2. **(2,5)** Seja $f : \mathbb{R}^2 \to \mathbb{R}$ uma função diferenciável tal que $x + 4y + 2z = 4$ é o plano tangente ao gráfico de f no ponto $(4, 3, f(4, 3))$. Seja

$$g(u, v) = vf(2uv, u^2 - v^2).$$

Encontre a equação do plano tangente ao gráfico de g no ponto $(2, 1, g(2, 1))$.

O plano procurado é $z - g(2,1) = \frac{\partial g}{\partial u}(2,1)(x-2) + \frac{\partial g}{\partial v}(2,1)(y-1)$

$x + 4y + 2z = 4$

$2z = -x - 4y + 4$

$z = -\frac{1}{2}x - 2y + 2$

$z = -\frac{1}{2}(x-4) - 2(y-3) + 2 - 2 - 6$

$z + 6 = -\frac{1}{2}(x-4) - 2(y-3) \Rightarrow \begin{cases} f(4,3) = -6 \\ \frac{\partial f}{\partial x}(4,3) = -\frac{1}{2} \\ \frac{\partial f}{\partial y}(4,3) = -2 \end{cases}$

$\frac{\partial g}{\partial u}(u,v) = v\left[\frac{\partial f}{\partial x}(2uv, u^2-v^2)\,2v + \frac{\partial f}{\partial y}(2uv, u^2-v^2)\,2u\right]$

$\frac{\partial g}{\partial u}(2,1) = 1\left[\frac{\partial f}{\partial x}(4,3).2 + \frac{\partial f}{\partial y}(4,3).4\right] = -\frac{1}{2}(2) - 2(4) = -1 - 8 = -9$

$\frac{\partial g}{\partial v}(u,v) = f(2uv, u^2-v^2) + v\left[\frac{\partial f}{\partial x}(2uv, u^2-v^2)\,2u + \frac{\partial f}{\partial y}(2uv, u^2-v^2)(-2v)\right]$

$\frac{\partial g}{\partial v}(2,1) = f(4,3) + 1\left[\frac{\partial f}{\partial x}(4,3).4 + \frac{\partial f}{\partial y}(4,3)(-2)\right] = -6 + \left[-\frac{1}{2}(4) - 2(-2)\right] = -4$

$g(2,1) = 1.f(4,3) = -6$

$\boxed{z + 6 = -9(x-2) - 4(y-1)}$ //