MAT2456 - Cálculo Diferencial e Integral para Engenharia IV

2º Semestre de 2019 - 2ª Lista de exercícios

1. Determine o intervalo máximo de convergência de cada uma das séries de potências abaixo:

a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{4^n} x^n$ b) $\displaystyle\sum_{n=1}^{\infty} n!\, x^n$ c) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^3+1}$

d) $\displaystyle\sum_{n=1}^{\infty} \frac{(3n)!}{(2n)!} x^n$ e) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-5)^n}{n3^n}$ f) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+1)^n}{(n+1)\ln^2(n+1)}$

g) $\displaystyle\sum_{n=1}^{\infty} \frac{10^n}{(2n)!}(x-7)^n$ h) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{e^n}(x-e)^n$ i) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}(x+3)^n$

j) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{(x-3)^n}{(2n+1)\sqrt{n+1}}$ k) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2}{4^n}(x-4)^{2n}$ l) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^2} x^n$

m) $\displaystyle\sum_{n=1}^{\infty} 2^n x^{n^2}$ n) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n4^n} x^n$

2. Obtenha o raio de convergência para as séries seguintes.

a) $\displaystyle\sum_{n=1}^{\infty} x^n \frac{(2n)!}{(n!)^2}$ b) $\displaystyle\sum_{n=1}^{\infty} x^{2n} \frac{(2n)!}{(n!)^2}$ c) $\displaystyle\sum_{n=1}^{\infty} x^{n^2} \frac{(2n)!}{(n!)^2}$ d) $\displaystyle\sum_{n=1}^{\infty} x^n \frac{n!}{n^n}$ e) $\displaystyle\sum_{n=1}^{\infty} x^{3n} \frac{n!}{n^n}$ f) $\displaystyle\sum_{n=1}^{\infty} x^{n!} \frac{n!}{n^n}$.

3. Determine o intervalo de convergência de:

a) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{(2+(-1)^n)^n}$ b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3n+2}{5n+7}\right)^n x^n$ c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2^n+3}{3^n+2}\right) x^n$ d) $\displaystyle\sum_{n=2}^{\infty} \frac{x^n}{\ln n}$

e) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+1)^n}{a^n+b^n}$, com $b > a > 0$.

4. Usando derivação e integração termo a termo, se necessário, determine as expansões em séries de potências em torno de $x_0 = 0$ das seguintes funções e os valores de x para os quais essas expansões são válidas:

a) $\dfrac{1}{x^2+25}$ *b*) $\operatorname{arctg}(2x)$ c) $\dfrac{1}{(1+x)^2}$ d) $\dfrac{1}{(1+x)^3}$ e) $\dfrac{2x}{1+x^4}$

f) $\ln(1+x)$ g) $\ln\left(\dfrac{1}{1+3x^2}\right)$ h) $\dfrac{x}{1+x-2x^2}$ i) $\displaystyle\int_0^x \frac{t}{1+t^5}dt$

5. Verifique que

a) $\displaystyle\ln\left(\frac{1+x}{1-x}\right) = 2\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1}, \ |x| < 1$ b) $\displaystyle\operatorname{arctg} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}, \ |x| \le 1.$

6. Obter uma expressão das somas das séries abaixo e os respectivos raios de convergência, usando derivação e integração termo a termo, se necessário.

a) $x + \dfrac{x^2}{2} + \dfrac{x^3}{3} + \cdots + \dfrac{x^n}{n} + \cdots$ b) $x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \cdots + (-1)^{n-1}\dfrac{x^n}{n} + \cdots$

c) $x + \dfrac{x^3}{3} + \dfrac{x^5}{5} + \cdots + \dfrac{x^{2n-1}}{2n-1} + \cdots$ d) $x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \cdots . + (-1)^{n-1}\dfrac{x^{2n-1}}{2n-1} + \cdots$

e) $1 + 2x + 3x^2 + \cdots + (n+1)x^n + \cdots$ f) $x + 2x^2 + 3x^3 + \cdots + nx^n + \cdots$

g) $x + 2x^3 + 3x^5 + \cdots + nx^{2n-1} + \cdots$ h) $\dfrac{x^2}{1\cdot 2} - \dfrac{x^3}{2\cdot 3} + \dfrac{x^4}{3\cdot 4} - \dfrac{x^5}{4\cdot 5} + \cdots$

i) $1 + 4x + 9x^2 + 16x^3 + \cdots$ j) $x + 2^3x^2 + 3^3x^3 + 4^3x^4 + \cdots$

k) $4 + 5x + 6x^2 + 7x^3 + \cdots$ l) $\dfrac{x^4}{4} + \dfrac{x^8}{8} + \dfrac{x^{12}}{12} + \cdots$

7. Utilizando as somas das séries obtidas no exercício anterior, calcule:

a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n2^n}$ b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{2^n}$ c) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{5^n(n-1)n}$.

8. Mostre que $\displaystyle\sum_{n=0}^{\infty} \frac{(n+1)(n+2)(n+3)}{6} x^n = \frac{1}{(1-x)^4}$.

9. Utilizando o desenvolvimento em série, obtenha um valor aproximado de

(a) e, com erro inferior a 10^{-5} (b) $\operatorname{sen} 1$, com erro inferior a 10^{-5} e a 10^{-7}
(c) $\ln 2$ e $\ln 3$, com erro inferior a 10^{-5} (d) $\operatorname{arctg}(1/2)$ e $\operatorname{arctg}(1/3)$, com erro inferior a 10^{-5}
(e) $\pi/4$, com erro inferior a 10^{-5}

10. Utilizando série de Taylor calcule $\displaystyle\frac{d^{320}\operatorname{arctg}}{dx^{320}}(0)$ e $\displaystyle\frac{d^{321}\operatorname{arctg}}{dx^{321}}(0)$.

11. Dê a série de Taylor de $f(x)$ centrada no 0, indicando os intervalos de convergência

a) $f(x) = x^2 e^x$ b) $f(x) = \ln(1-x^3)$ c) $f(x) = \operatorname{sen}(x^2)$
d) $f(x) = \cos^2 x$ e) $f(x) = x\cos(2x)$ f) $f(x) = x \operatorname{arctg} x$

12. Desenvolva em série de potências de x as seguintes funções, indicando os intervalos de convergência, e calcule $f(1)$ com erro inferior a 10^{-6}, sendo que as funções dos itens (a) e (c) são definidas em $t = 0$ como valendo 1:

a) $f(x) = \displaystyle\int_0^x \frac{\operatorname{sen} t}{t}\,dt$ b) $f(x) = \displaystyle\int_0^x e^{-t^2}\,dt$ c) $f(x) = \displaystyle\int_0^x \frac{\ln(1+t)}{t}\,dt$ d) $f(x) = \displaystyle\int_0^x \operatorname{sen}(t^2)\,dt$

13. Utilizando a expansão em série de potências das funções envolvidas, calcule os seguintes limites

(a) $\displaystyle\lim_{x\to 0} \frac{\operatorname{sen} x}{x}$ (b) $\displaystyle\lim_{x\to 0} \frac{1-\cos x}{x^2}$ (c) $\displaystyle\lim_{x\to 0} \frac{x-\operatorname{sen} x}{x^3}$

(d) $\displaystyle\lim_{x\to 0^+} \frac{\cos x - \left(1 - \frac{x^2}{2} + \frac{x^4}{4!}\right)}{x^\alpha}$ (e) $\displaystyle\lim_{x\to 0^+} \frac{\operatorname{sen} x - \left(x - \frac{x^3}{3!} + \frac{x^5}{5!}\right)}{x^\alpha}$

14. Ache a série de Fourier de $f(x)$, determine soma da série e faça os gráficos de f e da função soma da série encontrada:

a) $f(x) = \begin{cases} a, & -\pi < x \le 0 \\ b, & 0 < x \le \pi \end{cases}$ b) $f(x) = \begin{cases} ax, & -\pi < x \le 0 \\ bx, & 0 < x \le \pi \end{cases}$
c) $f(x) = |x|,\ -\pi < x \le \pi$ d) $f(x) = e^{ax},\ -\pi < x \le \pi,\ a \ne 0$
e) $f(x) = \operatorname{sen}(ax),\ -\pi < x \le \pi,\ a \notin \mathbb{Z}$ f) $f(x) = ax + b,\ -\pi < x \le \pi$
g) $f(x) = |\cos\ x|,\ -\pi < x \le \pi$

15. Ache a série de Fourier de senos de f, a série de cossenos de f, determine a soma de cada uma delas e faça os gráficos de f e da soma de cada uma das séries encontradas:

a) $f(x) = ax,\ 0 \le x \le \pi$ b) $f(x) = x^2,\ 0 \le x \le \pi$
c) $f(x) = ax + b,\ 0 \le x \le \pi$ d) $f(x) = \operatorname{sen}\ x,\ 0 \le x \le \pi$
e) $f(x) = |\cos\ x|,\ 0 \le x \le \pi$

16. Mostre que

a) $1 = \dfrac{4}{\pi}(\operatorname{sen}\ x + \dfrac{1}{3}\operatorname{sen}\ 3x + \dfrac{1}{5}\operatorname{sen}\ 5x + \cdots),\ 0 < x < \pi$;

b) $\pi - x = 2(\operatorname{sen}\ x + \dfrac{1}{2}\operatorname{sen}\ 2x + \dfrac{1}{3}\operatorname{sen}\ 3x + \cdots),\ 0 < x < \pi$;

c) $x^2 = \dfrac{\pi^2}{3} + 4(-\cos\ x + \dfrac{\cos\ 2x}{2^2} - \dfrac{\cos\ 3x}{3^2} + \cdots),\ -\pi < x < \pi$

d) $x(\pi - x) = \dfrac{\pi^2}{6} - (\cos\ 2x + \dfrac{\cos\ 4x}{2^2} + \dfrac{\cos\ 6x}{3^2} + \cdots),\ 0 < x < \pi$

e) $x(\pi - x) = \dfrac{8}{\pi}(\operatorname{sen}\ x + \dfrac{\operatorname{sen}\ 3x}{3^3} + \dfrac{\operatorname{sen}\ 5x}{5^3} + \cdots),\ 0 < x < \pi$

17. Verifique as seguintes igualdades, usando o exercício anterior

a) $\dfrac{\pi}{4} = 1 - \dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + \cdots + (-1)^n \dfrac{1}{2n+1} + \cdots$ b) $\dfrac{\pi^2}{6} = 1 + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \cdots + \dfrac{1}{n^2} + \cdots$

c) $\dfrac{\pi^3}{32} = \displaystyle\sum_{n=1}^{n} \dfrac{(-1)^{n-1}}{(2n-1)^3}$ d) $\dfrac{3\pi^3}{128}\sqrt{2} = 1 + \dfrac{1}{3^3} - \dfrac{1}{5^3} - \dfrac{1}{7^3} + \dfrac{1}{9^3} + \dfrac{1}{11^3} - \cdots$

e) $\dfrac{3\pi}{4} = \sqrt{2} + 1 + \dfrac{1}{3}\sqrt{2} - \dfrac{1}{5}\sqrt{2} - \dfrac{1}{3} - \dfrac{1}{7}\sqrt{2} + \dfrac{1}{9}\sqrt{2} + \dfrac{1}{5} + \dfrac{1}{11}\sqrt{2} - \dfrac{1}{13}\sqrt{2} - \cdots$

18. Calcule a soma das séries

a) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$ b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$

19. Determine c_1, c_2, c_3 de modo que as integrais abaixo assumam o menor valor possível:

a) $\displaystyle\int_{-\pi}^{\pi} [x - c_1 \operatorname{sen}\, x - c_2 \operatorname{sen}\, 2x - c_3 \operatorname{sen}\, 3x]^2 dx$ b) $\displaystyle\int_{-\pi}^{\pi} [x^2 - c_1]^2 dx$

c) $\displaystyle\int_{-\pi}^{\pi} [|\cos\, x| - c_1 - c_2 \operatorname{sen}\, x - c_3 \cos\, x]^2 dx$

20. Ache a série de Fourier da função $f(x)$ periódica de período 1 e que satisfaz $f(x) = x^2$ se $0 \leq x < 1$. Qual é a soma de série quando $x = 999/2$? E quando $x = 999$?

21. a) Obtenha a série de Fourier da função ímpar $f(x)$, periódica de período 4, e que satisfaça $f(x) = x$ se $0 \leq x \leq 1$ e $f(x) = f(2-x)$ se $1 \leq x < 2$.

b) Encontre $b_1, b_2, b_3, \ldots$ tais que $\displaystyle\sum_{n=1}^{\infty} b_n \operatorname{sen}\left(\frac{(2n-1)\pi x}{2}\right) = \begin{cases} x, & 0 \leq x \leq 1, \\ 2-x, & 1 < x < 2 \end{cases}$.

c) Encontre $c_1, c_2, c_3, \ldots$ tais que $\displaystyle\sum_{n=1}^{\infty} c_n \operatorname{sen}\left(\frac{(2n-1)\pi x}{2}\right) = \begin{cases} 1-x, & 0 \leq x \leq 1, \\ 1+x, & 1 < x < 2 \end{cases}$.

d) Quanto vale a soma de série do item c) para $x = 200$? E quando $x = 201$?

22. Obtenha constantes a e b tais que a série de senos de $f(x) = x^3 + ax$ em $[0, \pi]$ seja da forma

$$b \sum_{n=1}^{\infty} (-1)^n \frac{\operatorname{sen}(nx)}{n^3}$$

23. Usando a fórmula de Parseval prove que

(a) $\dfrac{\pi^4}{90} = \displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4}$ (b) $\dfrac{\pi^6}{945} = \displaystyle\sum_{n=1}^{\infty} \frac{1}{n^6}$

24. Calcule

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n-1)^4}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(4n^2-1)^2}$

25. (a) Dê fórmulas para as constantes a_n, $n \geq 0$, b_n, $n \geq 1$, tais que

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \operatorname{sen} nx) = x^2 e^x, \ -\pi < x < \pi$$

(b) Determine a soma da série para $x = \dfrac{11\pi}{2}$ e para $x = 11\pi$.

26. Encontre constantes $a_0, a_1, \ldots, a_n$ que minimizem a expressão

$$\int_0^{\pi} \left[x - \left(\frac{a_0}{2} + \sum_{k=1}^{n} a_k \cos kx \right) \right]^2 dx.$$

Respostas

1. a) $]-4,4[$; b) $\{0\}$; c) $[-1,1]$; d) $\{0\}$; e) $]2,8]$; f) $[-2,0]$; g) $\mathbb{R}$; h) $]0,2e[$; i) $]-3-e,-3+e[$; j) $[2,4]$; k) $]2,6[$; l) $[-1,1]$; m) $]-1,1[$; n) $[-4/3,4/3[$.

2. a) $R=1/4$; b) $R=1/2$; c) $R=1$; d) $R=e$; e) $R=\sqrt[3]{e}$; f) $R=1$.

3. a) $]-1,1[$; b) $]-5/3,5/3[$; c) $]-3/2,3/2[$; d) $[-1,1[$; e) $]-b-1,b-1[$.

4. (a) $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{5^{2n+2}},\ -5<x<5$
   (b) $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1}}{2n+1} x^{2n+1},\ -1/2 \le x \le 1/2$
   (c) $\sum_{n=0}^{\infty}(-1)^n (n+1)x^n,\ -1<x<1$
   (d) $\sum_{n=0}^{\infty} \frac{(-1)^n (n+2)(n+1)}{2}\ x^n, -1<x<1$
   (e) $2\left(\sum_{n=0}^{\infty}(-1)^n x^{4n+1}\right), -1<x<1$
   (f) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^n}{n}, -1<x\le 1$
   (g) $\sum_{n=1}^{\infty} \frac{(-3)^n}{n}\ x^{2n},\ \frac{-1}{\sqrt{3}} \le x \le \frac{1}{\sqrt{3}}$
   (h) $\sum_{n=1}^{\infty} \left(\frac{1-(-1)^n 2^n}{3}\right) x^n,\ \frac{-1}{2}<x<\frac{1}{2}$
   (i) $\sum_{n=0}^{\infty} \frac{(-1)^n}{5n+2}\ x^{5n+2},\ -1<x\le 1$

6. (a) $-\ln(1-x)$; (b) $\ln(1+x)$; (c) $\ln\sqrt{\dfrac{1+x}{1-x}}$; (d) $\operatorname{arctg} x$; (e) $\dfrac{1}{(1-x)^2}$; (f) $\dfrac{x}{(1-x)^2}$; (g) $\dfrac{x}{(1-x^2)^2}$; (h) $(1+x)\ln(1+x)-x$; (i) $\dfrac{1+x}{(1-x)^3}$; (j) $\dfrac{x+4x^2+x^3}{(1-x)^4}$; (k) $\dfrac{4-3x}{(1-x)^2}$; (l) $\frac{-1}{4}\ln(1-x^4)$.

7. $\ln 2$; 26; $\frac{6}{5}\ln\frac{6}{5}-\frac{1}{5}$.

9.
   (a) $e \approx \sum_{n=0}^{8} \frac{1}{n!} \approx 2.71827$
   (e) $\pi/4 \approx \sum_{n=0}^{49999} \frac{(-1)^n}{2n+1} \approx 0.78539$
   (b) $\operatorname{sen}(1) \approx \sum_{n=0}^{3} \frac{(-1)^n}{(2n+1)!} \approx 0.841468$ e $\operatorname{sen}(1) \approx \sum_{n=0}^{4} \frac{(-1)^n}{(2n+1)!} \approx 0.841471$
   (c) $\ln(2) \approx \sum_{n=0}^{10^5-1} \frac{(-1)^n}{n+1} \approx 0.69314$ e $\ln(3) \approx 2\sum_{n=0}^{6} \dfrac{(1/2)^{2n+1}}{2n+1} \approx 1.098607$
   (d) $\arctan(1/2) \approx \sum_{n=0}^{5} \frac{(-1)^n (1/2)^{2n+1}}{2n+1} \approx 0.463639$ e $\arctan(1/3) \approx \sum_{n=0}^{3} \frac{(-1)^n (1/3)^{2n+1}}{2n+1} \approx 0.321745$

10. 0 e $(320)!$.

11 .
   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{n+2}}{n!},\ x\in\mathbb{R}$
   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{x^{3n}}{n}, -1\le x<1$
   (c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!},\ x\in\mathbb{R}$
   (d) $\displaystyle 1+\frac{1}{2}\sum_{n=1}^{\infty} \frac{(-1)^n 2^{2n} x^{2n}}{(2n)!},\ x\in\mathbb{R}$
   (e) $\displaystyle\sum_{n=0}^{\infty} \frac{(2x)^{2n+1}}{n!},\ x\in\mathbb{R}$
   (f) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{(2n+1)},\ -1\le x\le 1$

12 .
   (a) $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)(2n+1)!},\ x\in\mathbb{R}$
   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)n!},\ x\in\mathbb{R}$
   (c) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1} \frac{x^n}{n^2},\ -1\le x\le 1$
   (d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+3}}{(4n+3)(2n+1)!},\ x\in\mathbb{R}$

13. (a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{6}$ (d) $-\frac{1}{6!}$, se $\alpha=6$; 0 , se $\alpha<6$; $-\infty$, se $\alpha>6$
   (e) $-\frac{1}{7!}$, se $\alpha=7$; 0 , se $\alpha<7$; $-\infty$, se $\alpha>7$

14. (a) $\frac{a+b}{2}+\frac{2}{\pi}(b-a)\sum_{n=1}^{\infty}\frac{\operatorname{sen}((2n-1)x)}{2n-1}$.

soma: a, se $(2k-1)\pi < x < 2k\pi$; b, se $2k\pi < x < (2k+1)\pi$; $\frac{a+b}{2}$, se $x = k\pi$, $k\in\mathbb{Z}$

(b) $\frac{b-a}{4}\pi+\frac{2}{\pi}(a-b)\sum_{n=1}^{\infty}\frac{\cos((2n-1)x)}{(2n-1)^2}-(a+b)\sum_{n=1}^{\infty}(-1)^n\frac{\operatorname{sen}(nx)}{n}$.

soma: ax, se $(2k-1)\pi < x \le 2k\pi$; bx, se $2k\pi \le x < (2k+1)\pi$; $\frac{b-a}{2}\pi$, se $x=(2k+1)\pi$, $k\in\mathbb{Z}$.

(c) $\frac{\pi}{2}-\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\cos((2n-1)x)}{(2n-1)^2}$

soma: $|x|$, se $-\pi\le x\le\pi$ e sua extensão periódica para $x\in\mathbb{R}$.

(d) $\frac{\sinh(a\pi)}{a\pi}+\frac{2\sinh(a\pi)}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2+a^2}(a\cos(nx)-n\operatorname{sen}(nx))$.

soma: e^{ax}, se $-\pi<x<\pi$; $\cosh(a\pi)$ se $x=\pm\pi$, e a sua extensão periódica para $x\in\mathbb{R}$

(e) $\frac{2\operatorname{sen}(a\pi)}{\pi}\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n}{n^2-a^2}\operatorname{sen}(nx)$

soma: $\operatorname{sen}(ax)$, para $-\pi<x<\pi$; 0 para $x=\pm\pi$ e a sua extensão periódica para $x\in\mathbb{R}$.

(f) $b+2a\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\operatorname{sen}(nx)}{n}$.

soma: $ax+b$, para $-\pi<x<\pi$; b, para $x=\pm\pi$, e sua extensão periódica, para $x\in\mathbb{R}$.

(g) $\frac{2}{\pi}\left(1+2\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\cos(2nx)}{4n^2-1}\right)$

soma : $|\cos x|$, para $x\in\mathbb{R}$

15. (a) $2a\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\operatorname{sen}(nx)}{n}$

soma : ax, para $-\pi<x<\pi$; 0 para $x=\pm\pi$, e sua extensão periódica para $x\in\mathbb{R}$.

$\frac{a}{2}\pi-\frac{4a}{\pi}\sum_{n=1}^{\infty}\frac{\cos((2n-1)x)}{(2n-1)^2}$,

soma : $a|x|$, para $-\pi\le x\le\pi$ e sua extensão periódica para $x\in\mathbb{R}$.

(b) $2\pi\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\operatorname{sen}(nx)}{n}-\frac{8}{\pi}\sum_{n=1}^{\infty}\frac{\operatorname{sen}((2n-1)x)}{(2n-1)^3}$.

soma : x^2 para $0\le x<\pi$; $-x^2$, para $-\pi\le x\le 0$; 0 para $x=\pm\pi$ e sua extensão periódica para $x\in\mathbb{R}$

$\frac{\pi^2}{3}+4\sum_{n=1}^{\infty}(-1)^n\frac{\cos(nx)}{n^2}$.

soma : x^2, para $-\pi\le x\le\pi$ e sua extensão periódica para $x\in\mathbb{R}$

(c) $\sum_{n=1}^{\infty}\frac{2}{n\pi}(b-(a\pi+b)(-1)^n)\operatorname{sen}(nx)$.

soma : $ax-b$, para $-\pi<x\le 0$; $ax+b$, para $0<x<\pi$, 0, para $x=\pm\pi$, 0 e sua extensão periódica para $x\in\mathbb{R}$.

$\frac{a\pi}{2}+b-\frac{4a}{\pi}\sum_{n=1}^{\infty}\frac{\cos((2n-1)x)}{(2n-1)^2}$.

soma : $a|x|+b$, para $-\pi\le x\le\pi$ e sua extensão periódica de $a|x|+b$ para $x\in\mathbb{R}$.

(d) $\operatorname{sen} x$,

soma : $\operatorname{sen} x$, para $x\in\mathbb{R}$;

$\frac{2}{\pi}(1-2\sum_{n=1}^{\infty}\frac{\cos(2nx)}{4n^2-1})$,

soma : $|\operatorname{sen} x|$ para $x\in\mathbb{R}$

(e) $\frac{2}{\pi}\operatorname{sen}\ x+\sum_{n=2}^{\infty}\frac{2n-1+(-1)^n}{\pi n(n-1)}\operatorname{sen}((2n-1)x)$,

soma : $-|\cos x|$ para $x\le 0$ e $|\cos x|$ para $x\ge 0$, $x\ne k\pi$ e 0 para $x=k\pi$, $k\in\mathbb{Z}$.

$\frac{2}{\pi}(1 + 2\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\cos(2nx)}{4n^2-1}$,
soma : $|\cos x|$, para $x \in \mathbb{R}$

16 .

(a) usar 14a) em $x = \pi/2$ (b) usar 14c) em $x = \pi$
(c) usar 14e) em $x = \pi/2$ (d) usar 14e) em $x_0 = \pi/4$
(e) usar 14b) em $x = \pi/4$

18. (a) $\frac{\pi^2}{8}$ b) $\frac{\pi^2}{12}$

19 .

(a) $c_1 = 2,\ c_2 = -1,\ c_3 = \frac{2}{3}$ (b) $c_1 = \frac{\pi^2}{3}$
(c) $c_1 = \frac{2}{\pi},\ c_2 = 0,\ c_3 = 0$

20. $\frac{1}{3} + \sum_{n=1}^{\infty}\left(\frac{1}{\pi^2 n^2}\cos(2n\pi x) - \frac{1}{\pi n}\operatorname{sen}(2n\pi x)\right)$ $S(999) = 1/2$ e $S(999/2) = 1/4$.

21 .

(a) $\frac{8}{\pi^2}\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)^2}\operatorname{sen}(\frac{(2k+1)\pi x}{2})$. (b) $b_n = \frac{8}{\pi^2}\frac{(-1)^{(n-1)}}{(2n-1)^2}$ para $n \geq 1$.
(c) $c_n = \frac{4}{(2n-1)\pi} + \frac{(-1)^n 8}{(2n-1)^2\pi^2}$, para $n \geq 1$. (d) $S(200) = S(0) = 0$; $S(201) = S(1) = 0$

22. $a = -\pi^2$, $b = 12$

23. (a) Use $f(x) = x^2$ (b) Use exercício 22.

24. (a) $\frac{\pi^4}{96}$ (b) $\frac{\pi^2}{16} - \frac{1}{2}$.

25. (b) $\frac{\pi^2}{4}e^{-\frac{\pi}{2}}$ e $\frac{\pi^2}{2}(e^{-\pi} + e^{\pi})$.

26. $a_0 = \pi$ e $a_n = \begin{cases} 0 & \text{se } n \text{ é par} \\ -\frac{4}{n^2} & \text{se } n \text{ é ímpar} \end{cases}$.