Inscrição No.: 64

Título em Português:	Introdução à teoria do grau topológico e aplicações à análise funcional e à teoria da bifurcação
Título em Inglês:	Introduction to topological degree theory and applications to functional analysis and bifurcation theory
Área de Pesquisa:	Análise
Palavras Chave:	grau topológico - teoremas de ponto fi - bifurcação
Ag. Financiadora do Projeto:	CNPq - PIBIC
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	Matemática
Validado em:	**07/09/2021**

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Resumo do Trabalho em inglês:

Introduction to topological degree theory and applications to functional analysis and to bifurcation theory.

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Objectives

This work had as main objective to introduce the student to Brouwer topological degree, and its applications in topology and analysis. Afterwards the work dealt with the extension of Brouwer topological degree to infinite dimension, its axiomatic construction and some modern extensions of the degree. Finally, this work aimed to study the algebraic topology approach to the degree in orientable compact manifolds. With respect to this theme, see the final comment.

Materials and Methods

The project was developed by the student Eduardo through the study of the texts suggested by the advisor and the search of other bibliographic material, autonomously made by the student. There where scheduled weekly meetings, face-to-face at the beginning, October 2019, and then remotely since the start of the pandemic. In each meeting, the student reported the progress made, the studied and proven results, the exercises found in the texts, showing possible doubts and comments. The meetings allowed a very productive and deep understanding of the discipline, offering the possibility to explore connections between the arguments in the work and other mathematical topics.

In the first moment the work consisted in a revision of preliminary concepts necessary to the work's development. Then the project was divided in five steps according to the following list.

1. Construction of the Brouwer topological degree for continuous functions between $\mathbb{R}^n$ spaces and its applications.
2. Extension of the Brouwer degree for compact perturbations of the identity in real Banach spaces (the Leray-Schauder degree).
3. Uniqueness of the degree and the axiomatic approach.
4. Modern versions of the degree, defined for locally compact perturbations of non-linear Fredholm operators between real Banach spaces.
5. Construction of the Brouwer degree for oriented compact manifolds, in the algebraic topology approach.

Results

The first part of this work was dedicated to the study of Brouwer degree. There were defined the admissible triples, which are triples (f, U, y) where $f : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous function, U is a bounded open set, and $y \in \mathbb{R}^n$ is such that $y \notin f(\partial U)$. The Brouwer degree is integer associated with each admissible triple that gives us information on the solutions in U of the equation $f(x) = y$. In the case where the triple is regular, that is that f is $\mathcal{C}^2$ and y is a regular value of f in U, the degree is defined by the formula

$$\deg(f, U, y) = \sum_{x \in f^{-1}(y) \cap U} \text{sgn } (f'(x)),$$

where sgn $(f'(x))$ is the signal of the determinant of the jacobian of f in x.

The importance of the degree resides in its proprieties, let us highlight some

Theorem 1 (Principal proprieties of the degree).
1. **Normalization** - Let $I : \mathbb{R}^n \to \mathbb{R}^n$ be the identity, and U a bounded open subset of $\mathbb{R}^n$. Then $\deg(I, U, y) = 1, \quad \forall y \in U$.
2. **Additivity** - Let (f, U, y) be an admissible triple for Brouwer degree. If $U_1, U_2 \subseteq U$ are disjoint open sets, such that $y \notin f(\overline{U} \backslash (U_1 \cup U_2))$, then $\deg(f, U, y) = \deg(f, U_1, y) + \deg(f, U_2, y)$.
3.**Homotopic invariance** - Let (f, U, y) be an admissible triple for Brouwer degree and $H : \overline{U} \times [0, 1] \to \mathbb{R}^n$ continuous, such that $H(x, \lambda) \neq y, \forall (x, \lambda) \in \partial U \times [0, 1]$. writing $H_\lambda = H(\cdot, \lambda)$ we have $\deg(H_\lambda, U, y)$ does not depend on λ.

Another equally important property is •**Existence** If $\deg(f, U, y) \neq 0$, then the equation $f(x) = y$ has some solution in U.

Among the degree's applications, we cite

Theorem 2 (**Brouwer fixed point theorem**)**.** Let B^n be the unit ball in $\mathbb{R}^n$, centered in the origin, and $f : \overline{B^n} \to \overline{B^n}$ continuous. Then there is a $x \in \overline{B^n}$, such that $f(x) = x$.

The second stage in the project consisted in extending the degree for functions between infinite dimensional spaces.In the Leray-Schauder dregree's case, the admissible triples are the triples $(I - K, U, y)$, such that I is the identity of a Banach space X, $K : X \to X$ is compact, $U \subseteq X$ is a bounded open set, and $y \notin (I - K)(U)$. The Leray-Schauder degree is defined through a process or reduction to the finite dimensional case and is, therefore, based on the Brouwer degree. Then, it was showed that this definition satisfies three proprieties that, made the necessary modifications, are analogous to the proprieties stated in the theorem 1. This proprieties were called *principal proprieties* as they have a important role in the axiomatic degree theory, which was studied in the next part of the work.

In the third part was addressed the axiomatic degree theory for functions from X to X, X a real Banach space. The theory consists in characterize the degree only by the three principal proprieties, cited before and it was proofed that there is at most one degree that satisfies them. The Leray-Schauder degree, like Brouwer, have many other proprieties that, in the axiomatic approach, are direct consequences of the three principal proprieties. In this part, it was studied what characterize the topological degree and which proprieties can be deduced from the three principal proprieties.

In the fourth part of the work, it was studied a more recent degree theory, for locally compact perturbations of Fredholm non-linear operators between real Banach spaces (see [4]). This theory, that extends the Leray-Schauder degree, have applications to differential equations. An interesting thing is that this theory deepens the concept of orientation for functions between infinite dimension Banach spaces. In [4] the authors proof that the uniqueness of this degree also holds for the considered class of functions.

The fifth and last part of this work consisted in the study of the degree, using tools from algebraic topology, in particular the homology theory. It was addressed the degree's construction for continuous functions between spheres of dimension n and there were verified the analogous to the three principal proprieties.

Conclusions

This work provided the contact of the student with various results and tools from general topology, algebraic topology and mathematical analysis. Besides, it was possible to study the topological degree in its various cases, including more modern degrees. This study resulted in a in-depth knowledge of the degree, its application, limitations and connections, theoretical and in relation to different mathematical areas.

Final Comment

Despite the title of our project, it wasn't studied bifurcation. In accordance with the advisor, the student choose to study the topological degree's construction done by means of algebraic topology, as he have more interest in this subject. The uniqueness of the degree, mentioned before, allows us to claim that these topological degree's, constructed by analysis and algebraic topology, actually coincide.

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Inscrição No.: 213

Título em Português:	Treinamento de Agentes Baseados em Lógica Robusta
Título em Inglês:	Agent Learning based on Robust Logic
Área de Pesquisa:	Metodologia e Técnicas da Computação
Palavras Chave:	Inteligência Artific - Computação - Lógica
Ag. Financiadora do Projeto:	CNPq - PIBIC
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	Ciência da Computação
Validado em:	**08/09/2021**

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USP - UNIVERSIDADE DE SÃO PAULO
29º Simpósio Internacional de Iniciação Científica

Inscrição No.: 213

Resumo do Trabalho em inglês:

AGENT LEARNING BASED ON ROBUST LOGIC

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Objectives

This research focuses on developing intelligent logic-based agents capable of performing well in deterministic zero-sum games - Tic-Tac-Toe in particular. Given the current scenario of Artificial Intelligence, in which a lot is demanded in performance, but little is understood about is done to reach the final result, the corresponding agent differs from the other ones by trying to learn about the game to be played through a set of hypotheses and heuristics previously formulated, thus generating solid and palpable knowledge about the environment. Using fundamental and structuring tools in logic and machine learning - First-order Logic and PAC-Learning respectively -, the agent to be developed aims at mastering the environment together with the possibility of knowing its action and decision limitations based on the set of hypotheses and heuristics provided and the sample size used for learning. In addition, it is also up to the agent the task of ensuring the increasing monotonicity of its win state identification in game as it is fed with more data, since its construction is based on models that provide some guarantees. In conclusion, the goal of this research is to prototype a first intelligent agent that identifies winning states with increasing skill depending on the data provided, and that can explain the reason for its choices given a previous class of hypotheses. In the near future, but which no longer fits exactly within the scope of this research plan, the implementation of agents with formal background like this could be tested in high-risk environments with intense human interference - such as strategic hospital decisions or predictions about COVID-19, for example - to guarantee good results with the proper explanations about what was decided.

Materials and Methods

The methodology used in this project follows a very natural line of investigation and research, keeping a simple and structuring cycle of study, application and evaluation. All phases of the project, therefore, were initiated by books, texts and articles reading to acquire knowledge, expand background and deepen content understanding, the very heart of an academic research project. After this first stage, all the knowledge acquired, along with the previous technical framework, will serve as the theoretical basis for the implementations and tests to be done, thus offering a little more reality and application to a project that once inhabited only the world of ideas. Once the second stage is completed, it is time to analyze the results, the last part towards the end of the cycle. In this step, expectations are aligned with the concrete and factual results obtained, bringing an aspect of sobriety to the project through a critical analysis of the work process and what was achieved. At the end of a cycle, a report will be written in order to well document the project's trajectory, listing, therefore, the successes and failures of the journey, and serving as a starting point for the next cycle to be open.

Results

The process of conducting academic research is diverse. The results arising from a scientific initiation are of the most varied nature and have

very enriching implications for the researcher's life. From the acquisition of knowledge and development of the technical-scientific repertoire, to the sophistication and formalization of academic writing, and the real implementation of a project, are examples of everyday achievements for this type of work. The first record of results, therefore, is dedicated to my development in the field of artificial intelligence and, in particular, theoretical machine learning. In addition to the more subtle result, the more palpable results are: an article named “What are the rules for Tic-Tac-Toe? A Theoretical Machine Learning Perspective” about the fundamentals of PAC-Learning, Agnostic PAC-Learning and VC-dimensions, together with the description of experiments carried out based on such models, the source codes produced during scientific initiation related to probabilistic study of the board states of Tic-Tac-Toe game (written in C), modeling and experimenting in PAC-Learning (written in C) with generation of histograms to evaluate the distribution of agents obtained (written in Python), modeling and experimenting in Agnostic PAC-Learning (written in C) with generation of histograms to evaluate the distribution of agents obtained (written in Python), modeling and experimenting in linear separators using VC-dimensions concepts (written in Python) with generation of illustrative surfaces found and histograms to evaluate the distribution of the agents obtained (written in Python), modeling and experimenting with non-linear separators using VC-dimensions intuitions (written in Python on an IPython notebook) with generation of histograms to evaluate the distribution of agents obtained (written in Python on an IPython notebook).

Conclusions

Once the fundamental concepts of PAC-Learning are properly introduced and formalized, we finally start the discussion about the positive and negative aspects of the technique. By remembering its weaknesses and extolling its richness, we explain the reason for this research. Beyond the intuitive premise of the framework, the especially strong points of the approach are the simple formalization and mainly the possibility to bound sample complexity needed for learning independently from the distribution one is working with. Although it seems at first that PAC-Learning could solve all of our problems in machine learning, modeling a PAC-Learnable hypothesis class and all other specifications needed for this tool to work can be a tough task. And even if we were able to build our model properly, we would still get a completely overdone sample complexity bound, which makes it hard to develop a scalable model. The VC dimension theory, then, changes the machine learning paradigm, making prior knowledge a feature, but not a necessity anymore. The appeal and value of this research, therefore, rely on better understanding the fundamentals of machine learning and trying to approximate modern machine learning techniques to the PAC-Learning field. Since society has achieved a new level of computational potential, these once obsolete methods emerge, not with the ambition to overcome all the contemporary machine learning approaches, but with the promise of deeper understanding of what is happening in the core of the most modern statistical frameworks.

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Inscrição No.: 909

Título em Português:	Dinâmica Topológica no Disco e na Esfera
Título em Inglês:	Discrete Dynamic Systems
Área de Pesquisa:	Matemática Aplicada
Palavras Chave:	Sistemas dinâmicos - Topologia - Dinâmica de aplicaçõ
Ag. Financiadora do Projeto:	CNPq - PIBIC
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	Matemática Aplicada
Validado em:	**08/09/2021**

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Resumo do Trabalho em inglês:

Topological Dynamics of the Disc and the Sphere

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Objectives

Topological Dynamics on surfaces is an important subject that can help us grasp natural systems such as the restricted three-body problem. On the other hand, these applications tend to need a pretty good amount of maturity and knowledge on the subject to be studied.

We studied two 'simple' papers, in order to get used to the ideas and arguments used in this field. The articles: The Theorem of Kerekjarto on Periodic Homeomorphisms of the Disc and Sphere[1] and Recurrent Surface Homeomorphisms[2] were really helpful in this sense.

Materials and Methods

Throughout the last year, under Sonia's supervision, we did weekly meetings where we discussed topics from the book An introduction to Chaotic Dynamical Systems[2], and other sources including books and papers. The participants were invited to do a presentation about the results and thoughts about the studied subject. This project, is a follow up of another project that started one year before this one, which means that the topics were a lot deeper than normally you would see in these kind of projects.

Results

We studied the dynamics of a homeomorphism $f : M \to M$ that is orientation preserving, in the case that M is the disc or the sphere.

Considering the first approach, with a more restricted set of conditions to our function. With the condition that the function is periodic, i.e., considering that f^n is periodic in n, we then conclude that, in the case of the circle, as well as the case of the sphere, our function is topologically Conjugated to euclidean isometries. These results can be found in reference[1].

In the second approach, considering a less restricted set of conditions for our function and looking only to the sphere, we suppose that f is recurrent in place of the periodic.

We then conclude that, in this case f has exactly two fixed points. Results can be found in reference[2].

Conclusions

Given that we are talking about function on topological manifolds, these results are actually pretty expressive. Guaranteeing, with these conditions, the existence of exactly two fixed points on the sphere as seen in the reference [2] by itself is a strong result.

Not only that, but the mathematical ideas in these papers, gave the necessary basis to go beyond the results presented here.

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Título em Português:	Indrodução a Teoria dos Números
Título em Inglês:	Introduction to Number Theory
Área de Pesquisa:	Álgebra
Palavras Chave:	Álgebra - Curvas Elípticas - Equações Diofantinas
Ag. Financiadora do Projeto:	CNPq - PIBIC
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	Matemática
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Resumo do Trabalho em inglês:

Introduction to Number Theory

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Objectives

The main subareas of this study are Number Theory. Besides understanding the general aspect of Elementary Number Theory, we aim to understand algebraic strutures like groups, rings and fields. We study some rings that have useful properties to solve Diophantine equations, like Gaussian Integer, Eisenstein Integers and Quadratic Extension. We also studied too in an introductory way about Elliptic Curves, which allow us to carry out a geometric approach to Diophantine equations.

Materials and Methods

The methodology used was the current one the Mathematics research, presentations were developed by the student, in which were discussed the results obtained after reading scientific texts.

Results

Based on [1], the Groups and Rings Theory was studied and we used [2] to understand Euclidean rings. From [4] and [6], the integers modulo n were understood, we prove the Chinese Remainder Theorem, which can be used to solve Diophantine equations. We proved that the division algorithm holds in Gaussian Integer, Eisenstein Integers and polynomial rings over a field. So, these rings have similar algebraic structures. After that, references [3] and [4] were used to understand elliptic curves theory. We saw that from a point on a elliptic curve it is possible to build other, which form an additive group.

Conclusions

We saw that there are some methods for finding integers solutions to a Diophantine equation. We can use unique factorization in Euclidean domains such as Gaussian Integer and Eisenstein Integer. In addition, we can write a Diophantine equation in the form of an elliptic curve and then use properties in elliptic curves to describe the solutions.

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Inscrição No.: 1024

Título em Português:	A.D.A - Assistente Distribuída Avançada -4- NLU
Título em Inglês:	A.D.A - Assistente Distribuída Avançada -4- NLU
Área de Pesquisa:	Metodologia e Técnicas da Computação
Palavras Chave:	IoT - Aprendizado de Máqui - Assistente Inteligen
Ag. Financiadora do Projeto:	CNPq - PIBITI
Projeto:	Iniciação Tecnológica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	Ciência da Computação
Validado em:	**13/09/2021**

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Objetives

A.D.A is a free software based distributed virtual assistant, designed to understand commands in portuguese and interact with Internet of Things devices [Freire et al. 2020].

For A.D.A. to be capable of executing the users commands, a natural language understanding component is needed to translate the users speech to a language or formal representation, allowing the users commands to be interpreted and executed by the assistant.

Materials and Methods

For that purpose, we studied concepts and techniques from natural language processing and deep learning. In particular, we tested and compared two solutions for natural language understanding: a model that uses a portuguese BERT model for contextual language representations[Souza et al. 2020], and the snips_nlu library by Snips Voice Platform [Coucke et al. 2018], that perform the tasks of intent classification and slot filling, i.e. identify the type of action desired by the user and the terms of the sentence corresponding to entities or objects relevant to that action.

We also studied neural network models capable of recognizing multiple intents per sentence, multiple values per slot, as well as multi-turn dialogue intent classification and dialogue management.

For testing, we implemented a BERT based model that performs joint intent classification and slot filling, baesd on the work of [Chen et al. 2019].

The snips_nlu library is available in Python and provides functions for instantiation, training and use of the model, returning a python dictionary with the intent and the slots already resolved.

We also studied the topics of multilabel intent classification, classification of multiple slot values per slot, as well as multiturn dialogue intent classification and dialogue state management.

Results

Both natural language understanding solutions obtained precision higher than 90% in both tasks. The BERT based model might be more adequate for less prectable commands or commands with more variable structure, such as internet search, and can also be used for a greater number of tasks, which makes it a better candidate for use in the A.D.A. assistant, considering a future increase in the assistant's capabilities.

Test results

Model	Intent	Slots
BERT + CRF	94.76	99.89
snips_nlu	100.0	96.74

Tabela 1. Performance of both model based on intent classification accuracy (%) and slot filling F1 score (%)

Since the snips_nlu library was created to work offline in embedded systems, it has a much lower memory usage and executes faster than the BERT based model. But, the library is designed to perform only the two tasks shown, which limits it's usage options.

Conclusions

We tested and compared two natural language understanding solutions, taht perform the tasks of intent classification and slot filling, with the goal of integrating them with the A.D.A. virtual assistant. The tests show that both the snips_nlu library and the BERT based model obtain good results and are therefore good options for use with A.D.A. depending on the context. The BERT based model is more wellsuited for less predictable commands, while the snips_nlu library uses less memory and is capable of executing faster.

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Inscrição No.: 1039

Título em Português:	A.D.A - Assistente Distribuída Avançada -2-
Título em Inglês:	A.D.A - Assistente Distribuída Avançada -2-
Área de Pesquisa:	Metodologia e Técnicas da Computação
Palavras Chave:	IoT - Aprendizado de Máqui - Assistente Inteligen
Ag. Financiadora do Projeto:	CNPq - PIBITI
Projeto:	Iniciação Tecnológica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	Ciência da Computação
Validado em:	**13/09/2021**

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Colaborador:

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Resumo do Trabalho em inglês:

A.D.A - Assistente Distribuída Avançada -2-

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Objectives

To develop a communication and control layer for intelligent devices to the Assistente Distribuída Avançada (A.D.A.) personal assistant, including both the software for embedded devices software and the API for accessing the resources. Specific objectives include using low-cost, easily obtainable hardware and highly adopted technologies, preferably free and open-source.

Materials and Methods

Espressif ESP8266 and ESP32 microcontrollers, both with wireless connections based on the IEEE 802.11 standard, were used as the hardware platform. The communication protocol chosen was the Hypertext Transfer Protocol (HTTP), with JavaScript Object Notation (JSON) as the data format.

An implementation was built using the star network topology, with a hub unit responsible for receiving the requests, analyzing the validity of the payload, sending them to the respective devices, and reporting the result to the personal assistant.

Concerning the network access point, we have implemented two solutions. The first integrate with an already existing network, suited for home use, and on the other, the hub also becomes an access point, which is appropriate for use in corporate environments. Figure 1 presents a diagram with both solutions.

To facilitate the discovery of newly connected devices without user intervention, the zero-configuration multicast DNS (mDNS) protocol was used. An initial set of predefined commands were proposed and designed. These commands are capable of realizing canonical functions such as activating and deactivating actuators connected to General Purpose Input/Output (GPIO) pins of the microcontroller, reading the state of those pins, and doing pulse-width modulation (PWM) on the output signal. Each request should contain the unique identifier of the target node device and one of the predefined commands, making the packet data size be up to 32 bytes. Packages with custom commands and eventually larger payload sizes are also supported.

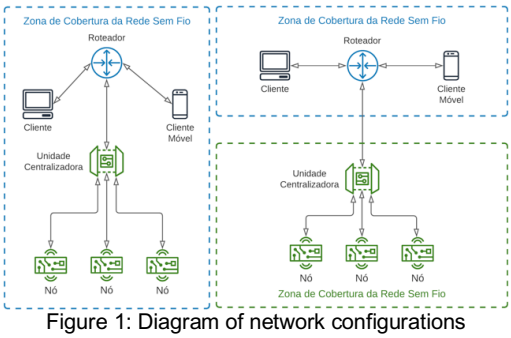

Figure 1: Diagram of network configurations

In addition to the predefined commands aimed at essential functions, an interface for defining new commands also was implemented, enabling the control of peripherals, including through I²C communication, Serial Peripheral Interface (SPI), and UART. Thereby, it's possible to integrate a higher range of peripherals into the system compared to other solutions.

Results

Functionality tests were conducted simulating the target uses. Among the activities performed are the activation and deactivation of an LED connected to the microcontroller, the reading of the LED status, and reading the output of a light sensor. To determine the average round-trip time (RTT) of the proposed solution, including the processing time of the requests, with intervals varying between one second and five minutes, were triggered, and 200 samples were collected. Figure 2 shows the average value for different packet sizes, excluding outlier values.

It is possible to notice that for the predefined commands, which cover a good amount of uses, the average time was below 120 milliseconds, with no noticeable delay between the execution of the request and the activation of the LED.

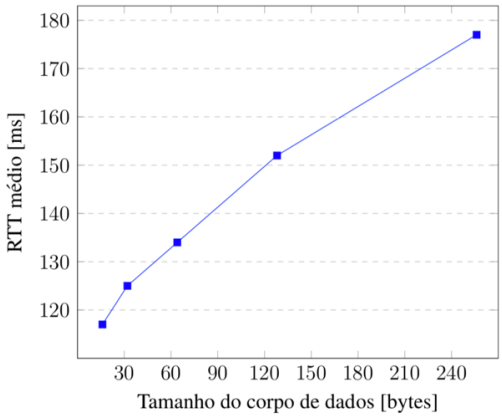

Figure 2: Average round-trip time on the target environment

Conclusions

The studies conducted in this research demonstrated the feasibility and efficiency of a control system for intelligent devices based on technologies that are already widespread, with low cost, and highly customizable. In this regard, the strategy of using a simple and with lower processing capacity hub, compared to that adopted by other solutions, proved to be adequate for managing smaller amounts of smart devices. Furthermore, the tests showed that it is possible to apply this infrastructure in a virtual assistant, where low latency is required.

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Inscrição No.: 1040

Título em Português:	A.D.A - Assistente Distribuída Avançada - Métodos para integrar WoT em uma assistente virtual
Título em Inglês:	A.D.A - Assistente Distribuída Avançada - Methods to integrate WoT into a Virtual Assistant
Área de Pesquisa:	Metodologia e Técnicas da Computação
Palavras Chave:	IoT - Sistema Distribuido - VPN
Ag. Financiadora do Projeto:	CNPq - PIBITI
Projeto:	Iniciação Tecnológica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	Ciência da Computação
Validado em:	**13/09/2021**

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Resumo do Trabalho em inglês:

METHODS TO INTEGRATE WOT INTO A VIRTUAL ASSISTANT

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Objectives

ADA, advanced distributed assistant, is a project started in 2019 at University of São Paulo to create an intelligent distributed personal assistant, which aims to interact with the user, in Portuguese, connecting him with devices through the Internet of Things[4]. Additionally, it aims to be a microservice-based and Open Source. The first phase of the project resulted in several articles about features, frameworks, API's and technologies to be used in the development of the virtual assistant [1][2][3][6][7].
In this work, the focus was the validation of methods for the interconnection between devices located in different environments, aiming to verify the possibility of using it with Web of Things gateway and Apache Kafka.

Materials and Methods

To reduce the limitations created by the frameworks used in A.D.A, technologies to distribute the system were revised, such as Virtual Private Network, remote access, SDP, cloud computing and Zero Trust Network Access. And with that, tests to control an LED took place from a computer in another environment in order to validate the communication.

Results

After testing the technologies, the ping averages for packet sending, practicality for remote interconnection and security for choosing the technology that best suited the project were analyzed and finally, VPN was defined, more specifically OpenVPN.

Conclusions

From the study carried out, it was possible to conclude that the advantages of using VPN over the others on ADA, having to interact with a LED in different places, such as at home and at work, using only one account to control everything in the same virtual network. In addition, another advantage is that the tool allows control without being in the same environment to change the state of the object, for example, turning on and off a light while traveling. One of the possibilities brought by distributed integration is the ease of sharing data between machines, in this case, access to objects and their state changes. With this, we also have that the interaction between user and device becomes simpler, as the control of the entire smart home is done by just one platform.

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Inscrição No.: 1758

Título em Português:	Esparsificação Espectral de Grafos
Título em Inglês:	Spectral Sparsification of Graphs
Área de Pesquisa:	Teoria da Computação
Palavras Chave:	aleatorização - grafos esparsos - matriz laplaciana
Ag. Financiadora do Projeto:	Sem fomento
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	Ciência da Computação
Validado em:	**13/09/2021**

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USP - UNIVERSIDADE DE SÃO PAULO
29º Simpósio Internacional de Iniciação Científica

Inscrição No.: 1758

Resumo do Trabalho em inglês:

Spectral Sparsification of Graphs

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Goals

Let $G = (V, E)$ be a graph and let w be a weight function on the edges. We define the weighted Laplacian matrix of G as:

$$L_G := \sum_{ij \in E} w_e (e_i - e_j)^\mathsf{T} (e_i - e_j). \qquad (1)$$

A spectral sparsifier of G is a sparse spanner subgraph H (with possibly new weights) such that:

$$x^\mathsf{T} L_G x \leq x^\mathsf{T} L_H x \leq (1+\varepsilon) x^\mathsf{T} L_G x \qquad (2)$$

holds for every $x \in \mathbb{R}^V$.

The goal of this undergraduate research project is to study efficient ways to construct spectral sparsifiers. Moreover, we also studied the connection between sparsifiers, expander graphs and Laplacian solvers. Another goal is to make Thiago proficient with recent and powerful tools and techniques from theoretical computer science (TCS).

Materials and Methods

The main method was the study of thesis, books and academic papers. The research papers were the central topic of study, while the books and thesis were mostly used to check definitions or to build a background before reading the papers.

Results

A range of algorithms to construct sparsifiers was studied, including randomized and deterministic approaches. We also studied an algorithm to solve a generalization of the original problem.

The first result was the construction in almost linear time by Spielman and Srivastava. The construction is based on a random sampling of the edges with probability proportional to the effective resistance of the edges. This algorithm results in a spectral sparsifier with $O(n \log n)$ edges.

The second result studied was a deterministic construction which results in a sparsifier with a linear number of edges. This result of Batson, Spielman and Srivastava was later generalized by de Carli Silva, Harvey and Sato for the problem of sparse sums of matrices.

The student also had contact with many important topics and techniques from TCS, like: dimensionality reduction, randomized algorithms and concentration lemmas. The student is now enrolled in the masters program of the department.

Conclusion

We conclude that there exists a diversity of algorithms to construct spectral sparsifiers. Furthermore, all algorithms studied in this project use interesting tools from TCS.

The research project was successfully completed. All goals were achieved and the results were satisfactory. The study of techniques and mathematical writing prepared Thiago for his graduate studies.

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Inscrição No.: 1808

Título em Português:	Teoria dos Grupos e o Teorema de Gabriel
Título em Inglês:	Group theory and Gabriel's Theorem
Área de Pesquisa:	Álgebra
Palavras Chave:	Grupos - Álgebras de Caminho - Teorema de Gabriel
Ag. Financiadora do Projeto:	FAPESP - Fundação de Amparo à Pesquisa do Estado de São Paulo
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	Matemática
Validado em:	**27/08/2021**

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Resumo do Trabalho em inglês:

Gabriel's theorem

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Objectives

The main goal of this project is to introduce the representations of finite dimensional algebras and study a Gabriel's theorem, for that was first made an introduction to abstract algebra, studying group theory. Given a finite-dimensional algebra A over an algebraically closed, basic and indecomposable field, our aim is to prove a characterization of A in terms of a structure called a quiver.

Materials and Methods

The methodology used in this project is the literature review on these subjects below:

- Group theory
  - Definition and examples of a group.
  - Subgroups and Lagrange's Theorem.
  - Homomorphism.
  - Normal subgroups and Quotient Groups.
  - Sylow's Theorem.

  In this part, the study was made under reference [4], and also [2].
- Gabriel's Theorem
  - Finite-dimensional algebra over an algebraically closed field
  - Module over a K-Algebra
  - Quivers and Path algebra
  - Gabriel's Theorem

  The study of these topics was made under the references [1], [3] and [5].

Under Eduardo's supervisor we did weekly meetings where we discussed the topics mentioned above. Besides, whenever necessary, students met virtually with the advisor to clear up questions and doubts.

Results

All listed topics of group theory and Gabriel's Theorem were studied. Here we will show the main notions studied.

Gabriel's Theorem

Let A be a finite dimensional, connected and basic algebra over an algebraically closed field K. Then there is a connected quiver Q_A and an algebra surjective homomorphism $\phi : KQ_A \to A$ such that $R_Q^m \subseteq ker\phi \subseteq R_Q^2$, and R_Q^m be the arrow ideal of the path algebra KQ, that contains all paths whose length is m. The quiver is unique, up to quiver isomorphism.

Basic Algebra

Let A be a K-algebra. This algebra A is called basic if there is an indecomposable projective decomposition $P_1, P_2, \ldots, P_t$:

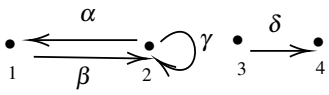
$$A \cong P_1 \oplus \cdots \oplus P_t, P_i \neq P_j, \text{ for } i \neq j$$

Note: It is no loss of generality, studying representations of algebras, assume that algebra is basic.

Quiver

A quiver (Q_0, Q_1, s, t) is a quadruple consisting of two sets: Q_0 (whose elements are called vertices) and Q_1 (whose elements are called arrows), and two maps s, t : $Q_1 \longrightarrow Q_0$ which associate to each arrow $\alpha \in Q_1$ its source $s(\alpha) \in Q_0$ and its target $t(\alpha) \in Q_0$, respectively. An arrow $\alpha \in Q_1$ of source $s(\alpha) = a$ and target $b = t(\alpha)$ is usually denoted by $\alpha : a \to b$. A quiver $Q = (Q_0, Q_1, s, t)$ is usually denoted briefly by $Q = (Q_0, Q_1)$ or even simply by Q.

Example 1.



For the quiver above, $Q_0 = \{1,2,3,4\}$ and $Q_1 = \{\alpha, \beta, \gamma, \delta\}$. Where $s(\alpha) = t(\beta) = 1$, $s(\beta) = t(\alpha) = s(\gamma) = t(\gamma) = 2$, $s(\delta) = 3$ and $t(\delta) = 4$.

Path Algebras

From now on all quivers will be finite. The path algebra KQ associated with Q as K-vector space has as its basis the set of all paths of length $n \geq 0$ in Q. For each vertice, there is a path length $n = 0$, called the trivial path associated denoted by e_i, such that $s(e_i) = t(e_i) = v_i$. For two paths $p = \alpha_1 \alpha_2 \ldots \alpha_r$ and $q = \beta_1 \beta_2 \ldots \beta_s$ in Q, the product is defined by

$$p \cdot q = \begin{cases} \alpha_1 \alpha_2 \ldots \alpha_r \beta_1 \beta_2 \ldots \beta_s, & \text{if } t(\alpha_r) = s(\beta_1), \\ 0 & \text{otherwise.} \end{cases}$$

The product of basis elements is then extended to arbitrary elements of KQ by distributivity.

Example 2. Let Q be the quiver:

The defining basis of the path algebra KQ is $\{e_1, \alpha, \alpha^2, \ldots\}$.So KQ is infinite dimensional. Let $K[x]$ be the e polynomial algebra over the field K. Thus KQ is isomorphic to $K[x]$ in one indeterminate x, the isomorphism being induced by the K-linear map:

$$\begin{aligned} \psi : KQ &\to K[x] \\ e_1 &\mapsto 1 \\ \alpha &\mapsto x \end{aligned}$$

Conclusion

The project has been completed in a way that the concepts of algebra have been well studied. Thus, students are building a solid path towards proving Gabriel's Theorem. Once this is done, we will study a second Gabriel's theorem that classifies algebras of finite representation type.

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Inscrição No.: 1809

Título em Português:	Teoria dos Grupos e o Teorema de Bezout para curvas
Título em Inglês:	Group Theory and Bezout's Theorem for curves
Área de Pesquisa:	Álgebra
Palavras Chave:	Grupos - Curvas algébricas - Teorema de Bezout
Ag. Financiadora do Projeto:	FAPESP - Fundação de Amparo à Pesquisa do Estado de São Paulo
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	Matemática
Validado em:	**27/08/2021**

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Resumo do Trabalho em inglês:

BÈZOUT'S THEOREM

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Objectives

The main objective of this project is to study Bèzout's Theorem. The final objective of our work is to understand and reproduce the proof of this important theorem. This understanding requires various prerequisites. We will approach, through bibliographical review, concepts such as affine and projective spaces, ideals, algebraically closed fields, as well as related concepts in algebraic geometry, for instance, varieties, multiplicities and curve components. The theorem states that, given two projective plane curves of degree d and d', defined in an algebraically closed field, with no common component, the number of intersecting points among the curves, counting their multiplicities, is dd'.

Methods and Procedures

The main methodology used in the pursuit of our objectives is the bibliographical review on themes related to Bèzout's Theorem, approaching the following, not exhaustive, list of topics: algebraic varieties, ideals, Hilbert's Nullstellensatz, local and coordinate rings, forms (homogeneous polynomials). We started the project by deepening ourselves in group theory, in order to get familiarized with abstract algebra concepts, which will be useful in reaching our final goal of understanding the theorem's proof. Moreover, we attend weekly seminars, under the supervision of our advisor Eduardo, where we present and discuss our results with another pair of students, whose studies are focused on Group Theory and Gabriel's Theorem. Such interactions have enabled us to have a broader view of the researched topic and have brought a complementary view to the one studied in the bibliography. The organized seminars consist of concepts explanations by the students themselves and by the advisor, being complemented by practical examples and exercises, solved during the encounters.

Results

We have already studied all the theoretical framework related to groups and affine spaces, highlighting that many of the found results can be extended to projective spaces, where Bèzout's Theorem is identified. More specifically, we are confident to approach propositions related to affine algebraic varieties and coordinate rings and to demonstrate Hilbert's Nullstellensatz.

Conclusions

Through the application of Bèzout's Theorem, given the intersection points of two curves in the projective space, we are able to identify, in many cases, their multiplicities without performing any calculations. The same theorem also enables us to quickly analyze the behavior of the curves in order to ensure we find all possible intersections, given the multiplicity of already found intersection points.

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**FAPESP scholarship since 01/01/2021.*

Inscrição No.: 2381

Título em Português:	Função Distância de Finsler e Frentes de Ondas
Título em Inglês:	finsler distance function and wavefronts
Área de Pesquisa:	Geometria e Topologia
Palavras Chave:	Geometria de Finsler - Função Distância - Frentes de Ondas
Ag. Financiadora do Projeto:	FAPESP - Fundação de Amparo à Pesquisa do Estado de São Paulo
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	
Validado em:	**08/09/2021**

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USP - UNIVERSIDADE DE SÃO PAULO
29º Simpósio Internacional de Iniciação Científica

Inscrição No.: 2381

Resumo do Trabalho em inglês:

Finsler Distance Function and Wavefronts

Guilherme Cerqueira Gonçalves

(processo 2021/00551-3, Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP).)

Orientador: Prof. Marcos M. Alexandrino

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Objectives

To study Finsler Geometry through the perspective of Foliations of Hypersurfaces. Investigate, in particular, foliations arriving from transnormal functions in Finsler manifolds (defined below), their properties of parallelism, equifocality and in general their similarities and differences from the case of Riemannian manifolds. This study keeps in mind some applications, for example wavefronts and wildfire.

Definition: A manifold (M,F) is called a Finsler Manifold if for each tangent space T_pM exists a Minkowski norm F_p, such that: (with, λ>0.)

$$(1)\ F:TM\to[0,\infty],\ \ with\ \ F(\lambda v)=\lambda F(v),$$

$$(2)\ g_v(u,w)=\frac{1}{2}\frac{\partial^2}{\partial t\,\partial s}F^2(v+su+tw)|_{s=t=0},$$

in (2), $g_v(u,w)$ is a metric that depends on the direction of v.

Definition: A smooth function f from a Finsler manifold (M, F) to the reals is called F-transnormal if exists a continuous real-valued function t with domain f(M), such that:

$$(3)\ \ F(\nabla f)^2=t(f)\ .$$

Materials and Methods

Study of books,papers and dissertations, whose principal reference is the paper [1] in the references section. Also, the student presented, to help in the process of study, weekly seminars to the advisor and other guest students. Those guests were in most cases graduate students from the same institute, invited by the advisor.

Results

Proposition: Let (M,F) be a smooth, compact and connected Finsler manifold and $f:M\to[a,b]$ a F-transnormal smooth function that satisfies equation (3), where t is a C^1 function in [a,b]. Suppose that:

(a) The level sets are connected spaces.
(b) The critical level sets $f^{-1}(a)$ and $f^{-1}(b)$ are submanifolds of codimension bigger than one.
(c) a and b are the only singular values of [a, b],
(d) The function t is such that:

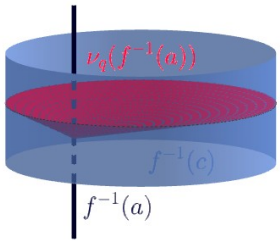
$$(4)\ \ t'(a)\neq 0\neq t'(b)\ .$$

Then, exists a Riemannian metric g in M such that f is a (Riemannian) transnormal function with respect to g. In particular, $\{f^{-1}(c)\}_{c\in[a,b]}$ is a Singular Riemannian Foliation in (M, g).

Theorem: Let (M,F) be a analytic, compact and connected Finsler manifold, let also

$f : M \to [a,b]$ be a F-transnormal analytic function. Suppose that the level sets are connected spaces, and that a and b are the only singular values in [a, b]. Then:

(a) The critical level sets $f^{-1}(a)$ and $f^{-1}(b)$ are submanifolds.
(b) The level sets are equidistant to one another, that is, $\{f^{-1}(c)\}_{c\in[a,b]}$ is a Finsler Partition. In particular, for each regular value c, $f^{-1}(c)$ is a past and future cylinder of each singular level set.



Picture 1: An exemple of level surface of a transnormal function in R3, where "a" is a singular value and "c" is a regular one, and the representation of the Finsler orthogonal space, in red.

Applications: The study of Transnormal Foliations can be used as a model for the behavior of wave fronts as proven in the paper [3], with that in mind, Finsler Geometry allows us to understand wave fronts in an anisotropic medium of propagation. Combined with this, according to [2] wildfire spread modelling uses anisotropic wave fronts because of the effect of the wind on the spreading of the flames and with these techniques it is possible to generalize the current models, as é possível generalizar os modelos atuais, as those can be perceived as finslerian transnormal foliations.

Conclusions

The study of Finsler Foliations allows the analysis of broader situations than those contemplated by the Riemannian Geometry (such as the study of foliations that are only parallel in one direction, ex: forward parallelism). There are, although, Riemannian phenomena that has its characterization in the Finsler setting (such as the theorem and proposition above that ensures, when the hypotheses are met, respectively a foliation to be parallel forward and backward similarly to the Riemannian case; the other ensures that a Singular Finsler Foliation can be perceived as a Riemannian one), given Analytical and Topological restrictions, showing new geometric phenomena to be studied and new applications such as those explained in the Applications part above.

"As opiniões, hipóteses e conclusões ou recomendações expressas neste material são de responsabilidade do(s) autor(es) e não necessariamente refletem a visão da FAPESP".

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Inscrição No.: 2429

Título em Português:	Previsões e identificação de momentos de compra e venda usando modelos de séries temporais no mercado financeiro
Título em Inglês:	Forecasts and identification of buying and selling moments using time series models in financial market
Área de Pesquisa:	Estatística
Palavras Chave:	SES - modelos ARIMA-APARCH - séries temporais
Ag. Financiadora do Projeto:	FAPESP - Fundação de Amparo à Pesquisa do Estado de São Paulo
Projeto:	Iniciação Científica
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Inscrição No.: 2429

Resumo do Trabalho em inglês:

Forcasts and identification of buying and selling moments using time series models and neural networks in financial market

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Objectives

The main goal is to identify turn points in the IBOVESPA stock price time series. The turn points identification is important as they usually correspond to stock buying or selling moments. We will make forecasts for different lags obtained from different techniques.

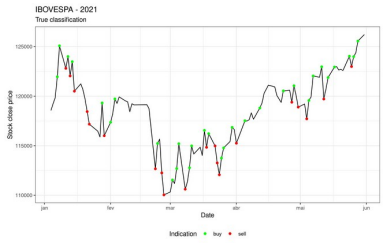
Materials and Methods

As a strategy to identify the buying and selling moments, we will fit the simple exponential smoothing (SES) (Morettin and Toloi, 2006) and the ARIMA model (Box and Jenkins, 1970) combined with the APARCH model (Ding, Granger and Engle, 1993) to deal with the conditional heteroskedasticity. For both techniques, the fit will be applied in the IBOVESPA daily log returns from may 28th 1993 through april 30th 2021. The ARIMA-APARCH parameters will be estimated by maximizing the likelihood, which requires the assumption of an error distribution, usually either the gaussian or the Student's-t or the skewed Student's-t distribution. We will make forecasts for the days of the month of may, 2021.

The procedure is to make 4 forecasts for each day of maym comparing all 4 of them to the log return value observed in the day previous to the one being forecasted. The first forcast for a determined day is made 4 days before, fitting all log return values data avaiable at that date. The remaining forecasts are are made similarly, with each one having one more sample value than the last. Before each forecast, the model is re-fitted with the updated avaiable data. Once we have all 4 forecasts from the SES fit, each one of them is compared to the observed log return value of the day previous to the one we are insterested in, and if all 4 forecasts values are greater (less) than that value, we consider that the next day will have an increase (decrease) in value, and therefore we label it as a buying (selling) day. When a day doesn't get the buying or selling label, we label that day as stable. The procedure for the ARIMA-APARCH model is quite similar, the difference being the metric of comparison, which instead of the forecast alone, is the forecast plus (minus) 0.005 times it's respective conditional volatility estimate. The hyperparameters of the ARIMA-APARCH model will be chosen based on the adequability for fitting all data until the last day of april 2021, with the adequability being verified through residuals analysis. Notice that, regardless of the applied technique, each day's labeling occurs in the day before, so there's time in the after market to make the intended transactions. The "true" label of each day of the time series was based on IBOVESPA's closing stock market price, with the label of buying (selling) day being assingned when the stock price increases (decreases) at least US$1000.00 in comparison to the previous day, or when there's a relative increase (decrease) of over 5% from the previous day. If none of the conditions above are met for a day, it is labeled as stable.

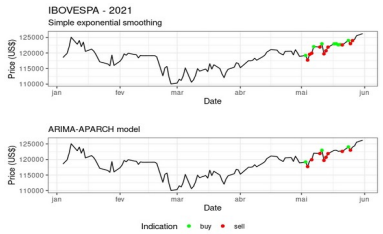
Results

For better visualization, the “stable” category won’t be shown in any of the following plots. Picture 1 ilustrates the “true” time series classification along the 2021 year.



Picture 1: 2021 IBOVESPA chart according to the classification method proposed

Picture 2 show predicted classifications in the month of may, according to the SES and ARIMA-APARCH techniques.



Picture 2: IBOVESPA chart with predicted classifications according to the SES and ARIMA-APARCH techniques

Confusion matrices, make it easy to evaluate the performance for each technique, prediction with the observed (“true”) label . Tables 1 and 2 display these matrices for the SES and ARIMA-APARCH techniques, respectively.

Table 1: Confusion matrix of SES’s performance

		Observed		
		Sell	Stable	Buy
Predicted	Sell	3	4	3
	Stable	0	0	0
	Buy	0	4	4

Table 2: Confusion matrix of ARIMA-APARCH’s performance

		Observed		
		Sell	Stable	Buy
Predicted	Sell	3	4	1
	Stable	0	3	4
	Buy	0	1	2

Conclusions

Analysing the results, we conclude that the SES is a more risky technique, always indicating a transaction (buy or sell), and thus it’s not a good technique to find stable days in the series. This technique had an accuracy of 38.89% considering 3 categories, and a 70% accuracy if we disregard the stable category (disregarding line 2 and column 2 of the confusion matrix). The ARIMA-APARCH model was a little better, as expected, achieving an accuracy score of 44.44% overall, and 83.33% considering only the buy and sell categories; both ARIMA-APARCH’s scores are an improvement compared to the SES’s, justifying the complexity gap between them. At last, perhaps there are procedures for classifying the “true labels” and predicting the label of each day in the time series that would be more appropriate to achieve the goals of this work.

Acknowledgements

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Special thanks to Professor Airlane Pereira Alencar, who co-supervised this work, and to the contributors Guilherme Frederico Rafare and Professor Marcelo de Souza Lauretto.

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Inscrição No.: 2436

Título em Português:	Residualidade em conjuntos de funções reais e complexas
Título em Inglês:	Residuality in sets of real and complex functions
Área de Pesquisa:	Análise
Palavras Chave:	Residualidade - Nunca diferenciável - Função holomorfa
Ag. Financiadora do Projeto:	Outros
Projeto:	Iniciação Científica
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Resumo do Trabalho em inglês:

RESIDUALITY IN SETS OF REAL AND COMPLEX FUNCTIONS

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Objectives

This work aims to show the residuality of two sets of functions: the continuous functions that are nowhere differentiable in [0,1] (*CND*) and the strongly annular functions (*FA*).
Considering the set C[0,1] of continuous functions in the interval [0,1], we have that it is a complete normed space, when equipped with the maximum norm. An element in *CND* is a function in C[0,1] that is nowhere differentiable.
A strongly annular function is a holomorphic function *f*, defined on the complex open unit disk (denoted by *D*), such that

$$\limsup_{r\to 1}\left(\min_{|z|=r}|f(z)|\right)=+\infty. \qquad \text{(I)}$$

Thus, *FA* is a subset of the space of the holomorphic functions in *D* (denoted by *H(D)*), in which we will use the compact-open topology, where the open neighborhoods of a function *f* are finite unions of sets of the form

$$\left\{g\in H(D):\sup_{K}|g-f|<r\right\} \qquad \text{(II)}$$

where K is a compact set contained in *D* and r>0.
H(D) with the compact-open topology is a complete and metrizable topological vector space.
The student had previously studied “large” sets from an algebraic point of view through concepts such as lineability and algebrability (including the *CND* set) and in this work we seek to understand the size of sets in another sense (the topological) through residuality. We say that a set is residual if its complement is a meager set (that is, a countable union of sets such that the inside of the closure of each one of them is empty).
Using Baire's theorem (in a complete metric space, every enumerable intersection of dense open subsets is still a dense subset) we obtain that residual sets in complete metric spaces can be considered “large” sets, thus providing a topological concept for talking about size of sets.

Materials and Methods

The student investigated the proofs contained in books [1] (where several topological and algebraic tools are addressed to analyze the size of different sets) and [3] (where the residuality of *CND* is demonstrated) and the article [2] (which proves several results about strongly annular functions), translating them because they are in English. Weekly meetings of one hour were then held (via Google Meet) in which seminars were presented to the supervisor about what was being studied by the student.

Results

Constructing, for each natural n, the set E_n of real continuous functions with domain [0,1] satisfying the following property

$$\text{“Exists } x \in \left[0, 1 - \frac{1}{n}\right] \text{ such that}$$

$$\frac{|f(x+h) - f(x)|}{h} \leq n, \text{ if } h \in [0, 1-x]\text{”} \quad \text{(III)}$$

we show that each E_n is closed with an empty interior and therefore the union of all of them is a meager set, thus obtaining that its complement in C[0,1] is a residual set (and a set of never differentiable functions). Therefore, the *CND* set is residual in C[0,1].
We show the residuality of the *FA* set in *H(D)*, by proving the residuality of the subsets of the form

$$\left\{ f \in H(D) : \lim_{\substack{|z| \to 1 \\ z \in C(\sigma)}} \frac{|f(z)|}{\varphi(z)} = +\infty \right\} \quad \text{(IV)}$$

where φ is a continuous function defined on D with positive real values, σ is a strictly increasing sequence in the interval (0.1) converging to 1 and C(σ) is the collection of all circles whose radii are the terms of σ. For this, we write each set of the form (IV) as the intersection of the sets

$$\bigcup_{m \geq n} \{ f \in H(D) : |f(z)| > n\varphi(z), \text{ if } |z| = r_m \} \quad \text{(V)}$$

(where n is natural and the r_m are the terms of σ), which we prove to be open and dense in *H(D)*.
We also verify that *CND* is spaceable (contains, except for the null element, an infinite-dimensional closed vector space) and *FA* is dense-lineable (contains, except for the null element, an infinite-dimensional dense vector space) and strongly algebrable (contains, less than zero, a free algebra generated by an infinite enumerable set), but we will not present these results in the present work.

Conclusions

It is not trivial that the sets *CND* and *FA* are non-empty. Until the exhibition of the Weierstrass's Monster in 1872, it was thought that a continuous function must have a derivative at least one point of its domain. On the other hand, it is observed that there is no holomorphic function *f*, defined in the open unit disk, satisfying

$$\lim_{r \to 1} \left(\min_{|z|=r} |f(z)| \right) = +\infty \quad \text{(VI)}$$

because of the Maximum Principle and the Uniform Continuation Principle.
We conclude, from the above results and a consequence of Baire's theorem, that these sets are not only non-empty but are “large” in a very precise (topological) sense.
In developing the project, the student was able to apply his knowledge of topology, real and complex analysis learned in undergraduate disciplines, in more advanced contexts that are not covered in such disciplines. The project also enabled the scholarship holder to learn mathematical analysis techniques and arguments, providing him with a background and a repertoire that will be important in his professional career.
The student wishes to thank CNPq for the financial support.

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Título em Português:	aplicação do modelo epidemiológico sir aos casos de infecção pela covid-19
Título em Inglês:	application of the epidemiological model SIR to cases of covid-19 infection
Área de Pesquisa:	Matemática Aplicada
Palavras Chave:	epidemiologia - covid-19 - modelo SIR
Ag. Financiadora do Projeto:	CNPq - PIBITI
Projeto:	Pré-iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
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USP - UNIVERSIDADE DE SÃO PAULO
29º Simpósio Internacional de Iniciação Científica

Inscrição No.: 2606

Resumo do Trabalho em inglês:

APPLICATION OF THE EPIDEMIOLOGICAL MODEL SIR TO CASES OF COVID-19 INFECTION

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Objectives

Project the spread of the Covid-19 virus using the SIR epidemiological model, with the purpose of comparing this projection of the pandemic to the real data.

Materials and Methods

The epidemiological model SIR was adopted, used to create projections of the spread of contagious diseases, taking into account the population divided into three groups: (S) Susceptible; (I) Infected; (R) Removed. [1]

Based on this model, a projection was created with the functionalities of the Microsoft Excel spreadsheet program, about the dissemination of Covid-19 in Brazil. Started on 2021/03/17, the simulation has its data updated in periods of 48 hours, this being the estimated time that an infected person can infect others.

The number of people infected by an infected individual, initially, is the basic reproduction ratio ($R0$). However, this number changes according to the susceptible rate of the population (Ts) and this value is updated by the effective reproduction ratio (Re). [2]

$$Re = R0 * Ts$$

$$Ts = \frac{S}{P}$$

Where P is the total population.

The equations complement each other and then divide into 2 groups. The first aims to establish the value of the total number of vaccinated (Tv) and the total number of infected (Ti), using calculations involving the average of vaccinated (Mv). Here are the first equations:

$$Tv = \sum_i Mv$$

$$Ti = \sum_i I_i$$

In I_i, *I* represents the number of infected and *i* corresponds to a period of time, where $i = 2,3,\ldots 30$;

The second group of equations defines, finally, the number of susceptible people (S), gathering all the results obtained.

$$S = P - R - I_i$$
$$I_i = I_{i-1} * Re$$
$$R = Tv + Ti$$

Results

The values obtained with the simulation showed a divergence in relation to the actual values. On May 16, 2021, the number of removed is 24,408,407. Comparing to the actual data, the results have a divergence of 64%, since the actual numbers correspond to 15,627,475 removed.

Conclusions

It was concluded that this epidemiological model proved to be efficient, but discrepant to the actual data. This was due to the criteria adopted for the realization of the projection. They are: people who received the first dose of the vaccine are immunized; the non-emergence of new variants of Covid-19 that quickly raised the numbers; measures such as lockdown, increased prevention measures and closing of public places, which could slow the growth of the pandemic, were not taken into account; people who have already been infected do not become infected again; the number of people vaccinated also grow every day, not counting on unforeseen events or lack of vaccines.

The advancement of technology is indispensable to combat this and other possible pandemics that may arise. Mathematics working to predict breadth and science working in combat are great allies to prevent the growth of infectious disease victims.

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Inscrição No.: 2608

Título em Português:	Estudo do contexto pandêmico a ser aplicado em aula para auxiliar na compreensão de conceitos matemáticos
Título em Inglês:	A study of the pandemic context to be applied in class to auxiliate the comprehension of mathematical concepts
Área de Pesquisa:	Matemática Aplicada
Palavras Chave:	conceitos - ferramentas - simulação
Ag. Financiadora do Projeto:	Sem fomento
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	
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USP - UNIVERSIDADE DE SÃO PAULO
29º Simpósio Internacional de Iniciação Científica

Inscrição No.: 2608

Resumo do Trabalho em inglês:

A STUDY OF THE PANDEMIC CONTEXT TO BE APPLIED IN CLASS TO AUXILIATE THE COMPREHENSION OF MATHEMATICAL CONCEPTS

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Objectives

Warn about the proportion of COVID-19, through mathematical simulations in spreadsheets, with computational resources according to the current pandemic scenario.

Materials and Methods

The methodology applied in the project is described in the following steps:

1. Review and study of the concepts of geometric progression, applied to the current context, and particularly the propagation of the new Coronavirus.
2. Examination of the Google Spreadsheets platform and exploration of its functions and formulas.
3. Analysis of the factors that make up the dissemination and transmission of COVID-19 and have a direct or indirect impact on the real outcome and its divergence from the simulation data. During the development of the simulation, we made some assumptions, among them: infected people couldn't be infected again; people who' ve been infected acquire immunity; and that when people take the first dosage, they also get immunity

4. Development of the simulation chart and estimation from the data referring to the pandemic in a given period, using the tools offered by Google Spreadsheets:

COVID-19 SIMULATION			
Date	Immunized population	Vaccinated population	Infected population
16/03/2021	20.438.965	8.919.356	11.519.609
19/04/2021	34.516.204	17.960.500	16.555.704
17/05/2021	47.184.840	25.406.148	21.778.692

COVID-19 REAL DATA			
Date	Immunized population	Vaccinated population	Infected population
16/03/2021	20.438.965	8.919.356	11.519.609
19/04/2021	38.754.414	24.776.701	13.977.713
17/05/2021	52.013.330	36.355.939	15.657.391

COVID-19 SIMULATION		
Date	Active transmitters	Susceptible population
16/03/2021	155.920	191.361.035
19/04/2021	401.439	177.283.796
17/05/2021	294.652	176.345.383

COVID-19 REAL DATA		
Date	Active transmitters	Susceptible population
16/03/2021	83.926	191.361.035
19/04/2021	30.624	173.045.586
17/05/2021	30.148	159.786.670

Conclusions

The dissemination of the virus in reality showed lower numbers than in the simulation, so we can see that vaccination and lockdowns have helped in decreasing the spread of the virus.

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Inscrição No.: 2814

Título em Português:	Algumas técnicas para o estudo do grupo de tranças
Título em Inglês:	some techniques for the study of the braid group
Área de Pesquisa:	Geometria e Topologia
Palavras Chave:	Tranças - Braid group - Configuration spaces
Ag. Financiadora do Projeto:	FAPESP - Fundação de Amparo à Pesquisa do Estado de São Paulo
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
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29º Simpósio Internacional de Iniciação Científica

Inscrição No.: 2814

Resumo do Trabalho em inglês:

SOME TECHNIQUES FOR THE STUDY OF THE BRAID GROUP

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Objectives

The goal of this project is the study of the fundamental properties of B_n, the Artin braid group of braids on n strings. In particular, we shall see that B_n is torsion-free.

Materials and Methods

If M is a manifold, define the n-th configuration space of M, $F_n(M)$, as the space $\{(x_1, \ldots, x_n) \in M^n : x_i \neq x_j \ \forall i \neq j\}$. The symmetric group Σ_n acts freely on $F_n(M)$, such that $C_n(M) := F_n(M)/\Sigma_n$ is said to be the n-th unordered configuration space of M. The main technique employed in this project consists of identifying B_n with $\pi_1(C_n(\mathbb{R}^2))$. From this, we make use of homotopy sequences and the theory of $K(G,1)$ spaces to obtain information about $\pi_1(C_n(\mathbb{R}^2))$.

Results

Denoting $F_{n,m}(\mathbb{R}^2)$ for $F_n(\mathbb{R}^2 \setminus \{q_1, \ldots, q_m\})$, we have that the projection $F_{n,m}(\mathbb{R}^2) \to F_{n-k,m}(\mathbb{R}^2)$ gives $F_{n,m}(\mathbb{R}^2)$ the structure of a fiber bundle (see [2]). By means of the associated homotopy sequences, it follows that $F_n(\mathbb{R}^2)$ is a space of type $K(G,1)$ for $G = \pi_1(F_n(\mathbb{R}^2))$. On the other hand, the projection $F_n(\mathbb{R}^2) \to C_n(\mathbb{R}^2)$ given by the quotient is a covering map, therefore we can utilize its homotopy sequence to observe that $C_n(\mathbb{R}^2)$ is a $K(B_n,1)$ space. From this, we have:

Theorem: The group B_n is torsion-free.

Indeed, suppose B_n has some subgroup isomorphic to $\mathbb{Z}_k$ and consider the connected covering space $p : X \to C_n(\mathbb{R}^2)$ such that $\pi_1(X) = \mathbb{Z}_k$. Since $C_n(\mathbb{R}^2)$ is a finite dimensional CW-complex (adapted from [1]), so is X, thus the latter's homology groups vanish for all but finitely many indices. On the other hand, we have that X is a $K(\mathbb{Z}_k,1)$ space, so it is homotopy equivalent to $S^\infty/\mathbb{Z}_k$. But $H_n(S^\infty/\mathbb{Z}_k) \neq 0$ for n arbitrarily large ([3]), a contradiction.

Conclusions

In this project, we not only proved an important property of B_n but also demonstrated the value of techniques from Algebraic Topology in the study of braids. The next step in the research consists of analyzing further properties of B_n, such as its usual presentation and its actions and representations of interest.

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[1] Recipient of grant #2021/02549-6, São Paulo Research Foundation (FAPESP).

Inscrição No.: 2877

Título em Português:	Localização em anéis não comutativos
Título em Inglês:	localization in noncommutative rings
Área de Pesquisa:	Álgebra
Palavras Chave:	localização - anel não comutativo - anéis de Ore
Ag. Financiadora do Projeto:	Sem fomento
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	
Validado em:	**08/09/2021**

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Resumo do Trabalho em inglês:

Localization in noncommutative rings

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Objectives

The objective of this project is the in-depth study of different approaches to the problem of the construction of fields and rings of fractions: the usual construction in commutative rings, its noncommutative version for Ore rings, limitations of the method in more general contexts and the Martindale version for prime rings.

Materials and Methods

The work was based on seminars and discussions about the results obtained by the student in weekly and virtual meetings, which were based on reading and studying bibliographic references.

Results

Every integral domain has a field of fractions. This fact is well known, as well as the following assertion: A commutative ring R can be embedded in a field if and only if R is a integral domain. In this project, generalizations of this construction were addressed for not necessarily commutative rings. Among the main constructions and results studied, we mention:

- It is said that a domain R is a right Ore domain if, for any $a, b \in R$ nonzero, $aR \cap bR \neq \{0\}$. Every right noetherian domain is a right Ore domain.
- Let R be any ring. Then R has a "skew field of fractions" T such that every element of T is expressed in the form ab^{-1}, with $a, b \in R$ and $b \neq 0$, if and only if R is a right Ore domain. Moreover, T is then determined up to a unique isomorphism over R.
- There exist domains that cannot be embedded in any ring skew field.
- The free associative algebra $R = C\langle u, v\rangle$ can be embedded in several skew fields that are not R-isomorphic.
- The symmetric Martindale ring of quotients, a construction applied to prime rings, generalizes the construction of the field of fractions and has many applications.

Conclusions

The generalization of the construction of fields of fractions for not necessarily commutative rings is a rather rich topic. Their study allows us to conclude that there exist domains that have unique rings of fractions, that there exist domains with several rings of fractions that are not isomorphic and, surprisingly, that there exist domains that cannot be embedded in any skew field. In addition to these facts, the symmetric Martindale ring of quotients proves to be an interesting alternative construction for this generalization.

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Inscrição No.: 2900

Título em Português:	Representações Modulares de Grupos Finitos
Título em Inglês:	modular representations of finite groups
Área de Pesquisa:	Álgebra
Palavras Chave:	Representações - Grupos finitos - Blocos cíclicos
Ag. Financiadora do Projeto:	Outros
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
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Colaborador:

Nome: Vitor de Oliveira Ferreira Instituição: Universidade de São Paulo

Resumo do Trabalho em inglês:

Modular Representations of Finite Groups

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Objectives

The concept of a group is present in many areas of Mathematics and it seeks to portray which symmetries a given object has and how they interact with each other. Groups can be defined abstractly, without mentioning the symmetries of a concrete object, but they naturally show up in examples acting in some set. When this set is a vector space and when the group acts by linear operators, this action is called a *representation*. Looking at the representations often allows one to discover new properties of the group and hence they are a powerful tool in the study of groups.

When Representation Theory began to settle down in the mathematical scenario, the vector spaces considered for representations were defined over the field of real numbers or over the field of complex numbers. However, the definition of a representation makes sense over any field. When the field in question has positive characteristic, the representations are entitled *modular*, an adjective which refers to the field of integers modulo a prime number.

The mathematician Richard Brauer was not the first to consider modular representations, but he was the first to perform a major progress in this theory, which is different and more complicated than the usual one. He intended to apply this new knowledge in the classification of the so-called finite simple groups. This plan was in fact achieved and other applications also emerged, even in the study of representations in characteristic zero.

The aim of the present work was to understand some of the tools used in the study of modular representations of finite groups, in particular two concepts introduced by Brauer: the one of a block of a group algebra and the one of a defect group of a block. The final goal was the description of blocks with cyclic defect groups, which is one of the greatest accomplishments of the theory.

Materials and Methods

Besides the individual study of the indicated bibliography, specially the book [1], weekly meetings were held with the presence of the supervisors and other three students, where the author exposed what he learned and his conclusions about the results discussed.

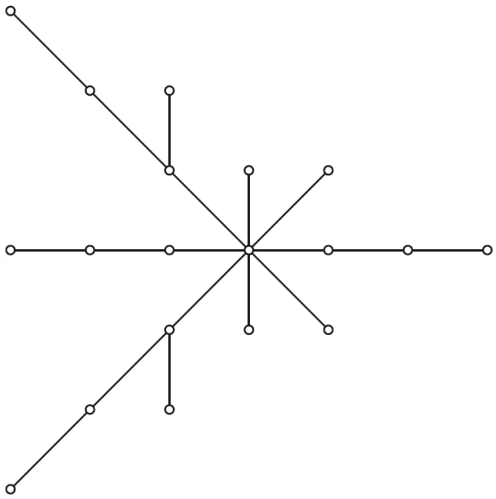
Results

Just as any integer number can be written as a product of prime numbers, every representation can be decomposed into the so-called *indecomposable* representations. In this way, the study of the representations of a group can be divided into two steps: classifying the indecomposable representations and understanding which indecomposable representations appear in a given representation.

When the base field has characteristic zero, every indecomposable representation is also *simple*, that is, it does not have proper nontrivial subrepresentations. This makes the study much easier than in the modular case, where this property does not hold. When the characteristic is positive, in most cases there are infinitely many indecomposable representations and there is little hope of understanding all of them. Therefore, the problem needs to be tackled differently.

One way of continuing the study is to isolate the representations which are more complex and to work on top of the remaining ones. For this purpose, it is possible to separate the representations into the so-called *blocks*. In some sense, two representations are in the same block if they are sufficiently compatible. Each block is provided with an invariant called the *defect group*. Intuitively, it is the defect group that measures how complicated the representations of a given block are. For example, when the defect group is trivial, the representations of the block behave as in characteristic zero and thus they are much easier to be analyzed.

The next difficulty level occurs when the block possesses a cyclic defect group. In this case, it is still possible to give a good description of the indecomposable representations. This description can be synthesized by the *Brauer tree* associated with the block, which is an acyclic graph embedded in the plane; see Picture 1 for an example. Each edge of the Brauer tree corresponds to a simple representation and the way the tree is embedded in the plane determines the structure of the indecomposable representations. There are also other interesting properties encoded by the Brauer tree. For example, its vertices correspond to simple representations in characteristic zero and the arrangement of the edges relates the properties in characteristic zero with those in positive characteristic.



Picture 1: Brauer tree of the principal block of the Thompson sporadic group in characteristic 19.

Conclusions

As this work progressed, it was possible to realize how rich the theory is and to learn a profound result describing the structure of blocks with cyclic defect groups. Therefore, the study carried out in this project served as a motivation and a good theoretical basis for a future deepening in the study of representations of finite groups.

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Inscrição No.: 2955

Título em Português:	Identificação de Peptídeos Antivirais Através de uma Arquitetura de Ensemble Deep Learning
Título em Inglês:	identification of antiviral peptides through an ensemble deep learning architecture
Área de Pesquisa:	Biotecnologia
Palavras Chave:	Bioinformática - Deep Learning - Peptídeos Antivirais
Ag. Financiadora do Projeto:	CNPq - PIBITI
Projeto:	Iniciação Tecnológica
Unidade de Apresentação:	Instituto de Matemática e Estatística
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Resumo do Trabalho em inglês:

Identification of Antiviral Peptides Through an Ensemble Deep Learning Architecture

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Objectives

Development of a Deep Learning methodology capable of predicting if a given peptide possesses antiviral activity. Perform a comparative analysis of the available methods for prediction of Antiviral Peptides (AVPs).

Materials and Methods

We gathered experimentally validated Antimicrobial Peptides (AMPs) from different databases found in the literature, resulting in 15,650 AMPs and 2818 AVPs. The data about peptides without antiviral activity was obtained through the *in silico* digestion of Uniprot proteins that were not secreted, not antiviral, not antimicrobial and not toxic. A homology reduction was performed with *cd-hit* in both sequence sets, which were then used to construct two datasets: a set with 9567 AMPs and 9567 non bioactive peptides and a set with 1695 AVPs and 1695 non bioactive peptides. These datasets were used as training data for the model in the AMP and AVP identification tasks respectively.

The predictor is composed of two main modules, both of which are independent machine learning models. The modules receive different information about the input sequence and perform their individual predictions about the bioactivity of the given sequence. These predictions are then linearly combined for the final result. The process of evaluating a sequence can be seen in Figure 1.

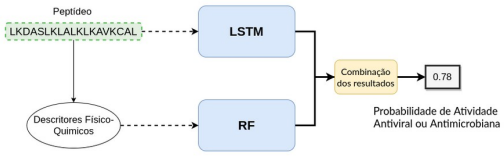

Figure 1: Evaluation of a given peptide by the model. The final result is the probability that a peptide has the expected type of activity.

A hiperparameter optimization step was performed utilizing the *optuna* library. The performance of 5 different feature sets as input for the Random Forest module was also evaluated.

Results

Figure 2 shows the results of the hyperparameter search utilizing each feature set.

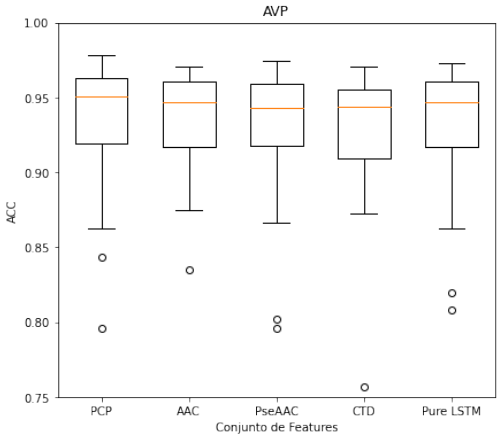

Figure 2: The model's Accuracy with each set of hyperparameters, split by feature set.

The best performing models obtained **96% Accuracy and 0.98 AUC** in the AVP identification task and **96% Accuracy and 0.99 AUC** in the AMP identification task in 10-fold cross validation test.

Tables 1 and 2 compare the performance of our model with other available tools utilizing the sequence sets before the pre-processing steps. We can see that our model outperforms existing tools in both cases.

Table 1: Correct and Incorrect Predictions in the experimentally validated AVP sequences (2818 sequences)

Tool	Correct Pred.	Incorrect Pred.
AntiVPP	1364 (48.4%)	1454 (51.6%)
Meta-iAVP	1494 (53.3%)	1309 (46.6%)
Our Model	2745 (97.4%)	73 (2.6%)

Tabela 2: Correct and Incorrect Predictions in the non bioactive peptide sequences (1279076 sequences)

Tool	Correct Pred.	Incorrect Pred.
AntiVPP	1007593 (78.7%)	271483 (21.3%)
Meta-iAVP	838772 (65.5%)	440003 (34.4%)
Our Model	1234716 (96.5%)	44322 (3.4%)

Conclusions

We have developed a new architecture for AVP identification based on sequence information, that outperforms existing models both in the correct AVP predictions and in the non bioactive peptide identifications. This tool can be used for identifying new AVPs, facilitating the discovery of new pharmacological principles.

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Inscrição No.: 2948

Título em Português:	Modelo de deep learning não-supervisionado para integração de variantes de splicing e CNVs em câncer de mama triplo negativo.
Título em Inglês:	unsupervised deep learning model to integrate splicing variants and cnvs in triple negative breast cancer.
Área de Pesquisa:	Metodologia e Técnicas da Computação
Palavras Chave:	Bioinformática - Deep Learning - Splicing Alternativo
Ag. Financiadora do Projeto:	CNPq - PIBITI
Projeto:	Iniciação Tecnológica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	
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Inscrição No.: 2948

Resumo do Trabalho em inglês:

Unsupervised deep learning model to integrate splicing variants and CNVs in triple negative breast cancer.

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Objectives

Breast cancer is the most frequently diagnosed type of cancer among women worldwide, with the triple-negative ductal-invasive (TNBC) subtype being the most aggressive and lethal. Therefore, it is important to identify molecular markers that can be useful for new treatments for breast cancer. The main goal is to establish a Deep Learning model based on autoencoder architecture for integration of biological data such as gene splicing variant expression, copy number variation (CNV) of a gene within a chromosome locus and protein-protein interaction (PPI). Using these data and the model, we intend to identify aberrant splicing molecular markers in TNBC.

Materials and Methods

We tested several splicing prediction and quantification models that have been well evaluated in the literature. CuffLinks, rMATS, StringTie and SplAdder were tested and compared. The methods showed a variation in the total number of variants predicted by splicing type, and the rMATS was the most effective in identifying and quantifying splicing variants.

After quantifying the splicing variants, we selected those whose p-value was lower than 0.1 in order to reduce the dimensionality of the predicted variants and not bias the model with non-significant data for further analyses.

An autoencoder approach is made up of encoders and decoders steps. Encoders work by encoding the data in a smaller dimension providing a more informative view of the data, while decoders aim to reconstruct the original data from this encoded data without loss of information.

Results

The rMATS was the method that identified the most significant splicing variants (Table 1), which confirms the choice of the method showing greater precision and robustness among the others analyzed. Exon Skipping was the most common type of variant identified in mammary tissues, being one of the most representative among the methods. While, according to the literature, Intron Retention is one of the most relevant in breast cancer, which reinforced the choice of rMATS, since it was the method with more occurrences. A large number of genes were observed with simultaneous occurrences of Exon Skipping and Multi Exon Skipping, the latter of which may be associated with cancer, requiring further studies. The model was implemented in python using the widely used torch library (1.8.1). We trained the model separately for 4 groups of data: Expression of splicing variants in cancer tissues

and normal tissues, copy number variation (CNV) in cancer tissues and normal tissues. The model consists of learning a single characteristic for each splicing variant or CNV, using as error the inconsistency with the PPI and the mean squared error with the reconstruction obtained using a decoder architecture. At the end of the training, the representation that has a characteristic for each splicing variant or CNV is stored for further analysis. Approaches to determine the best credibility score for identifying responsive splicing variants, based on these described characteristics, are being evaluated. After calculating the score, it is compared between the conditions of interest and the control, in order to identify which are the most important splicing variants.

Table 1: Summary of splicing types (ES: Exon Skip, A5SS: 5' Alternative, A3SS: 3' Alternative, MES: Multi Exon Skip, IR: Intron Retention) identified by rMATS.

Event	Total	Significant events
ES	92078	2197
A5SS	6296	338
A3SS	9716	439
MES	13767	2064
IR	6024	874

Conclusions

The rMATS was the most robust splicing predictor among the evaluated predictors, identifying a greater number of splicing variants with greater expression variation. The model under development will allow us to establish a way to obtain a single characteristic for each splicing variant, being possible to guide this characteristic through regularizations as performed with the mean squared error and the protein-protein interaction network. The preliminary model identified genes confirmed in the literature such as CD44, showing that the model is promising. The next steps include creating an automatic mechanism or criterion to validate the results of the identified splicing variants.

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Inscrição No.: 3011

Título em Português:	Estudo dos eventos de splicing alternativo e seu papel como mecanismo complementar em carrapatos
Título em Inglês:	alternative splicing events study and their role as a complementary mechanism in ticks
Área de Pesquisa:	Biotecnologia
Palavras Chave:	Splicing Alternativo - Carrapatos - Sistema Imune
Ag. Financiadora do Projeto:	CNPq - PIBITI
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	
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Resumo do Trabalho em inglês:

Alternative splicing events study and their role as a complementary mechanism in ticks

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Objectives

Study the different types of Alternative Splicing (AS) in ticks, to identify their associated role in the organism's immune system in response to the pathogen stress. Simultaneously we evaluate the best approaches to process this RNA sequence (RNA-Seq) data for AS in non model organisms. With those results we expect to develop a bioinformatics methodology for that evaluation, with the purpose to infer the AS biological functions.

Materials and Methods

We utilized the public RNA-seq datasets available at the National Center for Biotechnology (NCBI) of the tick *Ixode scapularis* species, with four biological replicates of a control sample and infected by lime disease.
In the pre-processing stage for quality control we utilized well established tools for this function, such as Fastx Toolkit and FastQC.
For the sequence map in the genome reference we employed the standard aligner HISAT2 and SAMtools tools for post-processing.
For the AS event prediction we compare two main programs, rMATS and SplAdder.
For the construction of the model that will infer the AS biological function given the analysis, for tick disease study, we use modern high level languages and respective libraries, like Python and/or R.

Results

There were used 8 RNA-seq samples (4 control and 4 treatments) from NCBI. The samples werer aligned with HISAT2 against the genome reference and we got an overall alignment rate with mean 53.52%.
Analyzing the given genome we counted 27,799 different genes, 292,816 different transcripts and 317,086 exons. Summarized in a mean of 10.54 transcripts per gene, and a mean of 1.08 exons per transcript.
Considering the results of the rMATS AS event prediction we got 171 exon skipping events, 21 mutex skipping, 133 alternative 3' and 122 alternative 5' events and 129 intron retention events. The predictions with SplAdder are under analysis.
Analyzing the predicted alternative splicing vents with rMATS one of the main product of the exons that were predicted to be involved in the AS were the serine and kinase genes, both molecules involved in the immune system of the tick, providing very promising results.

Conclusions

The results showing that serine and kinase functions are one of the main products of the AS events are appropriate with recent literature, since according to them one of the main pathways for the tick's immune system is the Janus kinase/signal transducer and activator of transcripts (JAK/STAT) for bacterial infections control.

Other important pathways appointed are the immune deficiency (IMD) also for bacterial, infection control and Toll and RNA interference (RNAi) for defense against viral infection.
It's notable that the alignment rate is small when compared to alignments made with model organisms. This revels a project characteristic that is to work with a smaller amount of data and an incomplete reference genome. Thus, we hope that with this project we also could improve the annotation of the reference genome.
Also, this leads us to develop models that are capable of dealing well with this reduced amount of information. Favoring models with a bayesian or classic statistical approaches, preliminary to more data-hungry models, like deep learning.

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Inscrição No.: 3038

Título em Português:	UM ESTUDO SOBRE APLICAÇÕES DE ÁLGEBRA LINEAR
Título em Inglês:	a study on linear algebra applications
Área de Pesquisa:	Matemática Aplicada
Palavras Chave:	Álgebra Linear - Matrizes - Aplicações
Ag. Financiadora do Projeto:	Outros
Projeto:	Iniciação Científica
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Resumo do Trabalho em inglês:

A STUDY ON LINEAR ALGEBRA APPLICATIONS

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Objectives

It is known that mathematics, since ancient times, is an extremely important tool for the development of technologies and solutions that help with problems and needs in the most different areas. Based on this principle, this study aimed to research, reflect and understand a little about the practical applicability of this science in people's daily lives, with a special focus on the field of linear algebra.

The applications of linear algebra studied in the period of this study were: Constructing Curves and Surfaces by Specified Points, Geometric Linear Programming, The oldest applications of Linear Algebra, Markov Chains, Strategy Games, Leontief Economic Models, Graph Theory and Administration of Forests.

In all the cases described above, the book [1] presented a mathematical modeling for the problem, involving matrices. More specifically, the following mathematical concepts were used: Linear Systems, Determinants, Analytical Geometry, Inequalities, Matrices, Matrix Operations, Limits, Probability Concepts and Matrix Algebra. In this way, the examples and theorems presented in the book for the solution of the proposed problems were studied.

Just as an example, and because of space limitations, we will describe in this summary the process of solving a problem in forest management.

Materials and Methods

The methodology used in this project was the usual one in mathematics. The scholarship student presented seminars to the group formed by the supervisor and other students of the Degree in Mathematics. The subjects were then discussed in the meetings among the participants. The seminars took place weekly, online, lasting around 1h - 1h30 minutes, and dealt with topics suggested by the supervisor, within the scope of the project. In addition to the expository presentations on the contents, exercises suggested by the bibliography were also developed.

Results

As noted, we will present in this summary, because of space limitations, how the mathematical modeling of the forest management problem takes place.
The application of linear algebra in forest management aims to find a matrix model that provides the best financial yield for the periodic felling of trees that are grouped into classes according to their height (different heights have different economic values).
Two very important definitions for this forest management policy are the definition of sustainable logging - which refers to the practice of cutting trees of varying sizes so that the rest have the same size configuration as the original forest - and the definition of optimal sustainable yield - which is precisely to find the

best sustainable cutting procedure so that the total economic value of all the trees removed is as high as possible. These two concepts together determine the greatest gain that can be obtained without decimating the forest.

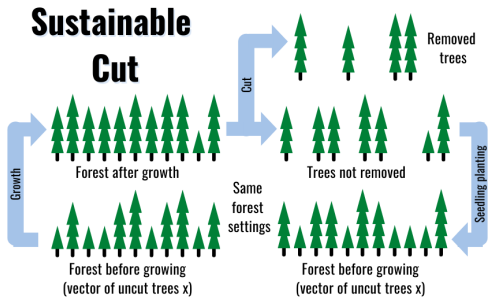

Figure 1: Sketch of a sustainable cut, based on [1]

Mathematically, equation (1) that characterizes a sustainable cutting condition is given by:

$$GX - Y + RY = X \qquad (1)$$

Where G is an $n\ x\ n$ growth matrix, GX in this equation will show the numbers of trees in the n classes after the growth period. Y is the column vector called the cut trees vector, where y_i $(i = 1, 2,..., n)$ is the number of trees of the i-th class that are removed. R is a replacement matrix $n\,x\,n$ and RY will be the configuration of trees planted after each cut. Finally, X is the column vector called the uncut vector, where x_i $(i = 1, 2,..., n)$ is the number of trees in the i-th class that survive the cuts.

After some algebraic manipulations, which are not convenient to show here, it was possible to arrive at two theorems that maximize the sustainable yield of a forest:

Theorem 1: Optimal sustainable yield is obtained by cutting down all trees of a specific height class and no trees of any other class.

Theorem 2: The optimal sustainable yield is the highest value found in equation (2)

$$\frac{p_q s}{\frac{1}{g_1}+\frac{1}{g_2}+...+\frac{1}{g_k-1}} \qquad (2)$$

with $k = 2, 3, ..., n$. The corresponding value of k is the class number that is completely cut off.

Conclusions

During the project, it was possible to learn several mathematical modeling techniques involving matrices and linear algebra to solve concrete and crucial problems for society, such as: vaccine distribution, traffic lights, forest management, economic models, among others. It was surprising to realize that Linear Algebra theory was decisive for several proposals for solving such problems. After having attended the Linear Algebra course at undergraduation, which has a more theoretical focus, this project provided a significant complement to the scholarship's education.

In addition, because the scholarship holder is a student of the Degree in Mathematics for future teachers, the topics studied in the project provided her with a greater repertoire of applications of mathematics in real life, which will be of great use in her future professional performance.

Furthermore, the student thanks CNPq for the opportunity to be a PICME scholarship holder.

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Inscrição No.: 3051

Título em Português:	O grau topológico e suas aplicações na economia
Título em Inglês:	topological degree and its applications in economy
Área de Pesquisa:	Matemática Aplicada
Palavras Chave:	Grau topológico - Equilíbrio geral - Economia
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29º Simpósio Internacional de Iniciação Científica

Inscrição No.: 3051

Resumo do Trabalho em inglês:

TOPOLOGICAL DEGREE AND ITS APPLICATIONS IN ECONOMY

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Objectives

The objective of this undergraduate research project, financed with the PUB grant nº 2297, was the study of the topological degree theory and its applications in the economic theory of general equilibrium. Thus, it was possible to prove the existence of economic equilibrium states in the case of complete markets.

The general equilibrium theory was the basis of a recent article [1], published in the Journal of Mathematical Economics, about the impacts caused on the economy by the COVID-19 pandemic. An important part of our work was dedicated to the study of the article mentioned above, which analyzes the positive and negative consequences on health and economic recovery caused by different speeds of reopening activities after a lockdown.

Materials and Methods

The subjects were studied through the bibliography indicated by the professor and with complementary materials researched by the student. Weekly meetings were held in a seminar format so that, at each meeting, the student reported the progress made, the results studied and demonstrated and the exercises found in the texts, presenting possible doubts and comments. The meetings allowed for a productive deepening of each subject, offering the opportunity to explore connections between the project's arguments and other mathematical and economic topics.

Results

The first part of the work was dedicated to the construction of the topological degree for functions between Euclidean spaces, called the *Brouwer degree*.

Let $f\colon R^n \to R^n$ be a continuous function and consider an equation of type

$$f(x) = y, \qquad (1)$$

where y is fixed. Given an open set $U \subseteq R^n$, the triple (f, U, y) is said *admissible* if f is proper on $\overline{U}$ and $y \in R^n \backslash f(\partial U)$. The Brouwer degree is an integer associated with the triple (f, U, y), answering questions about the existence of solutions to equation (1), as well as the quantity and localization of existing solutions. The degree's effectiveness is given by its properties, of which we will mention three:

Existence: Let (f, U, y) be an admissible triple and suppose that

$$\deg(f, U, y) \neq 0, \qquad (2)$$

then the equation (1) admits at least one solution in U.

Additivity: Let (f, U, y) be an admissible triple. If $U_1, U_2 \subseteq R^n$ are open and disjoint sets such that $(f^{-1}(y) \cap U) \subseteq (U_1 \cup U_2)$, then

$$\deg(f, U, y) = \deg(f, U_1, y) + \deg(f, U_2, y). \quad (3)$$

Homotopic invariance: Let $H: \overline{U} \times [0,1] \to R^n$ be continuous, with $H(x,\lambda) \neq y$, $\forall (x,\lambda) \in \partial U \times [0,1]$. Then,

$$\deg(H(\cdot,0), U, y) = \deg(H(\cdot,1), U, y). \quad (4)$$

The second part of the project was dedicated to the study of the economic theory of general equilibrium, whose foundations consist in mathematically modeled concepts in order to enable a quantitative study of the economy of a society. The model used in the book [3] is based on individuals/households who have wishes and desires for goods and services. The set of individuals in an economy is finite, as are the types of goods. A household has a preference for certain products and services, being able to choose a vector A of goods over another vector B, which leads us to define a *preference relation*. In this way, each household has their *consumption vector* based on their preferences, mathematically represented by a *utility function*.

For a consumption vector to satisfy the equilibrium of a given economy, it must maximize the utility function and, together with prices, satisfy the market clearing condition. In the project, it was studied the demonstration of the existence of equilibrium by using the topological degree in its version for functions between differentiable manifolds.

The last part of the work was dedicated to the application of the general equilibrium theory to an economic problem linked to the COVID-19 pandemic. It was approached an article [1] that analyzes the macroeconomic impact of the reopening speed of activities after a lockdown. The authors modeled economic variables such as productivity, final production, consumption and wealth and income inequalities. Furthermore, evolutions of the fraction of contaminated and immune individuals were formulated, generating a system of nonlinear differential equations. The solution of this system resulted in two equilibrium states, depending on the speed θ of reopening: for sufficiently small θ values, households do not experience the virus; higher values of θ, but not very high, have an impact on the economy and cause an increase in contagion, but allowing an equilibrium state to be reached. In case institutions decide for a very high speed of reopening, the number of infections diverges to $+\infty$ and, consequently, the economic damages are high. The authors emphasize that in the latter case the economy suffers permanent losses.

The article presents numerical simulations in order to analyze the evolution of the variables mentioned above for three different reopening speeds. The results showed that the best scenario is obtained for intermediate reopening speeds. For low speeds, the damage to the health of households is the smallest observed, however, the level of productivity is lower and the levels of inequality are high. It is noteworthy that the article was published in February 2021 and, therefore, cannot consider future events such as the effects of vaccination and new waves of contagion.

Conclusions

This work allowed the student to know the topological degree theory, an advanced tool for nonlinear analysis, and its applications to economic problems. In addition, issues such as topology, functional analysis and differentiable manifolds were addressed. The student developed autonomy in researching and reading scientific articles, as well as in understanding more elaborate mathematical demonstrations. The project will continue with the PUB grant nº 2501, focusing on the topological degree in infinite dimension.

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Inscrição No.: 3064

Título em Português:	a matemática como ferramenta para entendimento da pandemia
Título em Inglês:	mathematics as a tool for understanding the pandemic
Área de Pesquisa:	Matemática Aplicada
Palavras Chave:	pandemia - conceitos - progressão
Ag. Financiadora do Projeto:	CNPq - PIBIC
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Inscrição No.: 3064

Resumo do Trabalho em inglês:

MATHEMATICS AS A TOOL FOR UNDERSTANDING THE PANDEMIC

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Objectives

Translate mathematical epidemiological concepts to a language closer to high school students, using Geometric Progression (PG).

Materials and Methods

PG is a sequence of numbers Where each term, starting from the second, is obtained by multiplying the previous term by a fixed value, called ratio. Algebraically described by law:

$$a_n = a_1 \cdot q^{n-1}$$

Onde: a_n corresponds to the general term;

a_1 corresponds to the first term;

q corresponds to the reason;

n corresponds to the number of terms.

In a pandemic the spread of the disease follows a PG. In epidemiology, the reason for this PG is called the reproduction ratio (R_0), which refers to the number of individuals that, on average, can be infected from a single transmitter.

Another important term of epidemiology is the effective reprodução ratio (R) which takes into account that part of the population may not be susceptible to the disease. Algebraically it can be described:

$$R = R_0 \cdot S$$

Wave: R represents reproduction ratio;

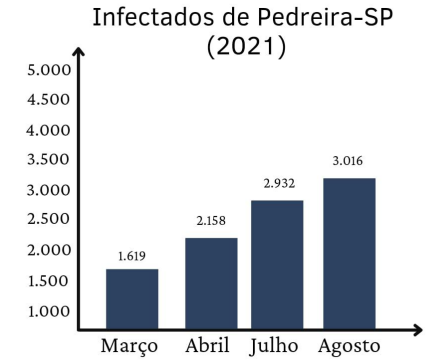
R_0 corresponds to reproduction ratio;

S corresponds to the susceptible population rate.

Results

Despite proving to be a useful tool to bring epidemiological concepts closer to the reality of high school students, PG alone is incapable of describing the evolution of a pandemic. The graph below shows the evolution of confirmed cases of Covid- 19 in the municipality of

Pedreira-SP over a few months of 2021.



Fonte: Boletins - Prefeitura Municipal de Pedreira

Note that the number of infected does not follow a PG, as the growth rate of the columns in the graph is not constant. This happens because the susceptible population tends to decrease over time, so the use of the basic reproduction ratio, which is a fixed value and varies as the susceptible population decreases.

Conclusions

Through the estimates made from simple calculations, it was noted the accelerated growth in the number of cases, where it was necessary to use preventive measures, together with mass vaccination, which after months of pandemic proved to be effective, having in view of the current decline of infection.

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Área de Pesquisa:	Álgebra
Palavras Chave:	Teoria dos Números - Álgebra Abstrata - Equações diofantinas
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Resumo do Trabalho em inglês:

Algebraic Number Theory

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Objectives

Number Theory is the branch of mathematics that studies the integers. One of its principal goals is the resolution of Diophantine equations. A Diophantine equation is simply an equation like any other, but we are only interested in its integer solutions. Number Theory stands out for presenting problems with extremely simple and concise statements that even a non-mathematician could understand, but which, in order to be solved, make use of advanced techniques from the most diverse areas of mathematics: abstract algebra, analysis, algebraic geometry, etc.

A great example of this is the famous Fermat's Last Theorem, whose statement is simply:

"Let n be an integer greater than 2. Then the equation

$$x^n + y^n = z^n \qquad (1)$$

has no solutions with x, y, z positive integers."

This problem, of simple statement, was proposed by Pierre de Fermat around 1637, and was solved only in 1994, by Andrew Wiles, making use of advanced results on elliptic curves that had been developed recently. Thus, it took a development of more than three hundred years in mathematics to solve a problem of extremely simple statement.

No wonder Number Theory is referred to by many as "The Queen of Mathematics", due to its central role in pure mathematics and its problems that stimulate the advancement of the most diverse branches of mathematics.

This work aims to study the bases of Algebraic Number Theory, the area of Number Theory that uses tools mainly from abstract algebra. Its main objective is to understand the so-called algebraic integer rings. These are objects that behave similarly to the set of integers, and a better understanding of their structures is fundamental in the study of Diophantine equations.

Materials and Methods

Besides the individual study of the indicated bibliography, especially the books [1] and [2], weekly meetings were held with the presence of the supervisors and other three students, where the author exposed what he learned and his conclusions about the results discussed.

Results

The set of integers is characterized by having several "good" properties: it is endowed with a sum and a multiplication that satisfy properties such as associativity, commutativity, distributivity, etc. We name this kind of structure a ring. Furthermore, the Fundamental Theorem of Arithmetic states that every integer can be uniquely written as the product of prime numbers. This makes the set of integers what we

call a Unique Factorization Domain (UFD). This allows us to solve several Diophantine equations through factorization.

Let us consider for example the Diophantine equation

$$x^2 - y^2 = 5 \quad (2)$$

We can factor the left-hand side of this equation using the “difference of two squares” identity, to get:

$$(x + y)(x - y) = 5 \quad (3)$$

As *5* is a prime number, using the Fundamental Theorem of Arithmetic we see that the only ways to write *5* as the product of two integers are:

$$5 = 5 \cdot 1 = 1 \cdot 5 = (-5) \cdot (-1) = (-1) \cdot (-5) \quad (4)$$

Hence our problem is reduced to solve four systems of two equation, from which we conclude that the only solutions of the Diophantine equation (2) are:

$$(x, y) = (3,2), (3, -2), (-3, -2), (-3,2). \quad (5)$$

Using the same procedure as above, we can find the solutions of a Diophantine equation of the form:

$$x^2 - y^2 = n, \quad (6)$$

where n is a fixed positive integer. A far more complicated problem is solving an equation of the form

$$x^2 + y^2 = n, \quad (7)$$

where n is a fixed positive integer. The difficulty that arises is that we cannot factor the left-hand side as the product of two integers, as we did before. Even so, we can “save” the idea of solving (2) with the factorization:

$$x^2 + y^2 = (x + iy)(x - iy), \quad (8)$$

where i is the square root of -1. The problem of this factorization is that the factors are not integers anymore. In fact, this is a factorization in a ring so-called ring of Gaussian integers. By studying this ring, we conclude that it has the same good properties as the set of integers, and from that we can solve the Diophantine equation (7) with the same procedure used in (2). Unfortunately, several algebraic integer rings do not possess these good properties. For example, the identity

$$6 = 2 \cdot 3 = (1 + \sqrt{-5})(1 - \sqrt{-5}) \quad (9)$$

tells us that the algebraic integer ring $\mathbb{Z}[\sqrt{-5}]$ does not possess the unique factorization property. In this work, several results were demonstrated that allow us to better understand algebraic integer rings, and consequently have more machinery to solve Diophantine equations. The main results demonstrated in this work were the Theorem on the Finiteness of the Class Number, which goes in the direction of trying to save the much desired unique factorization, and the Dirichlet’s Unit Theorem, which gives us information about the invertible elements of a ring of algebraic integers, that play a fundamental role in the whole theory.

Conclusions

This project served as an excelent introduction to the extensive branch of Algebraic Number Theory, by studying several important aspects of the structure of the rings of algebraic integers, such as their ideals and their groups of units, and also served to show how these ideas apply in practice, to solve diophantine equations and even problems in other areas, such as Wedderburn's Little Theorem.

Furthermore, this work motivated the continuation of studies in this area of research, laying the groundwork to the study of Valuation Theory and Class Field Theory.

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Inscrição No.: 3272

Título em Português:	Bases de Gröbner para Ideais Polinomiais
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Resumo do Trabalho em inglês:

Gröbner Bases for Polynomial Ideals

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Objectives

In Algebraic Geometry, the correspondence between varieties (Geometry) and polynomial ideals (Algebra) relates Geometry and Algebra. This project focused on the study of finite generating sets of with "good properties" of these ideals.
As an example, these properties should allow one to define in a simple way if a polinomial lies in an ideal. In this case, for polynomials in one variable, the Euclidean algorithm and the division algorithm are capable of solving the problem. But in the case of polynomials in more than one variable, these methods aren't viable: there isn't some algorithm like the first and an adaptation of the latter is possible, but it isn't enough to determine the membership. That's why "special generators" are required in these cases.

Materials and Methods

Considering the ring of polynomials in several variables and coefficients in a field, a subset of polynomials is an ideal of this ring (polynomial ideal) if it has the following properties: it's a nonempty set, it's closed under sum and multiples of its elements also lie in this subset, where a multiple of a polynomial f is the multiplication of f by any polinomial.
Besides that, a collection of polynomials is a generating set if any element of the ideal can be written as the sum of multiples of these polynomials. Analogously, it's possible to make the inverse procedure, creating an ideal from an arbitrary generating set.

For polynomials in one variable the ideals are principal, in which one polynomial is able to generate the entire ideal. The Euclidean Algorithm is enough to find this polynomial.
Also, any element of the ideal is a multiple of this generating polynomial; therefore, by the division algorithm, one can find if a generic polynomial is divisible by the generator.
In this case, if the remainder is zero, the polynomial lies in the ideal. If not, it doesn't, allowing us to conclude that the analysis of the remainder is enough to determine if a polynomial lies in an ideal.
In order to clarify the problem with more variables, let's take as example the polynomials g, h and f in variables x and y and real coefficients such that

$$g = xy - 1,\ h = y^2 - 1,\ f = xy^2 - x, \quad (1)$$

and let's take the ideal I generated by g and h. In this case it isn't possible to find just one generating polynomial. But, considering the ideal above, any element of I can be written as:

$$\varphi \cdot g + \psi \cdot h, \quad (2)$$

where φ and ψ are arbitrary polynomials. Thus if f lies in the ideal, it's possible to write it as the sum of multiples of g and h.
That's said the strategy applied to find if f lies in I consisted of the division of f by the generators. But then, there's a problem. If the division is made primarly by g and then h. The result is:

$$f = xy^2 - x = y \cdot g + 0 \cdot h + (-x + y). \quad (3)$$

In which the remainder is $-x+y$, but if f lied in the ideal, it would be written as shown in equation (2), in other words, the remainder would be zero. Following the logic of polynomials in one variable, one could conclude f doesn't lie in I.
But, if changed the division order, the result is:

$$f = xy^2 - x = 0 \cdot g + x \cdot h + 0. \quad (4)$$

As shown in (4), in reality, f lies in the ideal since it can be written as presented in (2).
Therefore, by changing the order of division, the remainder is changed and, with luck, this can be a way of determining whether or not a polynomial lies in an ideal. However, as seen in (3), it's proved that the division algorithm isn't enough to determine if a generic polynomial lies in an ideal.
Considering that, this project focused on finding a set of generators in which the order of the division didn't matter to determine if a polynomial lies in an ideal. Hence, the analysis of the remainder is enough to solve the membership problem.

Results

To begin, it was necessary to determine whether every polynomial ideal has a finite generating set. The study of monomial ideals enabled the demonstration for the monomial case (Dickson's Lemma). From that, it was possible to prove the Hilbert's Basis Theorem that ensures every polynomial ideal is finitely generated.
After that, it was defined the needed conditions in order for the generating set to display "good properties". The Gröbner bases were defined from the proof of Hilbert Basis Theorem. These bases are finite sets of generators of an ideal in which the problem described above doesn't happen. The remainder of the division of a polynomial by this set doesn't depend on the order chosen and this remainder will be zero if, and only if, the polynomial belongs to the ideal. Then, the set finally gets the "good properties" needed to use the division algorithm to determine if a polynomial lies in an ideal just by analyzing the remainder of the division of this polynomial by a Gröbner base of this ideal.
Finally, it was necessary to create an algorithm to find a Gröbner base for any ideal. The Buchberger Criterion, which enables one to identify if a generating set is a Gröbner base, was used to develop an algorithm (Buchberger's Algorithm) to find the set with "good properties".

Conclusions

The results show that the Gröbner bases display the properties that solve problems related to the polynomial ideals generated by polynomials with more than one variable. Besides being a tool for concrete problems - such as deciding whether a polynomial lies in an ideal or, if given two sets of polynomials, deciding whether the ideals generated by them are equal - Grobnër bases can be used to give an alternative proof of classical theorems, like Hilbert's Nullstellensatz, which relates algebraic varieties and polynomial ideals.

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Inscrição No.: 3886

Título em Português:	VIOLÊNCIA CONTRA MULHER
Título em Inglês:	VIOLENCE AGAINST WOMEN
Área de Pesquisa:	Matemática Aplicada
Palavras Chave:	Estatística - Violência - Mulher
Ag. Financiadora do Projeto:	CNPq - PIBIC
Projeto:	Pré-iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	
Validado em:	**13/09/2021**

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Resumo do Trabalho em inglês:

VIOLENCE AGAINST WOMEN

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Objectives

In this paper, we aim to present the results of field research carried out with high school students at the Professor Isaltino de Mello State School on violence against women in Brazil and to carry out a comprehensive analysis of the information on the subject, as well as the causes and consequences of the problem, and measures to combat the problem

Through several surveys carried out in governmental and non-governmental organizations, we concluded that violence against women reaches a large and growing number of cases in the country. As a result of this, the work aims to obtain a model that in turn highlights the disastrous results of current Brazilian society. Given these aspects, the work aims to present protective and preventive legislation against this type of violence. And present proposals to solve the social problem and raise awareness among the population.

Methods and Procedures

This work integrates the activities of the Scientific Pre-Initiation Project “Mathematics and models: promoting meaning for function concepts”, developed by a group of students from the Professor Isaltino de Mello State School. Initially, the group made a study of types of mathematical modeling, reviewing some specific contents. For the final work, the group chose a theme for research, data collection and analysis. The work methodology was carried out through numerous surveys carried out in a descriptive and exploratory manner. The research itself consists of a deepening of real data, mainly with a focus on bibliographic research, developed from results and facts dated on safe and reliable websites, as well as the field research carried out with the students of that school, by through a form made available by the WhatsApp groups, with the aim of raising awareness among readers.

Results

From the data collected through the questionnaire, we concluded that most voters suffered or witnessed some type of violence against women (psychological, physical, moral, patrimonial and sexual). Results attached in graphics:

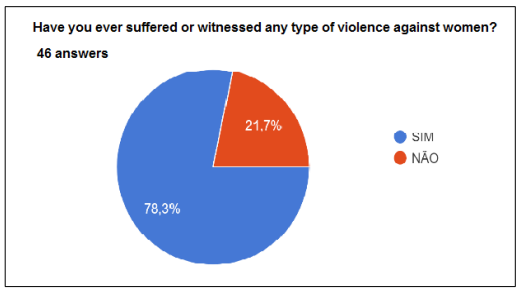

Figure 1: Case frequency graph.

Conclusions

Through the researches and the data obtained, we concluded that violence against women is present nowadays, this as a result of a patriarchal and sexist culture structurally installed in society. An increase in the indices of recent months was also evidenced, as a result of the current scenario in which Brazil is. From the readings and surveys carried out during the research, the normalization of sexist attitudes and acts became noticeable. Aggression entails numerous consequences for women, affecting their physical and mental health, as well as for society, affecting, for example, the victim's personal and professional life. Clearly, femicide cases are the most tragic consequences of this type of violence.

Due to the aspects observed, it was concluded that the feminist movement, NGOs, laws that protect women, such as the Maria da Penha Law, websites and applications that help victims are of paramount importance to minimize or even solve the problem .

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Inscrição No.: 3893

Título em Português:	PARTICIPAÇÃO E APROVEITAMENTO DAS AULAS ONLINE DURANTE A PANDEMIA COVID-19
Título em Inglês:	PARTICIPATION AND USE OF ONLINE CLASSES DURING THE COVID-19 PANDEMIC
Área de Pesquisa:	Matemática Aplicada
Palavras Chave:	Estatística - Ensino - Pandemia
Ag. Financiadora do Projeto:	CNPq - PIBIC
Projeto:	Pré-iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	
Validado em:	**13/09/2021**

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Resumo do Trabalho em inglês:

PARTICIPATION AND USE OF ONLINE CLASSES DURING THE COVID-19 PANDEMIC

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Objectives

The general objective is a study on the impacts of the pandemic, in relation to the learning development of high school students, and the process of emergency adaptation to remote education.
With this, contextualize the teaching that was provided, understand the frequency of students in online classes, analyze the factors that cause such results, so that there is a conclusion that actually shows how the participation and, especially, the achievement of these students during the pandemic was of COVID-19, in addition to theorizing possible methodologies of greater efficiency.
Above all, reflect on the consequences and conclude the best student learning, and the professionals' work, in relation to the classes.

Methods and Procedures

This work integrates the activities of the Scientific Pre-Initiation Project “Mathematics and models: promoting meaning for function concepts”, developed by students from the Instituto Técnico de Barueri Brasílio Flores de Azevedo. Initially, the group made a study of types of mathematical modeling, reviewing some specific contents. For the final work, the group chose a theme for research, data collection and analysis. Initially, research was carried out on information, texts and documents for theoretical foundations and greater support. Afterwards, a quantitative and qualitative popular survey was carried out in a technical high school to obtain data from a small sample. With these data, a separation of information was carried out and an analysis was carried out on the results obtained. The main research steps used are:

- Descriptive and basic research;
- Obtaining sample data through a quantitative-qualitative survey;
- Presentation of data and clarification of the same data;
- Study deepening.

Results

From the data obtained, the frequency of students in online classes is very low, with 20% to 25% of the total number of students in the analyzed classes, in a given week. Several factors for this low participation were presented (Figure 1), such as shaken mental/physical health, limited internet, difficulty in adapting, among other reasons that are also relevant.
About learning in online classes, students reveal low achievement, as shown in the graph (Figure 2). It's possible The problems and difficulties reported by students to participate in online classes, the teaching/learning methods applied can result in low achievement.

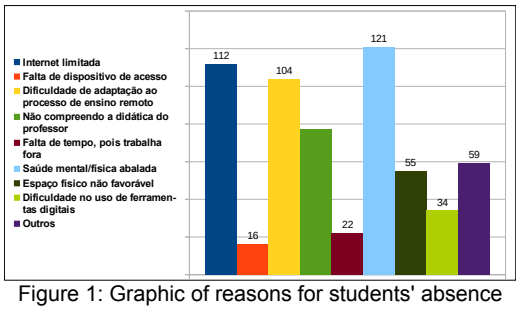

Figure 1: Graphic of reasons for students' absence from online classes

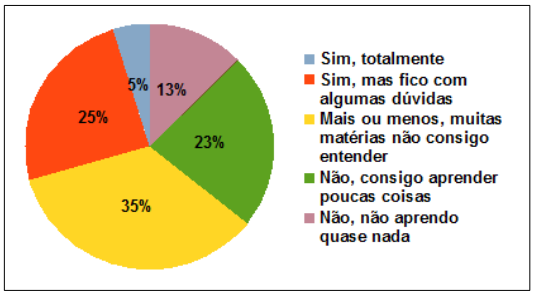

Figure 2: Graph on student opinion about learning in online classes

Conclusions

The Covid-19 pandemic brought disorganization and disorientation in the area of education in Brazil. Emergency remote teaching made even more visible the lack of preparation of teachers in relation to technological development increasingly applied to compulsory education and the excessive social inequality that Brazil has.

Distance learning (EaD), despite having been in existence for some years in the country, cannot be compared with emergency remote learning (ERE). The latter did not have teachers trained in specific technological and pedagogical knowledge, nor a basic functional structure, nor an adaptation of the teaching methodology applied for better student learning. In addition, quality distance education is limited and difficult for millions of students, given the data on internet access for students from public schools (IBGE).

The data collected in the survey reveal several factors that influence the low attendance of students in online classes, or that justify the reports of low achievement and learning, whether they are personal, technical and/or pedagogical.

The research carried out with students from an integrated high school in the city of Barueri pointed out all the causes mentioned above about the lack of student participation in classes and how this can affect a good continuous learning of students. With other research carried out used as a basis, the results are emblematic and reveal the current situation of education during the pandemic.

With so many difficulties in accessing and adapting, it is necessary that attitudes, in relation to the damage caused, are provided by institutions, in addition to technological preparation and psychological guidance for students and education professionals.

References

Several articles published on remote learning during the pandemic, available on the internet, were used. Data from surveys published by the IBGE (Brazilian Institute of Geography and Statistics) and UNICEF (United Nations Children's Fund), among others, were also used.

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Inscrição No.: 4041

Título em Português:	TEORIA DOS JOGOS APLICADA A UM JOGO DE BEISEBOL
Título em Inglês:	game theory applied in a baseball game
Área de Pesquisa:	Matemática Aplicada
Palavras Chave:	Teoria dos jogos - Conflito total - Modelagem
Ag. Financiadora do Projeto:	Outros
Projeto:	Iniciação Científica
Unidade de Apresentação:	Instituto de Matemática e Estatística
Departamento:	
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Unidade

USP - UNIVERSIDADE DE SÃO PAULO
29º Simpósio Internacional de Iniciação Científica

Inscrição No.: 4041

Resumo do Trabalho em inglês:

GAME THEORY APPLIED IN A BASEBALL GAME

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Objectives

Mathematical modelling helps in interpreting and solving a variety of problems. This research aims to understand and explore the use of game theory in an application. Specifically, a two players game involving a pitcher and a batter in a baseball game is modelled. It is assumed an ideal situation in which the players decisions are simultaneously taken and solely based on rationality. Detailed attention is paid to some possible scenarios where the best choices for each part are established. Also, an authorial contribution is given for a throw with three variations.

Methods and Procedures

The first step consists in determine the kind of conflict involved. In game theory, the conflict between two players can be characterized as a partial conflict or a total conflict, the difference being that in the first case one player can increase his payoff without disadvantage for the other one, whereas in the second case the gain of one player equals the loss of the other one.
The baseball duel fits in a total conflict case, and therefore the maximization and minimization of each player's payoff is relevant for the determination of the best choices for each one. For this, tools from linear programming were essential in the analysis of mixed strategies, where
both algebraic and geometric were employed.

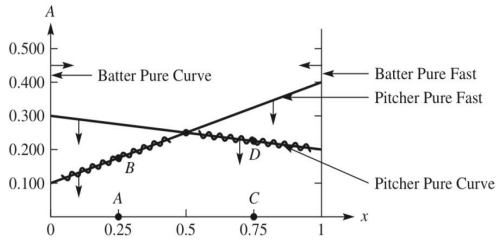

Figure 1 : Geometric representation for one of the analyses made for the batter
Reference : "A first mathematical modeling course "

Moreover, minmax and maxmin strategies were also used for pure strategies, giving different but interesting results.

Results

Initially, it was assumed that only fastballs or curves are throwed, and the average performance for each throw were given for the pitcher as well as for the batter. It was possible then to determine the mixed strategy which maximizes the performance of each player. Interesting enough, they are different for each player. As an extension of the problem presented in the bibliography, we created a third kind of throw (two stitches) and made a scheme to analyze the movement diagram and the possibility for the existence of a Nash equilibrium. The average performances of each player for this third throw were not chosen at

random but specified in the context of a possible real baseball game. In the future, real values
from the USA baseball database will be used to analyze a real situation.

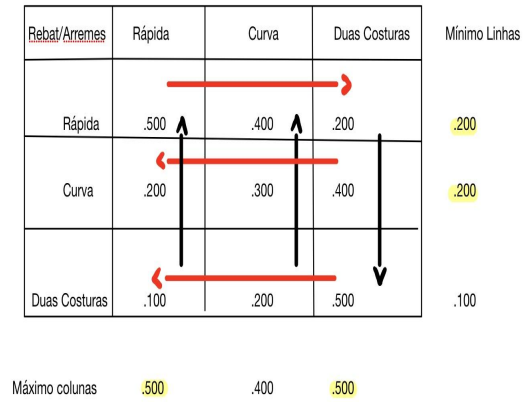

Figure 2: Movement diagram representation and a strategy that gives the maximum and minimum point in a pure strategy
Reference : Author

Conclusions

The following mixed strategies were derived for the two throws duel. For the batter, 50% of
the time fastballs and 50% of the time curves, whereas for the pitcher 75% fastballs and 25% curves. In the case of the three throws, the performance of the batter rested between 20 and 30 percent, according to the data. These values were obtained assuming that each player will rationally choose a mixed strategy in the two throws duel and a pure strategy in the three throws game. Moreover, no communication, mediation, or arbitrage are allowed.

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