3. **(1,5)** Seja $f(x,y) = (x^2+y^2)^{\frac{2}{3}}$. É a derivada parcial $\dfrac{\partial f}{\partial x}$ contínua em $(0,0)$? JUSTIFIQUE.

$$\frac{\partial f}{\partial x}(x,y) = \frac{2}{3}(x^2+y^2)^{-1/3} \cdot 2x \qquad \text{se } (x,y) \neq (0,0)$$

$$\frac{\partial f}{\partial x}(0,0) = \lim_{x\to 0} \frac{f(x,0)-f(0,0)}{x-0} = \lim_{x\to 0} \frac{x^{4/3}}{x} = \lim_{x\to 0} x^{1/3} = 0$$

Logo $$\frac{\partial f}{\partial x}(x,y) = \begin{cases} \dfrac{4}{3}\dfrac{x}{(x^2+y^2)^{1/3}} & \text{se } (x,y)\neq(0,0) \\ 0 & \text{se } (x,y)=(0,0) \end{cases}$$

Calcular

$$\lim_{(x,y)\to(0,0)} \frac{4}{3}\frac{x}{(x^2+y^2)^{1/3}} \cdot$$

$$\lim_{(x,y)\to(0,0)} \frac{4}{3}\frac{x}{(x^2+y^2)^{1/3}} \cdot \frac{(x^2+y^2)^{2/3}}{(x^2+y^2)^{2/3}} =$$

$$\lim_{(x,y)\to(0,0)} \frac{x}{x^2+y^2} \cdot (x^2+y^2)^{2/3} =$$

$$\lim_{(x,y)\to(0,0)} \frac{x}{\sqrt{x^2+y^2}} \cdot \frac{(x^2+y^2)^{2/3}}{(x^2+y^2)^{1/2}} =$$

$$\lim_{(x,y)\to(0,0)} \underbrace{\frac{x}{\sqrt{x^2+y^2}}}_{\text{limitada } (*)} \underbrace{(x^2+y^2)^{1/6}}_{\to 0} = 0 = \frac{\partial f}{\partial x}(0,0)$$

(*) pois $|x| \leq \sqrt{x^2+y^2}$

Logo $\dfrac{\partial f}{\partial x}$ é contínua em $(0,0)$.