [3], responsible for voting. Figure 1 shows an animation sequence generated by `Hashify` for the hash shown above.

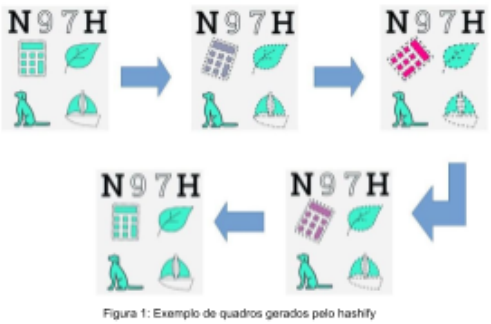

Figura 1: Exemplo de quadros gerados pelo hashify

Among the various stages of the `Helios` voting process, there is the audit stage. In this step, voters can check the election hash to confirm that they participated in the correct election and the ciphertext hash received when voting. Checks are performed by comparing strings. However, this comparison can easily confuse voters and lead them to make mistakes. As `Helios` is intended to be an online voting system for the general public, the audit mustn't be confusing, especially for people who do not have theoretical knowledge of cryptography. In this way, by converting the hashes into animations, we seek to increase security and confidence in the system by encouraging each voter to participate in the audit actively. Results from this project show that it is possible to compare `Helios` hashes using `Hashify` animations.

Methods and Procedures

As explained in the previous section, two free software were used: `Helios Voting` and `Hashify`. An instance provided by the developers of `Helios Voting` at https://vote.heliosvoting.org/ was used to create the elections and understand when and how the generated hashes are used. We identified the presence of two important hashes for voting: the election and the ballot. We tested the `Helios` hashes using the demo version of `Hashify`. After some tests, it was necessary to create a web system based on `Hashify`, capable of generating animations with the `Helios` hashes, since `Hashify` was developed to receive, as input, hashes of digital certificates (SHA-256 hashes) in hexadecimal representation and, although the hashes generated by `Helios Voting` are also hashes based on the SHA-256 hash function, they are presented to the person voting in base64

representation (string with 43 alphanumeric characters, '+' and '/').
In the developed web system, we receive a hash from `Helios` and convert it to a format accepted by `Hashify` that will generate its animation. This conversion was developed in two options: 1- convert character by character to a hexadecimal representation and 2- decode the hash from base64 representation to binary and then convert to hexadecimal. In both cases, the generated animations were specific for each hash; however, only in the second case it is guaranteed that all properties related to `Hashify`'s collision probability are maintained, given that the hash type would continue to be the same for which it was designed.

Results

The main results concern the web system developed to receive the two hashes generated by `Helios Voting` in a poll. Each one of these hashes generates a specific animation, which the voter must save for comparison after the election. Some requirements in terms of the user interface were implemented on this page so that it would serve the purpose of the project: Identification of which hash is currently being expected (first, it is informed that the hash of the election must be entered, then the hash of the ballot); reminder, in floating text, that the expected string must be 43 characters long and without spaces (the expected format of a SHA-256 hash in base64 representation); input validation (if the string is not 43 characters long, a visual feedback – the text box turns red – is presented to the voting person).

Figura 2: sistema web desenvolvido

Figure 2 shows a screenshot of generating an animation from a hash taken from `Helios Voting`. The "*Habilitar loop da Animação*" checkbox is to keep the animation in loop. The "*Decodificar Hash*" checkbox transforms the SHA-256 hash from base64 to binary and then to hexadecimal. When not selected, we treat the hash as a regular string and convert it to hexadecimal character by character. In the future, it is planned to remove this option once it has been proven whether 1- the conversion from base64 to hexadecimal is enough or 2- the `Hashify` algorithm works, collision-free, for 43-character hexadecimal strings. When pressed, the "*Próximo*" button will reload the page, this time requesting the hash of the ballot.

Conclusions

This project presented a proof of concept that it is possible to use `Hashify`, a software for generating animations from hashes, to facilitate the auditing of `Helios Voting`.
As the next steps, we intend to integrate the developed web system with `Helios Voting`, allowing the animations to be generated automatically at the time of voting, without the need to use a second web system, and sending them by e-mail so that the voting person can keep them stored for later audit. It is also intended to conduct research with human beings to assess whether visual hashes actually increase engagement in the election audit process.

Bibliographic References

[1] Tan, J., Bauer, L., Bonneau, J., Cranor, L. F., Thomas, J., and Ur, B. (2017). Can Unicorns Help Users Compare Crypto Key Fingerprints? In Proceedings of the ACM CHI'17, pgs 3787-3798.
[2] Ribeiro, J. M., Batista, D. M., and de Pina, J. C. (2020). hashify: Uma Ferramenta para Visualização de Hashes com Animações. In Proceedings of the Tools Session of the XX SBSeg.
[3] Adida, B. (2008). Helios: Web-based Open-Audit Voting. In Proceedings of the 17th USENIX Security Symposium (USENIX Security 08), pgs 335–348. USENIX.

Inscrição No.: 1373

Título em Português:	Introdução à Geometria Diferencial
Título em Inglês:	Introduction to Differential Geometry
Área de Pesquisa:	Geometria e Topologia
Palavras Chave:	Métricas Riemanniana - fibrados - Física Matemática
Ag. Financiadora do Projeto:	CNPq - PIBIC
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	
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Resumo do Trabalho em inglês:

Smooth manifods

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Objectives

The objective of the present work was to carry out a study on differential geometry, an area that is a theoretical basis on which many interisting theories are based, such as Albert Einstein's theory of general relativity. The main focus of the work was to study the theory of differentiable manifolds, to know the structures involved, examples and interesting results.

Materials and Methods

In order to achieve the desired objectives, during the work, the books on differential geometry that appear in the bibliographic references were studied. In addition, meetings were held with the supervisor and seminars on differential geometry were presented to other students on the topic.

Results

Basic notions were studied to define smooth manifolds, such as the notion of chart, topological manifold, atlas and maximal atlas.

Then we study the notion of differentiability in the context of smooth manifolds and the notion of tangent space to a manifold, always paying attention to the analogies between differential topology and the study of analysis in euclidean spaces. We have seen that many results studied in courses on analysis in euclidean spaces have analogous versions for differential topology, such as the inverse mapping theorem, the implicit mapping theorem, the rank theorem, the immersion theorem and the submersion theorem. We have seen some examples of smooth manifolds, such as smooth maps graphs, product manifolds, torus, sphere, real projective space and the Lie groups.

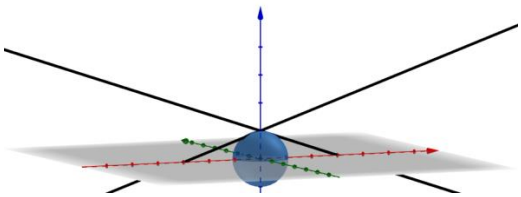

Figure 1: Stereographic projection

Vector bundles were studied, including tangent and cotangent bundles, later identified as particular cases of the tensor bundles. We also have been studied the concepts of vector fields and Lie bracket. A brief study on Lie algebras was made. Subsequently, we studied the theory of differential forms in the context of manifolds, a brief study was made on riemannian geometry and then the concept of partitions of unity was studied to build the theory of integration of differential forms and to prove the following theorem:

Theorem: Every smooth manifold admits a riemannian metric.

The final topic of the work was to study some strong results on smooth manifolds, such as Whitney theorem's:

Theorem: Every n-dimensional manfiold can be embedded in R^{2n}.

Conclusions

The work managed to open doors for the student to understand new subjects relatd to differential geometry, with an emphasis on mathematical physics. Using the knowledge adquired, it was possible to start a new path of studies, involving classical and quantum mechanics, general relativity and other theories of physics that use geometry in their formulation.

References

M.Lee, John. “Introduction to topological manifolds”. M.Lee, John. “Introduction to smooth manfiolds”. W. Tu, Loring. “An introduction to manifolds”. P. do Carmo, Manfredo. “Geometria diferencial de curvas e superfícies”. P. do Carmo, Manfredo. “Geometria riemanniana”.

Inscrição No.: 1766

Título em Português:	Classificação de sinais de EEG com modelos de regressão funcional
Título em Inglês:	Classification of EEG signals with functional regression models
Área de Pesquisa:	Probabilidade e Estatística Aplicadas
Palavras Chave:	Regressão Funcional - EEG - Wavelet e Fourier
Ag. Financiadora do Projeto:	Sem fomento
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	
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Resumo do Trabalho em inglês:

CLASSIFICATION OF EEG SIGNALS WITH FUNCTIONAL REGRESSION MODELS

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Objectives

Electroencephalography (EEG) is one of the best methods to assess cortical electrical activity. The EEG signal may be a result of spontaneous brain activity or may be related to sensory, motor and cognitive brain events [1]. This present research project aims to study signal processing techniques, such as Fourier transforms and Wavelet transforms, to decompose the EEG signal, and to evaluate the performance of Functional Regression models for predicting new data.

Materials and Methods

Spectral analysis allows the identification of interference sources and provides a quick and efficient way to identify the components of a signal. The bases of Fourier analysis are sine waves, and therefore the signal is analyzed as a whole. Wavelets decompose the signal into staggered and offset versions of its original Wavelet, they tend to be irregular and asymmetrical [2].

In functional logistic regression [3] the probability p_i of the occurrence of a binary event whose $Y_i = 1$ conditional on a functional predictor $X_i(t)$ and functional coefficient $\boldsymbol{B}(t)$ is expressed according to equation 1:

$$\mathrm{P}(\boldsymbol{Y}_i = 1|\boldsymbol{X}_i(t): t \in T) = \frac{e^{\alpha + \int_T X_i(t)\boldsymbol{B}(t)\,\boldsymbol{dt}}}{1+e^{\alpha + \int_T X_i(t)\boldsymbol{B}(t)\,\boldsymbol{dt}}} \qquad (1)$$

for $i = 1, \ldots, n$.

The use of basis and regularization functions in $X_i(t)$ contributes to reduce the bias induced by measurement error [4].

The quantitative evaluation of the research consists of the application of four different methods. Methodologies 1 and 2 are followed respectively by: Wavelet transform; application of the base function; functional logistic regression. For methodologies 3 and 4, the Fourier transform is adopted instead of the Wavelet. The first and third methodologies consider the Fourier basis function, while the second and fourth are applied to the Spline basis function. These approaches were applied to two EEG datasets related to emotion recognition and motor movement detection problems.

The Emotion base [5] consists of analyzing EEG signals from participants while they are playing rounds of games of chance. The subset provided contains aggregated observations from 23 participants. For our study, the objective will be to explain the potentials in the EEG signal for target conductivity corresponding to the monetary result (win or loss) at the end of each game round. The base contains a dimension of (184, 384) for the covariates and (1, 184) for the categorical variables. For modeling, we separated the base into training (70%) and testing (30%).

For *SelfRegulationSCP1* [6] the experiment with this dataset consists of evaluating whether the subject is increasing or decreasing his potential cortical slowness, that is, whether the subject has moved the cursor up or down. Recordings were made with 6 EEG channels at 256 Hz, which resulted in 896 samples per channel for each trial. The training base contains a dimension of (268, 896) for covariates and (1, 268) for the response variable, while for the test

base we have, respectively, (293, 896) and (1, 293).
To evaluate the performance of the adjusted models, some metrics derived from the confusion matrix were used, such as sensitivity, specificity and accuracy [7].

Results

The comparison of the performance of the models is summarized in Tables 1 and 2.

Table 1: Results in the Emotion set

	Emotion			
Methods	Sensitivity	Specificty	Accuracy	Accuracy IC 95%
1	0.77	0.58	0.68	(0.54, 0.80)
2	0.48	0.70	0.59	(0.45, 0.72)
3	0.39	0.46	0.43	(0.30, 0.57)
4	0.43	0.50	0.46	(0.33, 0.60)

Table 2: Results in the Self Regulation set.

	Self Regulation			
Methods	Sensitivity	Specificty	Accuracy	Accuracy IC 95%
1	0.88	0.88	0.87	(0.82, 0.91)
2	0.91	0.86	0.88	(0.84, 0.92)
3	0.52	0.51	0.52	(0.46, 0.57)
4	0.54	0.52	0.53	(0.47, 0.59)

It is notable that the best performance for the Emotion dataset in the test base was using methodology 1, that is, applying the Wavelet transform with the Fourier Base function (consisting of 8 components), a value of 68% accuracy. For the Self Regulation dataset, methodology 2 stands out with the best performance obtained, that is, applying the Wavelet transform with the Fourier Base function (composed of 4 components) and adjusting the Functional Logistic Regression for the channels 1, 2, 3, 4 and 5. The model's accuracy was 88% on the test basis.

Conclusions

The initial hypothesis of the research project is to evaluate the performance of functional regression models for predicting new data, using signal processing techniques, such as Fourier transforms and Wavelet transforms, to decompose the EEG signal. In this work we saw that the functional regression models with the aid of the Wavelet transform and Fourier transform are promising statistical and mathematical techniques for evaluating EEG data. Comparing the results of the Self Regulation set with those obtained in [2] using machine learning models, there is an increase of 2 percentage points in the accuracy result, which indicates that the approach of working with functional data by applying the regression models functional for EEG signals is satisfactory.

References

[1] I. Gannaz, “Classification of EEG recordings in auditory brain activity via a logistic functional linear regression model.”, p. 125–130, jun. 2014, Acessado: ago. 26, 2022. [Online]. Available: https://hal.archives-ouvertes.fr/hal-00830313

[2] L. Alípio, “Unraveling the Brain: a Quantitative Study of EEG Classification Techniques”, 2021.

[3] M. Febrero-Bande e M. O. de la Fuente, “Statistical Computing in Functional Data Analysis: The R Package fda.usc”, *J Stat Softw*, vol. 51, nº 4, p. 1–28, out. 2012, doi: 10.18637/JSS.V051.I04.

[4] J. S. Morris, “Functional Regression”, jun. 2014, Acessado: ago. 26, 2022. [Online]. Available: http://arxiv.org/abs/1406.4068

[5] “emotion: EEG and EMG recordings in a computerised gambling study in fdboost/FDboost: Boosting Functional Regression Models”. https://rdrr.io/github/fdboost/FDboost/man/emotion.html (acessado ago. 26, 2022).

[6] “Time Series Classification Website”. http://www.timeseriesclassification.com/description.php?Dataset=SelfRegulationSCP1 (acessado ago. 26, 2022).

[7] X. Deng, Q. Liu, Y. Deng, e S. Mahadevan, “An improved method to construct basic probability assignment based on the confusion matrix for classification problem”, *Inf Sci (N Y)*, vol. 340–341, p. 250–261, maio 2016, doi: 10.1016/J.INS.2016.01.033.

Inscrição No.: 1822

Título em Português:	Diagnóstico do COVID-19 nos registros de SRAG - Síndrome Respiratória Aguda Grave com base na Autópsia Verbal
Título em Inglês:	Diagnosis of COVID-19 in the records of SARS - Severe Acute Respiratory Syndrome based on Verbal Autopsy
Área de Pesquisa:	Estatística
Palavras Chave:	Autópsia Verbal - COVID-19 - Métodos de classific
Ag. Financiadora do Projeto:	Sem fomento
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
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Resumo do Trabalho em inglês:

Diagnosis of COVID-19 in SARS - Severe Acute Respiratory Syndrome records based on Verbal Autopsy

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Objectives

The present study aims to explore statistical methods of classification for application in cases of death from severe acute respiratory syndrome with an unspecified etiologic agent, differentiating between them caused or not by COVID-19.

Materials and Methods

The data used in the study refer to hospitalized cases of severe acute respiratory syndrome that were registered in 2020 and 2021, and were obtained through the SUS website. The complete database for 2020, available at https://dados.gov.br/dataset/bd-srag-2020, had 1,512,296 observations with 225 variables, but only were considered in the analysis individuals over 10 years of age and who died, reducing the database to a total of 284,915 observations. The 2021 database, obtained at https://dados.gov.br/dataset/bd-srag-2021, had 1,715,835 observations with 161 variables, leaving only 437,370 after applying the same restrictions. Thus, there are a total of 722,285 valid cases for analysis.

The available variables are obtained through an individual registration form, filled in by the patient, his companion or a member of the hospital team. It consists of questions about the symptoms that the patient has been showing, such as fever and loss of smell and taste, as well as historical aspects, such as whether the individual has traveled to a region at risk for contagion with the coronavirus or if had contact with an infected person.

This study aims to classify as Covid or Non-Covid for patients whose cause of death is not available for some reason. Before that, cases in which the cause of death are specified were used. A descriptive analysis of the data was performed followed by the application of the following statistical techniques (Hastie et al., 2017; James et al., 2014):

- Regressão logística
- Árvores de classificação
- Random forest
- KNN

For each of the methods, two models were built, one that includes the variable 'RT-PCR positive' and the other without, since it is understood that it is not always possible to count on this variable. The performances of the techniques were compared through the confusion matrix, considering the dataset partitioned as “training set” and “test set”. In the second phase, classification techniques were applied to the SARS dataset whose patients' cause of death were not specified.

Inspired by Duarte-Neto (2021), the results were further compared with the COVID-19 Rapid Mortality Surveillance (CRMS) algorithm.

Results

The different models and algorithms were compared using a set of metrics, specifically accuracy (A), sensitivity (S), specificity (E), positive predictive value (PPV) and negative predictive value (NPV).

Below are presented some of the results obtained.

Table 1: Metrics in the test set for models without the PCR variable.

Modelo	A	S	E	PPV	NVP
Logistic regression	0,59	0,58	0,64	0,85	0,30
Classification tree	0,61	0,64	0,52	0,82	0,29
Balanced random forest	0,78	0,99	0,02	0,78	0,55
Unbalanced random forest	0,60	0,58	0,66	0,86	0,31
Unbalanced KNN	0,78	0,98	0,04	0,78	0,44
Balanced KNN	0,58	0,57	0,63	0,85	0,30

Table 1 shows the results of models that do not take the PCR variable into account. The highest values for accuracy (0.78), sensitivity (0.99) and negative predictive value (0.55) were obtained by the random forest trained from the unbalanced data. However, the specificity for the model was the lowest among all, indicating that only 2% of the test set data that were Non-Covid were correctly classified.

Table 2: Metrics in the test set for models with the PCR variable.

Modelo	A	S	E	PPV	NVP
Logistic regression	0,75	0,68	0,96	0,98	0,46
Classification tree	0,72	0,64	0,99	0,99	0,44
Balanced random forest	0,82	0,93	0,44	0,85	0,63
Unbalanced random forest	0,75	0,69	0,96	0,88	0,47
Unbalanced KNN	0,81	0,91	0,46	0,86	0,60
Balanced KNN	0,75	0,69	0,94	0,98	0,47
CRMS	0,65	0,62	0,78	0,91	0,37

Table 2 shows the results of the models using the PCR variable, this time including the CRMS algorithm. In findings similar to those obtained for the models without the PCR variable, the highest values of accuracy (0.82), sensitivity (0.93) and negative predictive value (0.63) were obtained for the random forest with unbalanced data. The highest values for specificity (0.99) and positive predictive value (0.99) were obtained this time through the classification tree with balanced data and pruning.

Also noteworthy is the large increase in the specificity values of the models when the PCR variable is included in the analysis.

During the work, several models were tested, so that the results obtained in the test set, in addition to theoretical knowledge and works used as a reference, proved to be useful for choosing the best cut-off point (in the case of logistic regression and CRMS), as well as parameters for tree-based models and KNN. In addition, they helped to visualize the impact of using a balanced set of data to train the models.

Below are the predictions of a subset of the models tested for the 919 patients who did not have their cause of death identified.

Table 3: Predictions for deaths with unknown causes using models without the PCR variable.

Model	Covid	Non-Covid
Logistic regression	797	122
Classification tree	606	313

Balanced random forest	896	23
Unbalanced random forest	409	510
Unbalanced KNN	879	40
Balanced KNN	416	503

Analyzing the results obtained, it is interesting to see how, for the same model, the use of unbalanced data to perform the model training implies a greater tendency to classify a death case as caused by COVID-19.

Table 4: Predictions for deaths with unknown causes using models with the CRP variable.

Model	Covid	Non-Covid
Logistic regression	150	769
Classification tree	1	918
Balanced random forest	573	346
Unbalanced random forest	50	869
Unbalanced KNN	548	371
Balanced KNN	66	853
CRMS	298	621

For the models with the PCR variable, in addition to the similar results regarding the balancing of the data, it is also possible to note that the use of the variable causes the model to classify more deaths as Non-Covid.

Conclusions

The study allowed to explore different statistical classification methods, comparing the different metrics obtained in each one of them and their particularities. It was noticed how a very important variable can influence the models, as was the case with PCR, and what are the impacts of working with a highly unbalanced dataset within a classification context.

It was also possible, considering the models that used the PCR variable, to obtain results that were better than the CRMS.

As the last objective achieved, the study allowed, through the application of some of the algorithms for cases of death with an unknown cause, to become aware of the degree of underreporting of deaths from COVID-19 in patients with SARS.

References

- Duarte-Neto, A.N. et al., Rapid Mortality Surveillance of COVID-19 Using Verbal Autopsy. Int J Public Health, 05 October 2021
- Hastie, T., Tibshirani, R., Friedman, J. The Elements of Statistical Learning: Data Mining, Inference and Prediction (2017). Springer New York.
- James G, Wittenm D, Hastie T, Tibshirani R. An introduction to Statistical Learning: with Applications in R (2014). Springer New York.

Inscrição No.: 2440

Título em Português:	Co-modulação das variabilidades das frequências cardíacas durante um jogo de computador
Título em Inglês:	Co-modulation of heart rate variabilities during a computer game
Área de Pesquisa:	Metodologia e Técnicas da Computação
Palavras Chave:	VFC - séries temporais - classificação
Ag. Financiadora do Projeto:	Outros
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	
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Resumo do Trabalho em inglês:

Co-modulation of heart rate variability during a computer game

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Objectives

The main objectives of this project are to identify whether individuals can distinctly affect each other's physiological states and emotional experiences in the absence of direct communication during a game using heart rate variability (HRV) signals, to investigate autonomic reactions and responses, and to assess whether HRV synchrony is associated with the convergence of emotional response or not.

Materials and Methods

At least 20 pairs of volunteers will be recruited, i.e. 40 male healthy volunteers ranging in age between 18 and 40 years. The game Pong is used in the study, which consists of two paddles controlled by each player to hit an in-game ball back and forth. A player scores when the other one misses and the ball reaches the other side. During the experiment all data from the position of the paddles and the logs generated by the game are collected. The heart frequency data is also continuously recorded using the Polar 10 heart frequency sensor and a professional chest strap placed just below the participant's chest muscles.

The experiment starts when the participants are placed in distinct rooms not knowing about each other. They shall stay at rest for a few minutes so that their basal heart rate is recorded. Then the game begins and lasts for 20 minutes, separated into 4 random phases of 5 minutes each: 2 which the HRV data is collected in a match which participants play against each other and another 2 which volunteers play against the computer.

For the analyses, we implemented a pipeline in Python and R that performs signal correction to remove interference and performs the Granger causality test according to the definition formalized by Fujita et al (2010). By rejecting the test's null hypothesis, we verify Granger causality among the players' HRV.

Granger causality tests are also performed between the positions of the paddles in the game. In addition, Bootstrap tests for correlation between the players' HRV and the position of the paddles were conducted.

Results

To date, 5 repetitions of the experiment ready for analysis have been carried out and the p-values resulting from the tests are shown in Table 1. The rows identify the repetitions of the experiment with distinct pairs and the columns identify its phases. GH1-2 e GH2-1 correspond respectively to the Granger causality results from Player 1's to Player 2's HRV and from Player 2's to Player 1's HRV. Similarly, GP1-2 corresponds to the Granger test result from Player 1's to Player 2's HRV. PH and PP represent the Bootstrap test results for the correlation between the players' HRV and between the paddles respectively.

Furthermore, colors are used to identify whether it's a phase where players play with each other or not. The cells are coloured red when they do and blue when they don't. Darker color tones highlight the results considered

statistically significant with significance level set to 5%.

As expected, Granger's causality is verified between paddles in most phases in which players play together. As for the phases where they play against the computer, significant p-values are very rare. Moreover, for the majority of the experiments, Granger causality between players' HRV was verified at least from one player to another in not less than one of its two phases in which players interact.

Most of the Bootstrap tests' p-values aren't statistically significant, but when they are, Granger causality is usually also verified.

Table 1: Results from the experiments' hypothesis tests

	Phase 1						Phase 2						Phase 3						Phase 4					
Exp	GH1-2	GH2-1	PH	GP1-2	GP2-1	PP	GH1-2	GH2-1	PH	GP1-2	GP2-1	PP	GH1-2	GH2-1	PH	GP1-2	GP2-1	PP	GH1-2	GH2-1	PH	GP1-2	GP2-1	PP
1	0.202	0.046	0.516	0.000	0.025	0.109	0.214	0.706	0.896	0.722	0.557	0.130	0.977	0.277	0.584	0.005	0.007	0.139	0.375	0.176	0.261	0.997	0.408	0.121
2	0.129	0.695	0.039	0.424	0.181	0.532	0.449	0.980	0.759	0.922	0.373	0.350	0.499	0.850	0.284	0.121	0.079	0.916	0.723	0.406	0.302	0.000	0.054	0.007
3	0.018	0.456	0.003	0.776	0.100	0.773	0.993	0.216	0.004	0.002	0.020	0.764	0.272	0.637	0.872	0.583	0.140	0.577	0.050	0.045	0.079	0.546	0.006	0.008
4	0.013	0.097	0.193	0.002	0.021	0.358	0.059	0.473	0.905	0.086	0.568	0.617	0.974	0.270	0.412	0.223	0.303	0.382	0.232	0.442	0.114	0.409	0.601	0.430
5	0.525	0.695	0.521	0.932	0.296	0.350	0.449	0.465	0.890	0.003	0.146	0.021	0.130	0.529	0.831	0.048	0.049	0.022	0.721	0.727	0.469	0.584	0.243	0.484

Conclusions

The partial results support the hypothesis that individuals can affect each others' physiological states and converge emotional responses without direct communication. These results also support the hypothesis that the synchronization of the individuals' HRV is associated with this emotional convergence. However, it is still necessary to collect more data to reach stronger conclusions.

References

BRUDER, Martin et al. Emotional signals in nonverbal interaction: Dyadic facilitation and convergence in expressions, appraisals, and feelings. Cognition & emotion, v. 26, n. 3,p. 480-502, 2012.

FELDMAN, Ruth. On the origins of background emotions: From affect synchrony to symbolic expression. Emotion, v. 7, n. 3, p. 601, 2007a.

FERRER, E., & HELM, J. L. Dynamical systems modeling of physiological coregulation in dyadic interactions. International journal of psychophysiology, 88(3), 296-308, 2013.

FUJITA, A.; SATO, J. R.; KOJIMA, K.; GOMES, L. R.; NAGASAKI, M.; SOGAYAR, M. C.; MIYANO, S. Identification of Granger causality between gene sets. Journal of Bioinformatics and Computational Biology, 8(04), 679-701, 2010.

GOLLAND, Y.; ARZOUAN, Y.; LEVIT-BINNUN, N. The mere co-presence: Synchronization of autonomic signals and emotional responses across co-present individuals not engaged in direct interaction. PloS one, 10(5), e0125804, 2015.

GRANGER, C. W. Investigating causal relations by econometric models and cross-spectral methods. Econometrica: journal of the Econometric Society, 424-438, 1969.

LEVENSON, Robert W.; GOTTMAN, John M. Marital interaction: physiological linkage and affective exchange. Journal of personality and social psychology, v. 45, n. 3, p. 587, 1983.

Inscrição No.: 2434

Título em Português:	Co-modulação das variabilidades das frequências cardíacas durante a interação imitador-modelo
Título em Inglês:	Co-modulation of heart rate variabilities during imitator-model interaction
Área de Pesquisa:	Metodologia e Técnicas da Computação
Palavras Chave:	VFC - séries temporais - classificação
Ag. Financiadora do Projeto:	Outros
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
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Resumo do Trabalho em inglês:

Co-modulation of heart rate variability during imitator-model interaction

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Objectives

The project's goal is the development of a controlled social interaction situation between two willing participants in order to collect their heart rate data and analyze it to try and verify the co-modulation of the variability of the data acquired.

It was aiming to test if a physiological alteration, caused by the situation that was setup, could be verified through the analysis of the series generated, and how the both series are related.

Methods and Procedures

The project occured in 6 phases: bibliographic research, development of the experiment, development of a customized software, data collection, data cleaning and preparation and finally the analysis.
First, a detailed examination of the article published by Dumas et. al (2010) was conducted because the project's proposal was to develop an experiment analogous to the one created by them, but capture heart rate data instead.
After that, the idea of the experiment was adapted from the article mentioned to the conditions that were available at the moment. There would be two participants that would communicate only via gestures and without seeing each other directly. Two computers would be positioned in front of them to record and show their video to each other and the goal was that they took turns to imitate each other and vice versa. The experiment consisted in different phases to measure different states of the participants heart rates: resting, random gestures without their partner being able to see, spontaneous imitation (spontaneous gestures and the participants decide when to imitate each other) and a participant imitating its partner.
Following that, it was deemed necessary the implementation of a custom software so the data capture could happen as smoothly as possible and avoid tempering with the experiment. Two applications were developed that would communicate between themselves, where one of them would control the experiment, sending and receiving monitoring signals (Operator) and the other would receive the commands and execute them (Subject). The configuration of the experiment consisted in two applications Subject, one for each participant and one application Operator that would control and monitor the experiment, all of them connected through the network.
With the software implemented, 11 captures were performed, with 6 pairs of male subjects and 5 pairs of female subjects. The heart rate data was captured via a Polar H10 Sensor (https://www.polar.com/br/produtos/accessorios/transmissor_cardiaco_h10) which provides both heart rate data and R-R data.
After the collection of the data, the efforts were directed to compiling all of it in usable form and creating a general dataset that had all usable

data in its columns. First, the normalization of it had to be done because there were inconsistencies created by desynchronization of the devices between the two sensors due to the fact that they were sending data at different times. The processing phase focused on executing the interpolation of the data in two different ways (linear interpolation and nearest neighbor interpolation) in order to index each entrance in the final dataset by one integer second, effectively pairing both heart rate series. Besides that, it was necessary to annotate all experiments in order to section the videos in the exact phases of the experiment and obtain the time frames for the sections and its properties. The properties that were of interest were: if the participants were gesturing in sync, if they are in fact imitating each other, who is the imitator and who is the model. The software ELAN (https://archive.mpi.nl/tla/elan) was used and it generated an output file that was then parsed and the data obtained from it was added to the general dataset.
After the cleaning and preparation was done, the analysis phase was started. In it multiple descriptive statistics were obtained from all columns of the dataset and distribution tests were also applied to it. In particular, a PDC analysis was carried out on the heart rate series. The technique introduced by Baccala and Sameshina em 2001 which is able to analyze functional relationships between both series.

Results

Following the analysis phase, an extensive report was generated containing every result obtained. It contained both graphs generated by the function that implemented the PDC analysis and all the statistics extracted from the dataset columns. The results are still being analyzed and it was not yet possible to obtain any conclusions.

Bibliography

Guillaume Dumas, Jacqueline Nadel, Robert Soussignan, Jacques Martinerie, and Line Garnero. Inter-brain synchronization during social interaction.

PloS one, 5(8):e12166, 2010
Luiz Baccala and Koichi Sameshima. Partial directed coherence: A new concept in neural structure determination. Biological Cybernetics, 84:463–474, 05 2001.

Inscrição No.: 2438

Título em Português:	Co-modulação das variabilidade das frequências cardíacas e linguagem
Título em Inglês:	Co-modulation of heart rate variabilities and language
Área de Pesquisa:	Metodologia e Técnicas da Computação
Palavras Chave:	VFC - séries temporais - classificação
Ag. Financiadora do Projeto:	CNPq - PIBIC
Projeto:	Iniciação Científica
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Resumo do Trabalho em inglês:

Co-modulation of heart rate variability and language

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Objectives

Human beings have the unique ability to communicate and extract meaning through spoken and written language. And such language processing is one of the most complex cognitive tasks that humans routinely engage in.

To explore the heart rate and behavior of social interaction, this research project aims to assess the interaction between two or more individuals. In this work, we reviewed studies on neural mechanisms and markers of social interactions, however the research bias was based on performing a behavioral experiment via analysis of heart rate variability.

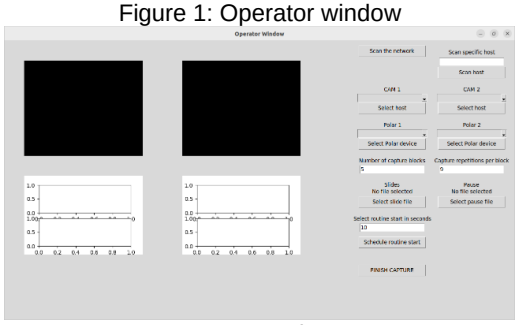
Methods and Procedures

In this work, we will reproduce the behavioral experiment described by Dikker et al. (2014), in which 10 speakers and 10 listeners will be selected, all men, right- handed and aged between 20 and 40 years.
All participants must have normal or corrected-to- normal vision, and no history of psychiatric or neurological disorders, free from any psychotropic medication.
During the experiment, those selected will be asked to view 45 hand-drawn color images depicting fictional scenes in which an animal or object performs an action on another animal or object (for example, a penguin hugging a star). Scenes will be built based on phrases that will be created through random combination of 45 transitive verbs and 90 nouns, denoting common objects, animals and foods. The speaker will be instructed to describe the images using declarative sentences, simple in the present continuous tense, with a single transitive verb and without adjectives or adverbial phrases.
Each image was assigned a predictability score, derived from an online questionnaire in which 40 volunteers described each of the 45 scenes with the description they considered most appropriate. None of these 40 volunteers will participate in the experiment. For each scene, they assigned a score to each participant, reflecting the percentage of participants who entered the same answer. Predictability is given by 1 minus the entropy value, with 0.0 being the least predictable and 1.0 the most predictable. Based on the distribution of predictability across items, items will be assigned one of two conditions: high predictability and low predictability.
We asked participants to indicate on a scale of 1 to 5 how certain they were that other people would also enter the exact same sentence. The sentences will be described by the speaking participants during the sessions of the face-to-face experiment, and will be simultaneously heard by the listeners during the capture. Then, for both the speaker (N = 10) and listeners (N = 10), we will present each image for 7.5 seconds, followed by 7.5 seconds of blank and then intermittent fixation crosses (375 ms on /off, 3 seconds total).
Then the display of the next image begins. Each participant will see a total of 45 trials in random order, distributed in five blocks. Each session will last approximately 45 minutes and in parallel, heart rate will be captured by Polar H10 electrocardiogram sensors.

Results

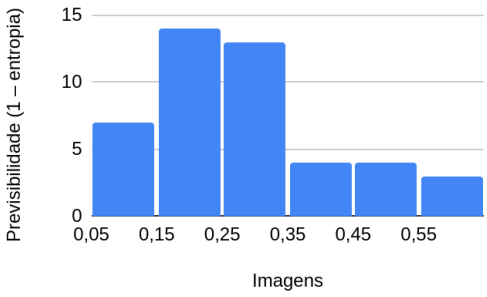
For the captures, a software was built, whose main characteristics are:
•It has an operator window, where the beginning of the capture is controlled.
•Allows you to establish the capture routine, image display time, pause, etc.
•Has windows for volunteers, where the start of the capture is controlled.
•Displays the images contained in a folder randomly.
•Saves the electrocardiogram signal.
•Automatically locates available cameras, sensors and electrocardiogram on localhost and other hosts on the network.

Figure 1: Operator window



Source: Authors

Figure 1 shows the capture of the operator window that will control the data acquisition in the face-to-face experiment.



Source: Authors

Figure 2 demonstrates the histogram constructed from the predictability coefficient calculated based on the online experiment.

Conclusions

So far the software has been implemented and the predictability indexes of the images have been calculated, the dataset will be captured from the second half of September.

Bibliographic references

1.Dikker, Suzanne, et al. "On the same wavelength: predictable language enhances speaker–listener brain-to- brain synchrony in posterior superior temporal gyrus." Journal of Neuroscience 34.18 (2014): 6267-6272.
2.Dale, Anders M., Bruce Fischl, and Martin I. Sereno. "Cortical surface- based analysis: I. Segmentation and surface reconstruction." Neuroimage 9.2 (1999): 179-194.
3.Dikker, Suzanne, et al. "Early occipital sensitivity to syntactic category is based on form typicality." Psychological Science 21.5 (2010): 629-634.
4. Sänger, Johanna, Ulman Lindenberger, and Viktor Müller. "Interactive brains, social minds." Communicative & integrative biology 4.6 (2011): 655-663.
5.Bergerbest, Dafna, Dara G. Ghahremani, and John DE Gabrieli. "Neural correlates of auditory repetition priming: reduced fMRI activation in the auditory cortex." Journal of Cognitive Neuroscience 16.6 (2004): 966-977.

Título em Português: Co-modulação das variabilidades das frequências cardíacas durante a coordenação do ritmo da fala
Título em Inglês: Co-modulation of heart rate variabilities during coordination of speech rhythm.
Área de Pesquisa: Metodologia e Técnicas da Computação
Palavras Chave: VFC - séries temporais - classificação
Ag. Financiadora do Projeto: USP - Programa Unificado de Bolsas
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USP - UNIVERSIDADE DE SÃO PAULO
30º Simpósio Internacional de Iniciação Científica

Inscrição No.: 2439

Resumo do Trabalho em inglês:

CO-MODULATION OF HEART RATE VARIABILITIES DURING COORDINATION OF SPEECH RHYTHM

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Objectives

Individual behavioral rhythms can be spontaneously synchronized during social interactions between humans via verbal and nonverbal communication. In a recent study, we observed a correlation between the synchronization of brain rhythm and speech rhythm (duration and pronunciation interval) between individuals during alternating speech tasks [1]. In that experiment, twenty pairs of participants alternately pronounced letters of the alphabet during electroencephalography (EEG). These tasks were conducted between pairs of volunteers (human-human tasks) and individually between each subject and a machine (human-machine tasks) in a predetermined sequence. The current research project proposes reproducing this study using a different biological marker. Here, we will verify whether synchronizations of heart rate variability (HRV) modulations arise when speech rhythms are synchronized between two subjects over verbal communication tasks. Finally, besides the numerous existing applications [2, 3], the fact that HRV can be measured noninvasively, the relatively low cost and ease of obtaining these data, and the existence of advanced and relatively simple statistical methods for the analysis of these experimental measures have made HRV the subject of intense investigation in the literature.

Methods and Procedures

The experiment consists of activities in which volunteers must alternately (and repeatedly) pronounce the letters of the alphabet A through G. At the same time; we collect their heart rate data (Fig. 1) with Polar H10 heart transmitters.
We conduct these tasks between the participants (human-human tasks) and individuals and equipment (human-machine tasks), with each session lasting 70 seconds and interrupted by 10 seconds. At the end of each of the sessions, a questionnaire will be applied in which individuals will be asked to rate the subjective aspects of comfort (Fig. 2), synchrony, speed, initiative, and humanity (of the machine only) of their experience on a scale of 1 to 5.
The rehearsal begins with a test in which recordings of the voices of both elements will be obtained. The audio obtained from this practice will then be filtered, separated, and restricted to files containing the pronunciation of a single phoneme. We have then two human-human stages, alternating the volunteer initiating the session. Sound, video, VFC, and subjective ratings will be obtained from these interactions.
With this, human-machine interactions begin. Each element must perform a total of 10 of these activities. The device with which the volunteers must interact consists of a monitor and speakers, through which phonemes will be reproduced in the voices of Electronic, Male, Female, Subject 1 (or 2), and Subject 2 (or 1). The phonemes reproduced in Subjects 1 and 2's voices will be from those obtained in the test session of the process. The participants will perform this practice alternately in batches of five sessions each. In the first two, we will use fixed time intervals of 1 second between the

end and the beginning of the machine's pronunciation of two consecutive phonemes.
In contrast, we will use random intervals of 800 to 1200 milliseconds in the last two batches. Audio, video and VFC data recordings should be obtained from these blocks. Once all the human-machine interactions are finished, the volunteers should perform two new human-human sessions, alternating the one to start the process.

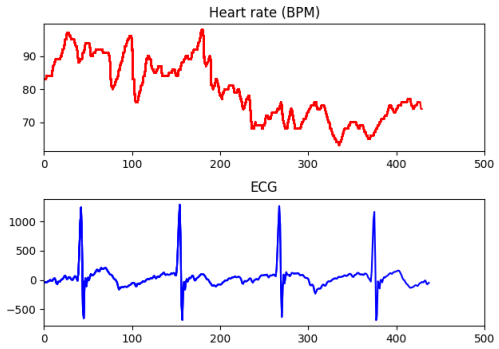

Figure 1: Plot of HRV data obtained from the heart transmitter.

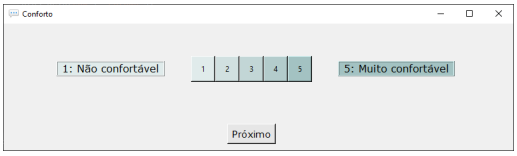

Figure 2: Subjective ratings of “confort”.

Results

The program necessary for conducting the experiment, as well as the experimental setup (Fig. 3), has been completed. In the illustrative image of the setup we can see a graphical representation of the HRV data to be obtained in each session and also the 5-point scale associated with the subjective rating of "comfort" to be performed at the end of each session.

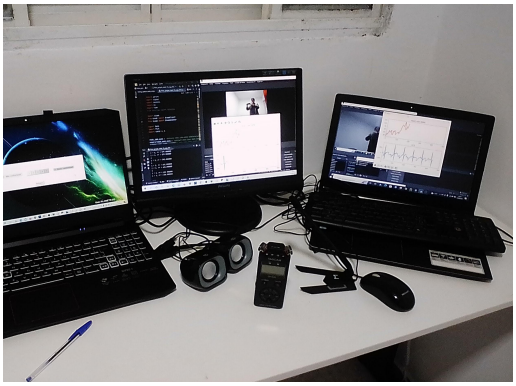

Figure 3: Experimental setup.

Conclusions

We emphasize the potential relevance of this study. Virtual realities have become popular due to the extensive technological advances in the last decades. They tend to become part of daily human life. Understanding how this technology will alter human interactions has been the subject of studies as in [4]. This project may contribute to understanding how human-machine interaction may or may not come to influence human relationships, in addition to finding differences or not in physiological responses in human and human-machine contacts through the promising biomarker VFC. Moreover, understanding the relationship between social interaction and physiological states, comodulation of HRVs, will allow us to propose new forms of therapy for disorders that affect social interaction, such as autistic spectrum disorder.

References

[1] KAWASAKI, Masahiro et al. Inter-brain synchronization during coordination of speech rhythm in human-to-human social interaction. Scientific reports, v. 3, n. 1, p. 1-8, 2013.

[2] ABREU, Luiz Carlos de. Heart rate variability as a functional marker of development. Journal of Human Growth and development, v. 22, n. 3, p. 279-282, 2012.

[3] VANDERLEI, Luiz Carlos Marques et al. Noções básicas de variabilidade da frequência cardíaca e sua aplicabilidade clínica. Brazilian Journal of Cardiovascular Surgery, v. 24, p. 205-217, 2009.

[4] DE CARVALHO PEREIRA, Itamar. Metaverso. 2009. Tese de Doutorado. UNIVERSIDADE DE BRASÍLIA.

Inscrição No.: 2678

Título em Português:	Estimação condicional da distribuição de passes em futebol
Título em Inglês:	Conditional estimation of the distribution of passes in soccer
Área de Pesquisa:	Probabilidade e Estatística Aplicadas
Palavras Chave:	Estatística - Probabilidade - Futebol
Ag. Financiadora do Projeto:	Sem fomento
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	
Validado em:	**07/09/2022**

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Orientador:

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Resumo do Trabalho em inglês:

Conditional estimation of the distribution of passes in soccer

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Objectives

Proper quantification of the expected value of a ball possession in a soccer match has modeling the evolution of the play as a necessary step, through transitions between the different possible states of the field.

The objective of this project is to study one of the forms of transition, through the distribution of passes in the field. For such goal, it is sought to estimate the bivariate density, considering the two dimensions of the soccer field, of the pass target conditionally to the state of the match, taking into account information such as the initial position of the pass and the location of the players.

Materials and Methods

The project was conducted through the comparison of different variations of the FlexCode conditional density estimator (Izbicki and Lee, 2017), based on the estimation of the density function through decomposition of the function into an orthogonal functional basis and estimation of the coefficients associated to the basis elements through point estimation.
A simulation study was performed in different contexts of bivariate conditional density estimation, with diverse dependency structures between the response and explanatory variables and between the two response variables, in order to choose an adequate loss function between four alternatives.
Once chosen the loss function, the performance of variations of the FlexCode estimator was compared for the density estimation of pass targets from a dataset of five U-20 soccer matches. In addition to different covariate choices between models, the use of the bivariate FlexCode, whose functional basis is given by the tensor product of univariate functional bases, was compared to modeling each marginal response variable through the univariate FlexCode estimator, with joint density given by the product of marginal densities.

Results

The simulation studies pointed to the following loss function

$$L(\hat{f}, X, Z) = \frac{1}{n}\Delta\sum_{i=1}^{n}\sum_{(s_j,s_k)\in S}\hat{f}((s_j, s_k) \mid x_i)\text{^}2 - 2\cdot\frac{1}{n}\sum_{\{i=1\}}^{n}\hat{f}((z_{i_1}, z_{i_2}) \mid x_i),$$

evaluated on a grid S with area Δ between the points composing it, proposed by Izbicki and Lee (2017), as the most adequate through comparison with visual inspection of the simulations.

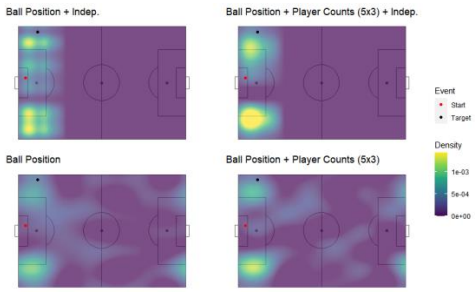

Figure 1: Comparison of four density estimation models for a pass

The best performing model with respect to the loss function between the options considered was the separate marginal estimation model with joint density given by the marginal densities' product, taking into account as covariables the

ball position in the horizontal and vertical dimensions of the field and the count of players from each team in fifteen 35 m x 23 m regions spaced uniformly on the field in 5x3 configuration. The results of including the covariates Ball (ball position in the horizontal and vertical coordinates) and Players (player count for each team in the regions) and of separate estimation for each marginal (Independence) or joint estimation (Bivariate) are available in Table 1 through mean loss in the test subset of the data.

Model	Loss
Uniform density	-0,00014
No covariables, Independence	-0,00020
Ball, Independence	-0,00066
Ball + Players, Independence	-0,00068
Ball, Bivariate	-0,00061
Ball + Players, Bivariate	-0,00064

Table 1: Mean losses in test subset of the data for the considered models

Conclusions

We conclude that the distribution of passes in the soccer field can be modeled using the state of the play. The inclusion of the ball position on the start of the pass results in a large improvement on the model. Additionally, the information of player counts per team in each region, even in regions of considerable area, presents an improvement over having only the ball position, in both forms of estimation. For both covariate sets, a better performance was obtained by estimating the conditional density for each variable separately than by estimating them jointly. We speculate that any improvements gained from modeling the dependency structure between the two dimensions of the response variable are smaller than a loss in marginal precision occurring in the joint estimation, as illustrated by Figure 1.

References

IZBICKI, R.; LEE, A. B. (2017). Converting high-dimensional regression to high-dimensional conditional density estimation. **Electronic Journal of Statistics, 11(2)**, 2800–2831.

Inscrição No.: 2967

Título em Português:	Algoritmos eficientes para estatísticas de rede baseadas em espectro
Título em Inglês:	Efficient algorithms for spectra-based network statistics
Área de Pesquisa:	Matemática da Computação
Palavras Chave:	Grafos Aleatórios - Densidade Espectral - Função Geradora
Ag. Financiadora do Projeto:	FAPESP - Fundação de Amparo à Pesquisa do Estado de São Paulo
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	
Validado em:	**08/09/2022**

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30º Simpósio Internacional de Iniciação Científica

Inscrição No.: 2967

Resumo do Trabalho em inglês:

Efficient algorithms for spectra-based network statistics

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Objectives

The graph's spectra contain essential structure information of complex real-world networks. By extracting the same spectra information from random Graphs Models, we can empirically test the goodness of fit between concrete and theoretical Random Graphs (e.g., Watts Strogatz (WS), Barabási Albert (BA), Erdős Rény (ER), and Geometric (GRG)). However, the computational bottleneck of this approach is the high computational cost $O(n^3)$ to compute all the eigenvalues of the n-dimensional adjacency matrix. Consequently, this approach is very limited in the context of big graphs, even whether they are sparse. Therefore, in this work, we implement and develop algorithms with alternative strategies to use the spectra information for network statistics and provide an open-source code of the methods in R.

Methods

The main result is a new random graph model parameter estimation method using the *Moment Generation function* of the *Laplacian Matrix,* which we combine with a new dimension reduction method. We show that the *Moment Generation function (*MGF*)* contains the relevant eigenvalue information for unifying the parameter of some random graph models. Furthermore, it is sufficient to approximate the MGF function with the traces of the matrices L and L^2 where L is the *Laplacian Matrix. Also, t*hese traces are estimated using *Hutchinson's Trace Estimator,* improving, even more, the computational efficiency. Finally, the shape of the approximation does not change with the size of the graph (e.g., **Figure 1-2** for the parameter p of the WS model), enabling a dimension reduction method. For estimating the parameter, we use a Monte Carlo method with the ternary search optimization algorithm [2]. Lastly, we also show a robust and efficient implementation of the well-known *Kernel Polynomial Method (KPM)* for estimating the *Spectral Density*.

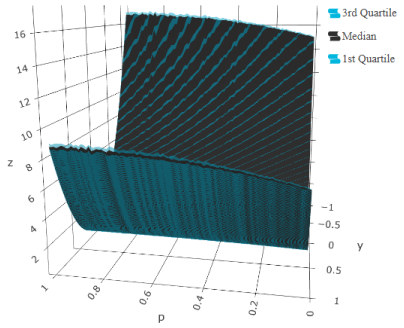

Figure 1: MGF approximation with L and L^2 from 50 samples of $WS\ (n = 100)$ for some values of p.

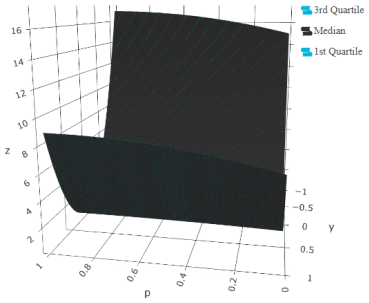

Figure 2: MGF approximation with L and L^2 from 50 samples of $WS\ (n = 10000)$ for some values of p.

Results

We first verify the KPM method and our implementation by estimating the spectral densities of some real-world networks. One

approximation is in **Figure 3,** and we can see that the KPM method achieves a good estimation of the spectral density.

Then, we test our parameter estimation methods with the ER, WS, GRG, and BA models, which produce consistent estimations with much less necessary computational time. For example, for the WS model in **Figure 4,** we can see that the *Moment Generating Function with Dimension Reduction* method produces consistent estimations of the parameter p. Furthermore, even for a WS graph with size $n = 10^6$, the computational method time simulated was between 61-71 seconds.

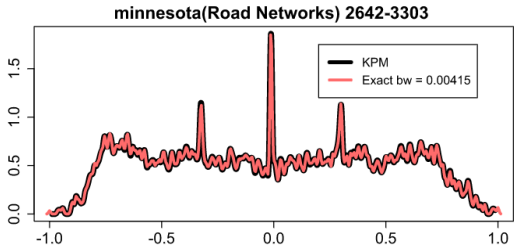

Figure 3: Spectral Density of a real-world network with "nº of vertices - nº of edges," calculated by the KPM method and by finding all the eigenvalues with the Gaussian Kernel.

Conclusion

Therefore, our work proposes a new and intuitive method to access the spectra information of theoretical/real-world graphs while being computationally efficient. Also, we can directly apply our methods to fitting observed graphs/networks to random graphs models [1-2] and to the problem of statistically comparing groups of graphs [3]. Furthermore, since most of the literature on the spectra of graphs focuses on *Spectral Distribution*, we hope our work gives a new perspective and advances the study area using the *Moment Generation function.*

Bibliography

[1] Takahashi, Daniel Yasumasa, et al. "Discriminating different classes of biological networks by analyzing the graphs spectra distribution." PloS one 7.12 (2012): e49949.

[2] Santos, Suzana de Siqueira, André Fujita, and Catherine Matias. "Spectral density of random graphs: convergence properties and application in model fitting." Journal of Complex Networks 9.6 (2021): cnab041.

[3] Fujita, Andre, et al. "A semi-parametric statistical test to compare complex networks." Journal of Complex Networks 8.2 (2020): cnz028.

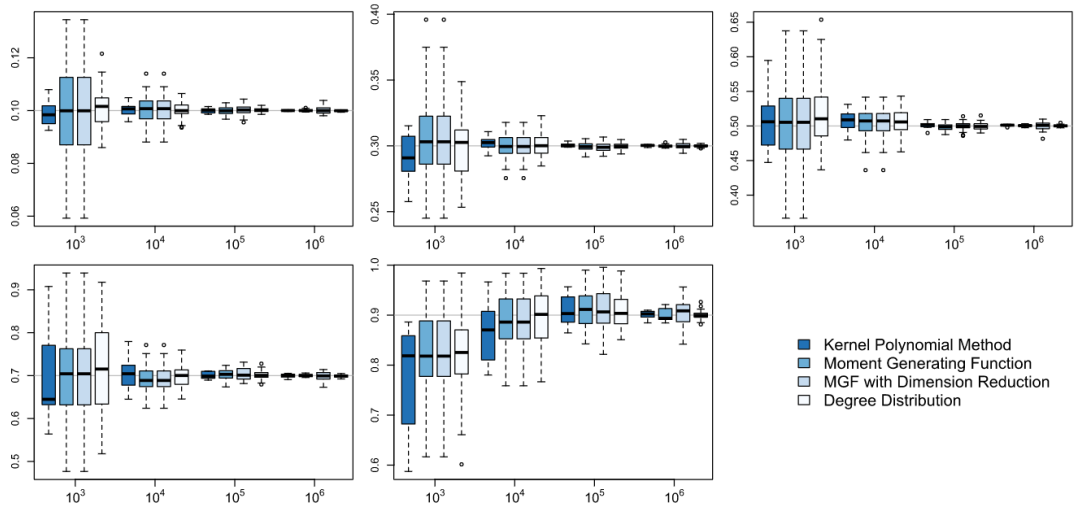

Figure 4: For each number of vertices $n = 10^3, 10^4, 10^5, 10^6$ and for each parameter $p = 0.1, 0.3, 0.5, 0.7, 0.9$ we sample 50 graphs from the WS model and apply different parameter estimation methods. We use the MGF with Dimension Reduction method for $n = 10^5$ and $n = 10^6$, both mapped to the dimension $\eta = 5{\times}10^3$.

Inscrição No.: 3296

Título em Português:	A matemática no contexto da educação escolar quilombola a partir de um levantamento bibliográfico no Portal de Periódicos da CAPES
Título em Inglês:	mathematics in the context of quilombola school education from a bibliographical survey on the capes periodical portal
Área de Pesquisa:	Currículo
Palavras Chave:	Educação quilombola - Estudo bibliográfico - Currículo
Ag. Financiadora do Projeto:	USP - Programa Unificado de Bolsas
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	
Validado em:	**06/09/2022**

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Resumo do Trabalho em inglês:

Mathematics in the context of quilombola school education from a bibliographical survey on the CAPES Periodical Portal

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Objectives

Presentation of the preliminary results of a bibliographical survey on the CAPES Periodical Portal, carried out with the purpose of understanding, in part, the terms and the development of Brazilian academic production on mathematics and its teaching in the context of Quilombola School Education. This text is the result of a scientific initiation project, which is inserted as an unfolding of the project "Saberes em diálogo: community, school and university in the construction of quilombola school education in Barra do Turvo-SP".

Materials and Methods

The process began with a literature review as a systematic method of bibliographic survey in the CAPES Periodical Portal about the Brazilian production that deals with mathematics and Quilombola School Education. We considered as "relevant results" the articles in Portuguese and published in peer-reviewed journals and that presented key terms in their title and/or abstract. During the process, 15 productions were analyzed.

Results

From the cut made by the survey, the 15 selected texts were classified into three major themes: "legislation", "culture and traditions", and "teaching practices", being possible for a text to be classified in more than one division. The next step was to identify which articles were highlighted in each classification.
Thus, for the category "Legislation", 8 articles pertinent to the theme were found and the publication (PAULA, NAZÁRIO, 2017) was considered a highlight. As for the theme "Culture and traditions", among the 9 articles in this category, the text "Jongo and Quilombola School Education" (MAROUN, 2016) was highlighted. For the last category (with 7 articles), two subdivisions were created: "report of activities in the quilombola school context" and "report on the pedagogical work in the context of quilombola school education". In the first subdivision, 6 texts are found, while in the second, only one, being the article "Quilombola teacher training and the Ethnomathematics Program: rethinking Mathematics teaching processes" (JESUS, SOUZA, 2018).
The concern of these numbers lies in the lack of didactic and scientific support for teachers who teach quilombola students. In order to respond to the demand of a compulsory education for these children, the research for pedagogical works and reports of activities becomes a necessary movement on the part of these teachers, who may find it difficult to find these materials due to the small number of these texts.

Conclusions

Considering the general context in which the scientific initiation is located (Saberes em Diálogo Project), it is interesting to compare the innovative themes that the extension course provided by the collaborating professors of the Project to the teachers of Barra do Turvo-SP who attend the children of the city's quilombola communities brought in comparison to the themes present in the articles surveyed. Thus, a table summarizing the innovations was schematized:

Table 1: Comparison of themes between the extension course and selected articles

Practices or contexts present only in the articles mapped	Practices or contexts present only in the course	Practices or contexts present in both
Typical Dances (Jongo)	Games (Sona and Flongodo) Symmetry and reflection (Sona) Probability (Sona, Buzios) Adobe Construction Tissue Dyeing Basketry Patterns	Games (Shisima, Tsoro Yematatu, Mancala) Units of Measure

Thus, it is interesting to highlight the novelties that the training offered under the project brought compared to the topics found in the articles. In general, it is perceived that the theme games is usually a practical content for teachers to work with in quilombola contexts, but that there are several options of topics to be worked with students, in order to guarantee, as required by law, that children from quilombos receive education territorially referenced in their culture and experience.

References

BRAZIL, Law nº10.639, of January 9, 2003. Alters Law nº 9.394, of 20 December 1996, that establishes the directives and bases of national education to include the theme "Afro-Brazilian History and Culture" in the official curriculum of the Education Network, and makes other provisions. *Diário Oficial da União*, Brasília, 10 jan. 2003. Available at < http://www.planalto.gov.br/ccivil_03/leis/2003/l10.639.htm#:~:text=LEI%20No%2010.639%2C%20DE%209%20DE%20JANEIRO%20DE%202003.&text=Altera%20a%20Lei%20no,%22%2C%20e%20d%C3%A1%20outras%20provid%C3%AAncias.>. Accessed 27 May 2022.

COPPE, C. Saberes e fazeres etnomatemáticos de matriz africana. Rio de Janeiro: CEAP, 2012.

D'AMBROSIO, U. *Etnomatemática* - a link between traditions and modernity. Belo Horizonte: Autêntica Editora, 2001.

._____. Ethnomathematics: past and future. *Revemop*, 2, 2020, p.1-14.

GALVÃO, M. C. B. O levantamento bibliográfico e a pesquisa científica. Fundamentos de Epidemiologia [s.n.], 2011. Available at: < http://www2.eerp.usp.br/nepien/disponibilizararquivos/levantamento_bibliografico_cristianegalv.pdf> Accessed on 20 abr. 2022.

JESUS, E. L. F. de; SOUZA, R. B. Quilombola teacher training and the Ethnomathematics Program: rethinking mathematics teaching processes. *Brazilian Journal of Field Education,* v.3, n.3, p.1064-1087, Sep/Dec, 2018. Available at: <https://sistemas.uft.edu.br/periodicos/index.php/campo/article/view/5099>. Accessed 06 Dec. 2021.

MAROUN, K. Jongo and Quilombola School Education: dialogues in the field of curriculum. *Cadernos De Pesquisa*. v.46, n.160, p.484-502, apr/jun, 2016. Available at: < https://www.scielo.br/j/cp/a/TSCQ8j3k3pXq3V59dWmSXPk/?lang=pt#>. Accessed on 03 mar. 2022.

PAULA, E. de; NAZÁRIO, R. Entre o quilombo e a educação infantil: o (não) lugar das crianças quilombolas na política educacional brasileira; *Revista Poiésis,* Tubarão, v.11, n.19, p.96-111, jan/jun, 2017. Available at: < https://portaldeperiodicos.animaeducacao.com.br/index.php/Poiesis/article/view/4774>. Accessed 03 Mar. 2022.

Inscrição No.: 3639

Título em Português:	modelagem sir do r0 durante a pandemia covid-19
Título em Inglês:	SIR R0 MODELING DURING PANDEMIC COVID-19
Área de Pesquisa:	Matemática Aplicada
Palavras Chave:	COVID-19 - R0 - suscetíveis
Ag. Financiadora do Projeto:	Outros
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
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Validado em:	**15/09/2022**

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Resumo do Trabalho em inglês:

SIR R0 MODELING DURING PANDEMIC COVID-19

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Key words: COVID-19 R_0, susceptible.

Objectives

In order to promote national hygiene and raise awareness among the population, a project was developed with the aid of the history of contamination by COVID-19, made available by the municipality of Pedreira, which consists of determining the R_0, which in epidemiology represents the average of individuals who are infected by a single contaminated during the period of transmissibility of the virus.

The main purpose of the development of the work is strictly observation. Like this

when determining the R_0, at an estimated value based on the data provided by the Municipal Department of Health of Pedreira, it is possible to paragonate the result obtained with those disclosed by the health agencies, both national and international.

Materials and Methods

From the meeting of data obtained in the covid-19 newsletter of the city of Pedreira, it was possible to attach 466 days of information about the viral disease in a Microsoft Excel spreadsheet, containing the daily number of deaths, positive, new positive, growth rate of the positive, suspected, new suspects, growth rate of suspects, participation of the two doses of mandatory vaccine, in addition to the additional dose, thus calculating those susceptible to the virus.

Then, the references were reduced at five-day intervals, and with this, it was possible to use the following formula and achieve the objective of calculating the R_0.

$$R_0 = \frac{\ln(s(\infty))^{-1}}{1 - s(\infty)}$$

Where, s(∞) is the fraction of susceptible when the pandemic no longer presents significant growth of the infected. And ln, the natural logarithm.

Thus, using mathematical methods in conjunction with official data updated every day, lasting two and a half years, it can be concluded that the pandemic demonstrated irregularity of contamination and contagion. Therefore, the community demonstrates vulnerability to an outbreak of viral disease and thus arouses uncertainty about the future of the local population.

Results

The estimated value of R_0may be influenced by the size of the population and the proportion of susceptible people at the beginning, by the infection of the organism and

by the rate of disappearance of cases due to cure and recovery or death. Taking into account all the factors mentioned above and using the equation in question, it is concluded that the result of infection transmitted by a single individual at the time of stability of disease contagion is approximately 1.7257. As the average is greater than 1, the contamination will dissipate exponentially, and irregularly as can be observed in the period between January 10 to February 16, 2022 represented reduced in the table below.

Figure 1: Evolution of the COVID-19 infection rate in 2022

date 2022	positive	new positive	growth rate
10/jan	3269	137	652,38%
14/jan	3456	137	136,50%
19/jan	3692	236	126,20%
24/jan	3706	14	5,93%
28/jan	3738	32	228,57%
02/fev	3839	91	284,38%
07/fev	4033	204	224,18%
11/fev	4314	281	137,75%
16/fev	4567	253	90,04%

PMP, 2022

Conclusions

From brief analysis the results obtained, it is a fact that the spread of the virus occurred in an agile way, at least R_0 be greater than 1, and without an explicit pattern. The development of this research becomes important in a pandemic context experienced throughout the planet, in order to improve the epidemiological modeling SIR (Susceptible - Infected - Recovered) which predicts the spread of a contagious disease and the interaction of the individuals involved.

References

Aronson, J., *et. al.* When will it be over? An introduction to viral reproduction numbers, R_0 and Re. On behalf of the Oxford COVID-19 Evidence Service Team Centre for Evidence-Based Medicine. Nuffield Department of Primary Care Health Sciences University of Oxford. Disponível em http://www.cebm.net/oxford-covid-19/ Acesso em 06 de setembro de 2022.

Ke, R. *et. al.* Estimating the reproductive number R0 of SARS-CoV-2 in the United States and eight European countries and implications for vaccination. Journal of Theoretical Biology. Disponível em: journal homepage: www.elsevier.com/locate/yjtbi Acesso em 06 de setembro de 2022.

ONU. Organização das Nações Unidas. OMS cita chegada do covid-19 ao Brasil e diz que países têm janela de oportunidade contra o vírus. Disponível em: https://news.un.org/pt/story/2020/02/1705481 Acesso em 25 de agosto 2022.

Pedreira, Prefeitura Municipal de. Secretaria de Saúde. Dados sobre Evolução da taxa de infecção pela COVID-19 em 2022. Pedreira, 2022.

THIEME, H. Matemática em Biologia Populacional. Princeton University Press, 2003. Disponível em https://www.jstor.org/stable/j.ctv301f9v. Acesso em 08 de setembro de 2022.

Inscrição No.: 3661

Título em Português:	Teoremas de Goldie e Anéis Noetherianos Não Comutativos
Título em Inglês:	goldie's theorems and noncommutative noetherian rings
Área de Pesquisa:	Álgebra
Palavras Chave:	anel - noetheriano - não comutativo
Ag. Financiadora do Projeto:	Sem fomento
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	
Validado em:	**09/09/2022**

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Resumo do Trabalho em inglês:

Goldie's Theorems and Noncommutative Noetherian Rings

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Objectives

The objective of this project was the study of noncommutative noetherian rings aiming understand the Goldie's theorems (about the characterization of rings that have a ring of semisimple fractions). The main results studies are due to A. Goldie in 1958 and were key to the accelerated development of Noetherian Ring Theory during the second half of the 20th century.

Materials and Methods

The methodology used was the current one in Mathematics reasearch, where we developed seminars weekly presented by the student with supervisor supervision for the purpose of discuss the results obtained by reading of scientific texts.

Results

Given a ring Q, a right order in Q is any subring $R \subseteq Q$ such that every regular element, that is, na element that is non-zero-divisor, of R is invertible in Q and every element of Q has the form ab^{-1} for some $a, b \in R$ with b regular. It is said that a ring R is right Goldie if the right regular module R_R has finite rank and R has ascending chain conditions on right annihilators. As an important exemple of right Goldie rings, we have all the right noetherian rings. So the main Goldie's theoreme guarantees that a ring R is a right order in a semisimple ring if and only if R is a semiprime right Goldie ring. If R is a semiprime right Goldie ring, then any semisimple ring in which R is a right order is calle a right Goldie quotient ring of R.

Conclusions

In commutative ring theory, a technique widely used is to pass from a commutative ring R for a prime quotient ring R/P and then to the quocient field of R/P. However, in the noncommutative context, the rings, even noetherians, need not have quotients that are domains. But, in particular, Goldie's theorem assures us that if P is a prime ideal in a ring noetherian R we can build a quotient ring that is and artinian. Thus, we won the possibility to study rings noetherians from the analysis of a ring semisimples, namely the Goldie quotient of R/N, where N is the prime radical of R. We can then pass information from R to R/N and mainly trying to get information of R from R/N.

References

1. Coutinho, S. C., McConnell, J. C.. The quest for quocient rings (of noncommutative noetehrian rings), Amer. Math. Monthly **110** (2003) 298-313.
2. Goodearl, K. R., Warfield, R. B.. na Introduction to noncommutative Noetherian Rings, Cambrige Universaty Press, Cambrige, 1989.

Inscrição No.: 3809

Título em Português:	EVOLUÇÃO DO PREÇO DA CESTA BÁSICA EM PEDREIRA-SP
Título em Inglês:	EVOLUTION OF THE PRICE OF THE FOOD BASKET IN PEDREIRA-SP
Área de Pesquisa:	Matemática Aplicada
Palavras Chave:	Cesta básica - Custo - Estatísticas
Ag. Financiadora do Projeto:	Outros
Projeto:	Pré-iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
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Resumo do Trabalho em inglês:

EVOLUTION OF THE PRICE OF THE FOOD BASKET IN PEDREIRA-SP

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Key words: Food basket, cost, statistics

Objectives

This work is part of a permanent project of the Quarry Education Study and Research Center (NEPEP) that aims to monitor the evolution of food basket prices in the municipality of Pedreira-SP. The aim of the current research was to monitor monthly the increase or decrease in the values of the food basket during the months of January to August 2022.

For this it is necessary to define what is a food basket, because the products consumed may vary according to the culture from region to region.

Materials and Methods

The definition of food basket established by DIEESE was adopted and a monthly survey of products considered extremely important as rice, beans, sugar, oil, coffee, flour, bread, meat, butter, vegetable and fruit, from January to August 2022 was carried out. As a vegetable was adopted as standard, the tomato and the standard fruit chosen was banana, in three different supermarkets in the city of Pedreira, thus calculating the amount established for region 1 of the DIEESE methodology for each product at its given place of purchase. Using tools such as Microsoft Excel for the creation of spreadsheets containing the collected data and the aid of mathematical methods for the development of graphs and averages of increase and decrease of the total values of the basket.

Results

The results obtained were stable between January and February, falls between March and April and, in July, there was a large increase in values. It can also be noticed increases in the costs of the food basket in the months of May, June and August. The average cost of the food basket over the eight months of study was R$ 536.69.

Graph 1: Quarry Food Basket

Fonte: Own authorship, 2022

It is also noted, throughout the data collection period, an accumulated percentage increase of 13.63% in the cost of the food basket.

Conclusions

From the research, the total cost of the food basket was defined in each establishment, and it may be noted that there was an instability in the monthly values. The analysis of this window from January to August 2022 provides the visualization of the fall and the increase that food costs had during this period. It is also possible to notice the effects of inflation on food prices, which certainly affects the cost of living of the municipality's population, especially the poorest population.

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References

DIEESE, Metodologia da Cesta, 2009, Disponível em: <https://www.dieese.org.br/metodologia/metodologiaCestaBasica.pdf>. Access in: September 07, 2022

Inscrição No.: 3814

Título em Português:	DESENVOLVIMENTO DE ALGORITMO DE CRIPTOGRAFIA RSA EM LINGUAGEM PYTHON
Título em Inglês:	DEVELOPMENT OF RSA ENCRYPTION ALGORITHM IN PYTHON LANGUAGE
Área de Pesquisa:	Matemática Aplicada
Palavras Chave:	RSA - Criptografia - Python
Ag. Financiadora do Projeto:	Outros
Projeto:	Iniciação Científica
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Resumo do Trabalho em inglês:

DEVELOPMENT OF RSA ENCRYPTION ALGORITHM IN PYTHON LANGUAGE

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Key words: RSA, Encryption, Python

Objectives

Materials and Methods

Based on the research, the history of cryptography was studied, from the most basic cryptographies used in the past to arriving at RSA, number theory and the mathematical bases that are used in RSA encryption. So, using the Python programming language, a program was developed that encrypts and decrypts alphanumeric messages.

RSA needs a public key and a private one; the public is used to encrypt messages and the private one to decrypt them.

To generate a public key, you need two prime numbers P and Q. It is crucial that they be really large, for the computer take time to factor them.

After determining the numbers P and Q, it is necessary to calculate N, which is obtained according to the following formula:

$$N = P * Q \qquad (i)$$

After defining the N of P and Q, the Euler's totient function, also called phi, is used.

$$Phi(N) = (P-1) * (Q-1) \qquad (ii)$$

Then another random number E is found, which should follow the condition described below and also be prime to each other with Phi(N)

$$1 < E < Phi(N) \qquad (iii)$$

The public key is composed of N and E. So, using the ASCII table, you can turn the message into code.

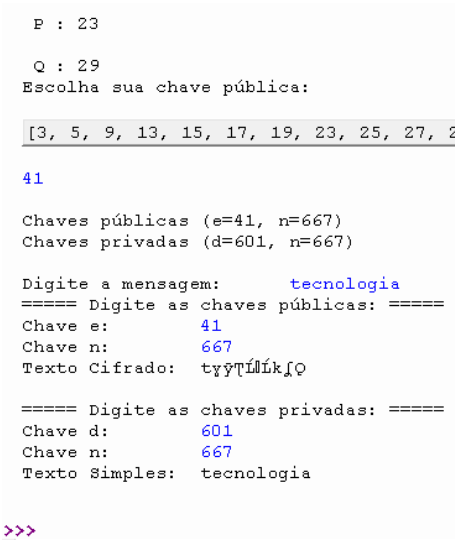
After that, you will find the private key that will be used to decrypt the message, this key is called D and is found following the equation:

$$D * E \ mod \ Phi(N) \equiv 1 \qquad (iv)$$

To decrypt the message, you must raise each message number by D and do the modular operation by N. Finally, the ASCII table is used again to convert the values obtained into alphanumeric characters.

In the following image you can see the program running in the Thonny editor interface itself for Python. In the example used, the message chosen for encryption was the word technology.

Figure 1. Demonstration of encryption and decryption of the program created in Python.



Own authorship, 2022.

RSA is safe because, even if the attacker has the private key, it would be necessary to discover the prime numbers P and Q to calculate the N, and then discover the Phi(N). Since RSA uses extremely large numbers, it is necessary to factor them, which, with current technology, demands high computational cost.

Results

It was possible to increment RSA in Python creating a program that encrypts and decrypts a particular message.

This programming language was used because it is versatile, popular and easy to learn, facilitating the understanding of the program.

Conclusions

The research validated the efficacy, methodology and calculations of RSA encryption. This method has been demonstrated using a program developed through the Python programming language. Encryption has been used since ancient Greece, it has been important for secure communication and has evolved a lot over time, but with the appearance of the internet and computing a change was needed to make it even more effective, so came RSA encryption.

References

CASTRO, F. Criptografia RSA: uma abordagem para professores do ensino básico. UFRGS, 2014.

PYTHON. Python software Foundation. The Python Standard Library, 2022, Disponível em: <https://docs.python.org/3/library/index.htm> Access September, 08, 2022.

Título em Português:	Transição de Fase no Modelo de Ising com Campo Externo Aleatório
Título em Inglês:	phase transition in ising model with random external field
Área de Pesquisa:	Matemática Aplicada
Palavras Chave:	Ising - Mecânica Estatística - Transição de Fase
Ag. Financiadora do Projeto:	FAPESP - Fundação de Amparo à Pesquisa do Estado de São Paulo
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
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USP - UNIVERSIDADE DE SÃO PAULO
30º Simpósio Internacional de Iniciação Científica

Inscrição No.: 3887

Resumo do Trabalho em inglês:

Phase Transition in Ising Model with Random External Field

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Objectives

Proving the existence (or absence) of phase transition is one of the primary goals of statistical mechanics. It is known, experimentally, that some materials exhibit spontaneous magnetism and that they lose this characteristic at high temperatures (a phenomenon known as ferromagnetism), this being an example of a phase transition. Due to the complexity of dealing with these materials realistically, several simplified models have been proposed over the last 100 years, the most important of which is the so-called Ising model. It is a well-known fact that the model does not exhibit phase transition in one dimension, but exhibits such behavior in any other dimensions. A slightly more realistic model in some cases is the so-called random field Ising model (RFIM), which considers an external magnetic field assuming random values at each vertex of the crystal (lattice).

This model is used, for example, to account for possible misalignments or impurities in the crystal or to model metallic alloys, which are made up of more than one type of element. In 1975, Imry and Ma [1] argued that the model has phase transition in dimensions greater or equal to three. This claim was challenged in 1979 by an argument due to Parisi and Sourlas [2], who proposed that the phase transition would only occur in dimension greater or equal to four. Investigating the phase transition in this model and the solution to the presented controversy is the main objective of the present work.

Materials and Methods

The aforementioned controversy was finally resolved by Bricmont and Kupiainen [3] in 1988, who rigorously proved, using advanced group renormalization techniques, that Imry and Ma were correct. The methodology used by Bricmont-Kupiainen, in addition to presenting strong technical barriers due to the high complexity, also proved to be not very fruitful in being extended to other models. In the present work, we follow the alternative argument recently formulated by Ding and Zhuang [4], in 2021, which uses well-known tools from statistical mechanics, such as geometric objects called contours, introduced by Peierls to prove phase transition in the case d > 1. Results from Talagrand's “Majorizing Measure Theory” [5] and arguments due to Fischer-Fröhlich-Spencer [6] are also used.

Results

The random field Ising model is said to have phase transition if the Gibbs measures at infinite volume with boundary conditions + and - (denominated by

$$\mu^{+}_{\beta,\epsilon h} \qquad \text{e} \qquad \mu^{-}_{\beta,\epsilon h} \tag{1}$$

respectively) differ with probability 1. The main result of the work is to show the existence of this phase transition:

Theorem 1. *For $d \geqslant 3$, there exists constants $T, c > 0$ such that, for all $0 \leqslant T, \epsilon \leqslant c$,*

$$\mu^{+}_{\beta,\epsilon h} \neq \mu^{-}_{\beta,\epsilon h} \tag{2}$$

for almost all h.

Conclusions

The argument presented by Ding and Zhuang was able to prove the existence of phase transition in the RFIM, using more familiar arguments in statistical mechanics when compared with the complex methods of Bricmont and Kupiainen. Due to its lower complexity, the argument can also be used in similar models, such as the Potts model, and adapted for use in more complicated models, such as the long-range random field Ising model.

References

[1] Y. Imry and S.-K. Ma. Random-field instability of the ordered state of continuous symmetry. Phys.Rev. Lett., 35:1399–1401, Nov 1975.

[2] Parisi, G. and Sourlas, N. Random magnetic fields, supersymmetry and negative dimensions. Phys. Rev. Lett., 43(11):744-5, 1979.

[3] J. Bricmont and A. Kupiainen. Phase transition in the 3d random field Ising model. Comm. Math. Phys., 116(4):539–572, 1988

[4] J. Ding and Z. Zhuang. Long Range Order for Random Field Ising Model, Comm. Pure Appl. Math. *To appear*. arxiv: 2110.04531, 2022.

[5] M. Talagrand. Majorizing measures: the generic chaining. Ann. Probab., 24(3):1049–1103, 1996

[6] D. S. Fisher, J. Fröhlich, and T. Spencer. The Ising model in a random magnetic field. J. Statist. Phys., 34(5-6):863-870, 1984.

Inscrição No.: 4113

Título em Português:	TÉCNICAS COMPUTACIONAIS PARA A RECUPERAÇÃO DE INFORMAÇÃO MUSICAL EM REPERTÓRIOS INDÍGENAS BRASILEIROS
Título em Inglês:	computational techniques for music information retrieval in native brazilian repertoires
Área de Pesquisa:	Metodologia e Técnicas da Computação
Palavras Chave:	computação musical - repertório indígena - etnomusicologia
Ag. Financiadora do Projeto:	USP - Programa Unificado de Bolsas
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
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Resumo do Trabalho em inglês:

COMPUTATIONAL TECHNIQUES FOR MUSIC INFORMATION RETRIEVAL IN NATIVE BRAZILIAN REPERTOIRES

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Objectives

Some music styles have characteristic rhythmic patterns which may be easily identified by any listener. When transcribing these patterns, approximations of durations are frequently done in order to simplify the symbolic representation in standard music notation. Therefore, it is common to find small variances between the symbolic representation and the musical performance. When these variances are seen systematically, it is possible to investigate the characteristics of the music style, like expressiveness, which is frequently inaccessible through symbolic notation. The field that investigates these variances is called microtiming [1].

In this work, we make an analysis of similar rhythmic patterns following the methodology made by Fabien Gouyon in "*Microtiming in 'Samba de Roda' — Preliminary experiments with polyphonic audio*" [1]. The objective of this work is to investigate a piece of the Parixara repertoire, a music and dance style that is present in both Wapichana and Macuxi cultures [2], two native Brazilian cultures from Roraima, analyzing variances in "microtiming" in rhythmic patterns found in the score.

Materials and Methods

The chosen piece is a Macuxi Parixara interpreted by Manaaka and Yauyo, which has its recording and score at the project Panton Pia' website[1]. To find the beginning of each measure, we used an algorithm included in the "Vamp" plugin collection, an option add-on in "Sonic Visualizer" [3]. After applying the algorithm, there was a manual adjustment to guarantee that the marked onsets indeed corresponded to the rattle beats. As the presence of the rattle is extremely strong in the signal, we used Spleeter [4], a Python library that separates up to five tracks, each associated to (i) vocal line, (ii) percussion, (iii) bass, (iv) piano and (v) other sources. We only used the vocal line for the analyses, even though the separated vocal track still has a part of the rattle sound in the back. The next step was to define an Onset Detection Function (ODF) to apply to the signal. Following Fabien Gouyon's methodologies [1], we applied the "complex spectral difference" function [5] to the whole signal, with frames sized 23.2 ms and 11.6 ms as hop size. In order to compare, we also applied the same method with another ODF, the HFC (High Frequency Content) [6]. We used the Essentia Python library [7] to apply these functions and generate the graphs.

To compare the different measures associated to the same rhythmic pattern, it is necessary that the corresponding novelty functions are realigned to the onsets based on the rattle beats.

This way, we resampled each extract of the novelty function (the ODF), marked by the rattle beats, using 100 evenly spaced points and cubic interpolation.

The song measures were divided into 5 groups, characterized by the same rhythmic pattern, or really small variations of it.

Results

[1] See http://pantonpia.com.br/.

We generated graphs for each measure with both ODFs, the complex spectral difference and HFC in order to compare patterns. In each group, one image shows the novelty functions' profiles of all measures belonging to it, as well as the average profile, allowing observations regarding not only individual variations but also coincident pulses from other measures.

Because of space, here we show only two examples of measure groups, a complex (group 4) and a simple one (group 5):

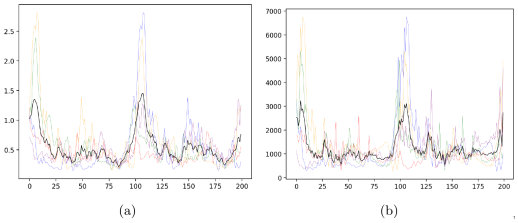

Figura 1: Measure group 4's profiles (colorful lines) and its average profile (black line). (a) Complex spectral difference (b) HFC.

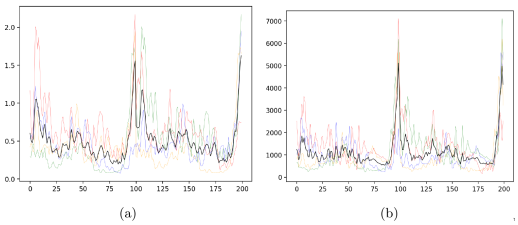

Figura 2: Measure group 5's profiles (colorful lines) and its average profile (black line). (a) Complex spectral difference (b) HFC.

We can observe that the groups with more complex rhythmic patterns (2, 3 and 4) do not have well defined peaks at the novelty functions' profiles, apart from the rattle beats, at the beginning and middle of the measure. Yet for groups 1 and 5, where the rhythmic pattern is simpler, the novelty functions' profiles' peaks seem to, at least partially, reproduce the rhythmic patterns observed in the score. When comparing the groups' graphs from both of the functions one-to-one, it is possible to see that the complex spectral difference defines sharper peaks than the ones from the HFC function.

Conclusions

The results we obtained indicate the necessity of further research, since the novelty functions did not show the rhythmic pattern of each measure type. The less complex repertoire from Gouyon's work may explain his clearer results. Voice segmentation techniques could also be explored in order to localize rhythmic events. Our work also focuses on helping disseminate the Parixara music style and the use of computational techniques to analyze native Brazilian repertoires, presenting itself as a starting point for future research.

References

[1] Fabien Gouyon. "Microtiming in 'Samba de Roda' — Preliminary experiments with polyphonic audio". At: Proc. SBCM, 2007.

[2] Felipe Munhoz Martins Fernandes. "Do parixara ao forró, do forró ao "parixara": uma trajetória musical". Master's Diss. Universidade Federal de São Carlos, 2015.

[3] Chris Cannam, Christian Landone e Mark Sandler. "Sonic Visualiser: An Open Source Application for Viewing, Analysing, and Annotating Music Audio Files". At: Proc. ACM Multimedia Intl. Conf., pp. 1467–1468, 2010.

[4] Romain Hennequin et al. "Spleeter: a fast and efficient music source separation tool with pre-trained models". At: Journal of Open Source Software, 5(50), p. 2154, 2020.

[5] Meinard Müller. "Fundamentals of Music Processing: Audio, Analysis, Algoritms, Applications", Springer, 2015.

[6] Paul Masri and Andrew Bateman. "Improved Modelling of Attack Transients in Music Analysis-Resynthesis". At: Proc. ICMC, 1996.

[7] Music Technology Group - Universitat Pompeu Fabra. Essentia Library. https://github.com/MTG/essentia, 2019.

Inscrição No.: 4307

Título em Português:	Definição de Caos em Dinâmica de Aplicações
Título em Inglês:	Definition of Chaos in Applications Dynamics
Área de Pesquisa:	Matemática Aplicada
Palavras Chave:	sistemas dinâmicos - caos - dinâmica discreta
Ag. Financiadora do Projeto:	USP - Programa Unificado de Bolsas
Projeto:	Iniciação Científica
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Resumo do Trabalho em inglês:

Definition of Chaos in Applications Dynamics

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Objectives

This work's purpose is to explore the most used definition of chaos in the area of dynamical systems, given by [1]. The concept of chaos is widely used in many areas such as, physics, engineering, economy and mathematics itself, but in general the intuitive notion doesn't abide by the criteria for a truly chaotic system.

In popular culture, chaos is informally seen as something disorganized, hard to grasp or comprehend. This concept can be related to one of the criteria for chaos: sensitivity to initial conditions, which is, in informal terms, a system that, when submitted to small changes, has an important shift in behavior.

In physics, it is common to treat the terms chaotic and sensitive to initial conditions as synonyms, but that is not always correct. Therefor, one of the prime objectives of this work is to explore if there are situations in which sensitivity to initial conditions is enough to make a system chaotic or if a physical system always needs to abide by other criteria to be chaotic according to the definition of [1].

Thus, it is intended to explore the cases in which the conditions for chaos are not completely independent and expose related examples.

Materials and Methods

The studies of the research group this work is based on were divided in weekly meetings with the purpose of discussing the topics of the main reference book [1], which were individually studied during the week, with the supervisor Sônia Regina Leite Garcia.

In this period of time, the group studied the basic concepts of discrete dynamical systems that were of great importance for the comprehension of the definition of chaos. The discussion regarding when the definition's criteria are in fact independent were not explored on a deeper level during the group sessions, but treated as an individual project developed by reading papers [2] and [3] and a masters degree theses [4].

Results

For the discussion of the definition of chaos, it is necessary to first understand the following definitions:

Definition 1: If $f:V\to V$, then $x\in V$ is said to be a periodic point if there is $n\in\mathbb{N}$ such that $f^n(x)=n$. We say that x is a periodic point of period m if this is the smallest natural number such that $f^m(x)=x$.

Definition 2: $f:V\to V$ is said to be topologically transitive if for every $U,W\subset V$ open sets, there is $k\in\mathbb{Z}$ such that $f^k(U)\cap W\neq\varnothing$.

Definition 3: $f:V\to V$ has sensitivity to initial conditions if there is $\delta>0$ such that, for every $x\in V$ and for every neighborhood N of x, there is a $y\in N$ and $n\geqslant 0$ such that $|f^n(x)-f^n(y)|>\delta$.

Finally, here follows the definition of chaos:

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Definition 4: $f : V \to V$ is a chaotic map if:

1. f has sensitivity to initial conditions.
2. f is topologically transitive.
3. The set of periodic points of f is dense in V.

It is possible, however, to prove that in certain circumstances the criteria in definition 4 are not independent. Bellow are some theorems whose demonstrations are to be discussed in the SIICUSP project:

Theorem 1: Let V be a metric space $f : V \to V$ be topologically transitive with a dense set of periodic points. Then, f has sensitivity to initial conditions.

Theorem 2: Let $I \subset R$ be an interval (not necessarily limited). Then, if $f : V \to V$ is topologically transitive,

(i) f has sensitivity to initial conditions.

(ii) It's set of periodic points is dense in I.

These theorems have major implications in areas like physics, since it is common to work with metric spaces and intervals. Notice that, in the theorem 2 criteria, chaos is a synonym for topological transitivity, not for sensitivity to initial conditions.

There are also results that relate functions with periodic points of period 3 to chaos, which is also intended to be explored in the SIICUSP project.

Conclusions

Theorems 1 and 2 are very strong affirmations and, particularly the first one has a lot of implications in applied areas, that study dynamical systems in metric spaces. Therefor, there is a lot to explore beyond what was mentioned here.

References

[1]Devaney, R. L. An introduction to Chaotic Dynamical Systems. Westview Press, 1986.

[2]Michel Vellekoop, Raoul Berglund, On Intervals, Transitivity = Chaos, The American Mathematical Monthly, 1994.

[3] Tien-Yien Li, James A. Yorke, Period Three Implies Chaos, The American Mathematical Monthly, 1975.

[4] ARAUJO, T. **Caos em dinâmica topológica.** Tese (Mestrado em Matemática aplicada) – Instituto de Matemática e Estatística, Universidade de São Paulo. São Paulo, p. 56. 2013.

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Inscrição No.: 4379

Título em Português:	Métodos topológicos na análise não linear: Grau topológico e índice de Conley
Título em Inglês:	topological methods in nonlinear analysis: topological degree and conley index
Área de Pesquisa:	Análise
Palavras Chave:	grau topológico - índice de Conley - análise não linear
Ag. Financiadora do Projeto:	USP - Programa Unificado de Bolsas
Projeto:	Iniciação Científica
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USP - UNIVERSIDADE DE SÃO PAULO
30º Simpósio Internacional de Iniciação Científica

Inscrição No.: 4379

Resumo do Trabalho em inglês:

TOPOLOGICAL METHODS IN NONLINEAR ANALYSIS: TOPOLOGICAL DEGREE AND CONLEY INDEX

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Objectives

This undergraduate research work is a continuation of the PUB project nº 2297 and aimed to study: (i) the theory of the topological degree of Leray-Schauder (LS-degree); (ii) a concept of degree, introduced in [1], for Fredholm maps of index zero between real Banach spaces; (iii) a connection between the LS-degree and the Conley index theory for flows in infinite-dimensional Hilbert spaces [3]. This subject was developed together with the student Sérgio Henrique Maciel, who dedicated the first part of his work to the Conley index theory.

Materials and Methods

Topics (i) and (ii) were investigated through books and articles indicated by the advisor and researched by the student. Weekly meetings were organized in a seminar format with the professor to report on the progress made, and monthly meetings with Sérgio and the advisor for the students to inform the progress of the research.

Results

In the first stage of the project, the LS-degree for functions between Banach spaces of possibly infinite dimension was deepened. The construction uses the Brouwer degree for functions on R^n. Consider a continuous function $f\colon R^n \to R^n$, an open and bounded subset U of R^n and a point $y \in R^n$. The triple (f, U, y) is said to be *admissible* if $y \notin f(\partial U)$. The Brouwer degree is an integer associated to the triple (f, U, y) and gives information about the solutions of the equation $f(x) = y$ in U. The main properties of Brouwer degree are listed below.

Existência: Let (f, U, y) be admissible and $\deg_B(f, U, y) \neq 0$. Then, the equation $f(x) = y$ admits at least one solution in U.

Aditividade: Let (f, U, y) be admissible. If $V, W \subseteq R^n$ are open, disjoint and such that $(f^{-1}(y) \cap U) \subseteq (V \cup W)$, then

$$\deg_B(f, U, y) = \deg_B(f, V, y) + \deg_B(f, W, y). \quad (1)$$

Invariância homotópica: Let $H\colon \overline{U} \times [0,1] \to R^n$ be continuous, with $H(x, \lambda) \neq y$, $\forall (x, \lambda) \in \partial U \times [0,1]$. Then,

$$\deg_B(H(\cdot, 0), U, y) = \deg_B(H(\cdot, 1), U, y). \quad (2)$$

The LS-degree is defined for $f = I - T : \overline{U} \to E$ functions, where E is a Banach space, $U \subseteq E$ is open and bounded, I is the identity of E and T is completely continuous. Consider f as above and a point $y \notin f(\partial U)$. Analogously to the Brouwer degree, the triple (f, U, y) is said to be *admissible.* The distance $\rho = dist(y, f(\partial U))$ é positive and f is proper. From functional analysis, there exists $T'\colon \overline{U} \to X$, with finite-dimensional image, such that $||T(x) - T'(x)|| < \rho, \forall x \in \overline{U}$. Consider now $U' = U \cap E'$, where E' is a finite-dimensional space that contains $T'(\overline{U})$ and the point y. The LS-degree is defined by

$$\deg_{LS}(f, U, y) := \deg_B(I - T', U', y). \quad (3)$$

The above formula does not depend on the choice of T' or E'. The Brouwer degree properties, listed above, hold for the LS-degree with the necessary adaptations.

The second part of the project was dedicated to the study of a recent extension of the LS-degree for Fredholm maps of index zero between Banach spaces [1]. Such a notion is based on a concept of orientation for these maps. This orientation is defined, primarily, for a Fredholm linear and continuous operator of index zero, $L: E \to F$. It extends in infinite dimension the concept of a matrix determinant and, if the L is an oriented isomorphism, it allows to associate a signal to L: $sign(L) := +1$, if the zero operator between E and F is an element of the orientation of L; $sign(L) := -1$, otherwise.

The set of isomorphisms from E to F is open in the linear operators space $L(E,F)$, therefore an orientation of an operator L induces an orientation on the sufficiently close operators. This allows you to define an *orientation* of a Fredholm nonlinear function of index zero $f: E \to F$, as an orientation of the differentials $Df(x)$, $x \in E$. Now, let $f: E \to F$ be oriented, $y \in F$ a point and $U \subseteq E$ be open. The triple (f, U, y) is said to be *admissible* if $f^{-1}(y) \cap U$ is compact. If y is a regular value for f in U, the degree is defined by

$$\deg(f, U, y) := \sum_{x \in f^{-1}(y) \cap U} sign\, Df(x), \qquad (4)$$

where $f^{-1}(y) \cap U$ is a finite set. If y is a critical value, if z is a regular value sufficiently close of y and W is a convenient neighborhood of $f^{-1}(y)$, then $\deg(f, W, z)$ does not depend on z and W. Therefore, we define

$$\deg(f, U, y) := \deg(f, W, z). \qquad (5)$$

This degree concept satisfies all the properties normally found in degree theory, analogously to the LS-degree.

The final part of the work, together with Sérgio, was dedicated to the study of a connection between the LS-degree and the Conley index, which is a topological invariant, generally defined for flows of dynamical systems in R^n. If S is an invariant set for a flow and N is an isolating neighborhood of S, the Conley index $h(s, \phi)$ is represented by homology groups and gives information about topological properties of S.

In infinite dimension, let H be a real and separable Hilbert space and $L: H \to H$ be a isomorphism such that: $H = \bigoplus_{n=0}^{\infty} H_n$, where H_n are orthogonal, L-invariant and of finite dimension. Consider a function $f: \Omega \to H$ of the form $f(x) = Lx + K(x)$, where $\Omega \subseteq H$ is open, K is C^1 and completely continuous and ϕ is the flow generated by the equation $x' = -f(x)$. Let S be an invariant set and N its isolating neighborhood. By the invariance of L on H_n, the Conley indices of the flow projections on $\bigoplus_{i=0}^{n} H_i$ become definitely constant and such value defines an extension in infinite dimension, called *LS-index*, denoted by $h_{LS}(S, \phi)$, [3]. The main result is the following.

Theorem: *In the above notations, the following equality holds:*

$$\chi(h_{LS}(S, \phi)) = \deg_{LS}(I + L^{-1}K, int(N), 0), \qquad (6)$$

where $\chi(h_{LS}(S, \phi^t))$ is the Euler characteristic of the N invariant set LS-index.

Conclusions

Topological degree theory gives qualitative information about the solutions of nonlinear equations (e.g., differential equations). The importance of degree lies in the properties of existence and location of solutions, as well as in the homotopy invariance, facilitating the calculation of a more complicated degree. The Conley index gives information about properties of invariant sets in dynamical systems. Thus, it is possible to make a comparison between the solutions of a nonlinear equation and the invariant set of a dynamical system, as both satisfy properties of existence and homotopic invariance, in addition to other properties.

References

[1] Benevieri P. & Furi, M. *A simple notion of orientability for Fredholm maps of index zero between Banach manifolds and degree theory*. Ann. Sci. Math. Québec, **22** (1998), 131-148.

[2] Lloyd, N. G. *Degree theory.* Cambridge University Press, 1978.

[3] Styborski, M. *Conley index in Hilbert spaces and the Leray-Schauder degree.* Topol. Methods Nonlinear Anal., **33** (2009), 131-148.

Inscrição No.: 4443

Título em Português:	Métodos topológicos na análise não linear: índice de Conley e suas aplicações às equações diferenciais e aos sistemas dinâmicos
Título em Inglês:	topological methods in non-linear analysis: conley index and its applications to differential equations and dynamical systems
Área de Pesquisa:	Geometria e Topologia
Palavras Chave:	Conley index - index-degree - Dynamical systems
Ag. Financiadora do Projeto:	CNPq - PIBIC
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	
Validado em:	**09/09/2022**

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Resumo do Trabalho em inglês:

Topological methods in non-linear analysis: Conley index and its applications to differential equations and dynamical systems

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Objectives

The project had two objectives: the first was to study the Conley index, a topological invariant that gives us information about invariant sets in dynamical systems. The second one, that worked for the last three months of activity, was the analysis of the relations between the Conley index and the Leray-Schauder topological degree. This part was developed together with the student Renato Borlino Casagrandi.

Materials and Methods

The project was supplied with texts and other complementary materials by the advisor. Weekly meetings had been held where the student Sérgio presented seminars reporting the advances in the research, discussing questions, conjectures and problems.
Monthly meetings had been held in the first nine months of the work in which the two students shared their respective advances. Within the last three months, the joint sessions became weekly and the students worked together on the same topic.

Results

The Conley index theory deals with flows in topological spaces. Given a topological space X, a *flow* is defined as a continuous function $\phi\colon \mathbb{R}\times X\to X$ satisfying $\phi(0,x)=x, \forall x\in X$ and $\phi(t,\phi(s,x))=\phi(t+s,x)$, $\forall t,s\in\mathbb{R}$.
The set $\mathcal{O}(x,\phi):=\{\phi(t,x), t\in\mathbb{R}\}=:\phi(\mathbb{R},x)$ is called an *orbit* of x. A set $S\subseteq X$ is *invariant* if $\phi(\mathbb{R},S)=S$. A compact set $N\subseteq X$ is said to be an *isolating neighbourhood* of S if its maximal invariant set $Inv(N):=\{x\in N:\phi(\mathbb{R},x)\subseteq N\}$ is contained in the interior of N, $Int(N)$. In this case, S is called an *isolated invariant set*, which will be denoted by IIS.
The invariant sets are remarkable in dynamical systems, but often difficult to be studied. Alternatively, seeking within the isolating neighbouhoods is usually more efficient. This could lead us to valuable information about the invariant sets theirselves.

The definition of the Conley index is based on the following concept.

Definition 1. Let S be an IIS of a flow. An *index pair* for S is a pair (N,L) of compact sets, with $L\subseteq N\subseteq X$, such that

$i)$ $S=Inv(\overline{N\backslash L})$ and $N\backslash L$ is a neighbourhood of S;
$ii)$ $\forall x\in L$ and $\forall t>0$, if $\phi([0,t],x)\subseteq N$, then $\phi([0,t],x)\subseteq L$;
$iii)$ $\forall x\in N$ and $\forall t>0$, if $\phi(t,x)\notin N$, then there is $t'\in[0,t]$ such that $\phi([0,t'],x)\subseteq N$ and $\phi(t',x)\in L$.

Conley proves in [C] that every IIS has an index pair and this property allows us to define the Conley index.

Definition 2. Let S be an IIS for the flow ϕ and (N,L) an index pair for S. We define the homotopic *Conley index* of S as the homotopy class of the pointed space $(N/L,\ [L])$:

$$h(S,\phi):=[(N/L),[L]].$$

We also define the homological *Conley index* of S as the sequence of homology groups

$$CH_n(S,\phi):=H_n(N/L,[L]),\ \forall n\in\mathbb{N}.$$

Analogously, one could work in a cohmomological framework for the index.

In [C], Conley shows that the index is independent of the choice of an index pair. In addition, for every IIS, there will always be some index pair (N, L) such that $H_n(N/L, [L]) = H_n(N, L)$.
The Conley index gives us information about the IIS. For example, if $h(S, \phi) \neq 0$, then $S \neq \emptyset$. Here, $h(S, \phi) \neq 0$ means that N/L is not contractible. Furthermore, the index is invariant to flow homotopies. This property allows us to simplify the study of a flow ϕ by analysing a simpler homotopic flow ψ.
The index also gives us information about the possible decompositions of an IIS into the union of invariant subsets and proves the existence of atractors and repellers.

In the second part of the project, in collaboration with Renato, the relation between an extension of the Conley index for flows in Hilbert spaces of infinite dimension and the Leray-Schauder degree was studied. The Leray-Schauder degree is a topological tool to investigate solutions of equations of the form $f(x) = y$ in Banach spaces. The following result relates the Brouwer degree with the Conley index.

Let $F: \Omega \subseteq \mathbb{R}^n \to \mathbb{R}^n$ be locally Lipschitz, ϕ the gradient flow of F, S an IIS and N its isolating neighbourhood. Then

$$\chi(h(S, \phi)) = deg_B(F, Int(N), 0), \quad (1)$$

where χ stands for the Euler characteristics and deg_B denotes the Brouwer degree.

In 2009, Styborski, [St], gets an extension in infinite dimension for the above equality. He introduces the *LS-index*, an extension of the Conley index for Hilbert spaces of infinite dimension. Its description can be summarized as follows.

Let $L: H \to H$ be an isomorphism and $H = \bigoplus_{n=0}^{\infty} H_n$, where each H_n is L-invariant, orthogonal to any other and finite dimensional. Let $f: H \to H$ be defined as $f = L + K$, with K completely continuous and ϕ the gradient flow of f. Fixed S, an IIS, and its isolating neighbourhood, N, due to the invariance of H_n, $N \cap \bigoplus_{i=0}^{n} H_i$ is a isolating neighbourhood of $S \cap \bigoplus_{i=0}^{n} H_i$ for all natural n big enough. We take the sequence of the Conley index of each one of these IISs, which become definetely constant. This value is conceived as the *LS -index* of S, denoted as $h_{LS}(S, \phi)$.

Theorem 1 (Styborski). It follows that

$$\chi(h_{LS}(S, \phi)) = deg_{LS}(I + L^{-1}K, Int(N), 0), \quad (2)$$

where deg_{LS} stands for the Leray-Schauder degree.

Conclusions

The Conley index turns out to be a valuable tool for studying dynamical systems. It allows us to understand properties of invariant sets such as their stability and decomposability of these sets. Strong relations between the Conley index and the Brouwer degree have been proved, as shown in the formula (1) and its generalization to infinite-dimensional case in formula (2). The analogies between the index and the degree are of great interest: they study, respectively, IIS and solutions for non-linear differential equations by means of algebraic properties of suitable neighbourhoods of the objects of interest. It is crucial to highlight that both the index and the degree share some properties, such as the homotopy invariance, additivity and existece, the latter dectecting IIS if the index is non-trivial and solutions if the degree is non-zerol.

References

[C] CONLEY, C. *Isolated invariant sets and the morse index*. AMS, Providence, 1976.

[M] MISCHAIKOW, K. *The Conley index theory: a brief introduction*. Banach center publications. Warszawa, 1999.

[MM] MISCHAIKOW, K.; MROZEK, M. *Conley index*. Ch. 9. Handbook of dynamical system, VOL. 2. Elsevier, Amsterdam, 2002.

[S] SALOMON, D. *Connected simple systems and the Conley index of isolated invariant sets.* Transactions of the AMS. v. 291, n.1, 1985.

[St] STYBORSKI, M. *Conley index in Hilbert spaces and the Leray-Schauder degree*. Topol. Methods Nonlinear Anal., 33, 2009, 131-148.