Local search and trajectory metaheuristics for the flexible job shop scheduling problem with nonlinear routes and position-based learning*

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Abstract

This paper considers the flexible job shop scheduling problem with nonlinear routes, a production environment with a wide range of relevant practical applications, especially in today's on-demand printing industry. In order to approximate the problem of real-world applications, we consider the influence of a position-based learning effect on the processing time of the operations. The goal is to minimize makespan. In the present work, we are concerned with the development of effective and efficient methods for its solution. For this purpose, a local search method and four trajectory metaheuristics are considered. In the local search, we show that the classical strategy of reallocating only those operations that are part of the critical path can miss better-quality neighbors, as opposed to what happens in the case where there is no learning effect. Consequently, we introduce an alternative type of neighborhood reduction that eliminates only neighbors that are not better than the current solution. Additionally, we analyze the application of the classical strategy on top of the new reduction. Through experimentation, we verify that it significantly shrinks the size of the neighborhood, thereby increasing efficiency, with minimal loss of effectiveness. Extensive numerical experiments are performed. Statistical tests confirm that tabu search based on the reduced neighborhood, when applied to large-sized instances, outperforms the other three metaheuristics, namely iterated local search, greedy randomized adaptive search, and simulated annealing. Experiments on classical instances with linear routes only show that the introduced methods also stand out in relation to methods from the literature. All methods, instances, and solutions are freely available.

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1 Introduction

The flexible job shop (FJS) with nonlinear routes is a production environment with a wide range of relevant practical applications, especially in the today's on-demand printing industry [39, 40]. Today, companies in the print-on-demand business must deal with customized production and prioritize on-time delivery in an effort to meet their customers' needs. In this context, production activities are organized in flexible workshops to better manage the execution of the wide range of tasks demanded. Other businesses that fit into this production environment include the glass industry [1], the mold industry [23], the scheduling of aircraft support operations in flight decks [50], the scheduling of repairing orders in automobile collision repair shops [2], and the construction of production programs for steelmaking [20]. It is therefore important that the solving methods are prepared to cope with the most diverse characteristics of the actual problems encountered in this production environment. One such factor is the learning effect, i.e. how the processing time of an operation varies with the number of times it is executed. Naturally, using processing times that are not entirely consistent with reality can lead to inaccurate schedules and result in significant economic losses.

The Online Printing Shop (OPS) scheduling problem is a challenging real-world FJS scheduling problem that appears in the printing industry nowadays. Recent studies of the OPS [39, 40] addressed various complexities, including machine downtime, resumable operations, and sequence-dependent setup times. However, they overlooked a crucial aspect of real-world operations: the involvement of human operators in tasks such as job setup times (loading paper, adjusting ink levels, and configuring the press), binding and finishing operations (folding, cutting, and binding), color management and quality control (adjusting color settings, detecting defects, and making quality control decisions), makeready times (preparing a press for a new job, including cleaning, adjusting, and testing), and digital printing (file preparation, color management, and press settings). These tasks are subject to the learning effect, where repeated execution of the same operation leads to efficiency gains. It is worth noting that, according to [11], learning is not a one-time event, but rather an ongoing and dynamic process that evolves throughout the planning horizon of the scheduling process.

The FJS scheduling problem is an extension of the classical job shop (JS) scheduling problem in which each operation can be processed by one within a set of machines instead of a single machine. This characteristic is known as routing flexibility. Two additional features are considered in the present work: nonlinear routes and learning effect. In the FJS with linear routes only, there exists the concept of a task, which consists of a set of operations that must respect a sequential order of execution (first the first operation, then the second, then the third, etc). The nonlinear routes feature consists in considering that the precedences between the operations of a same task are given by an arbitrary directed acyclic graph (DAG). In a classical scheduling problem, given an operation and a machine that can process this operation, a fixed processing time is given that corresponds to the time demanded by the machine to process the operation. The learning effect refers to the real-world phenomenon where an individual improves their performance and speed through repetition, as they learn and refine their skills by executing a task multiple times. In this work we consider a learning function that depends on the position that an operation occupies within the list of operations executed by a machine, i.e. a position-based learning effect function. The goal is to minimize makespan.

A recent literature review on the FJS scheduling problem was done in [17], while a recent review of the FJS with nonlinear routes was included in [4]. For applications in real-world problems see [1, 2, 8, 20, 23, 39, 40, 50] and for development of methods for the FJS with nonlinear routes and a variety of additional features see [9, 10, 14, 24, 32, 33, 34, 46].

The impact on a worker's qualification, by repeated processing of an operation, and the resulting reduction in the operation's processing time was investigated in [19] in 1957. Since the publication of the pioneer works [11, 15, 28] that introduced the concept of learning effect in scheduling problems nearly two decades ago, a large number of papers have been published devoted to this subject. Surveys and classifications can be found in [6, 12, 30, 43]. However, few papers address the FJS scheduling problem with learning effect and, to the best of our knowledge, none of them consider nonlinear routes. These papers are reviewed below.

In [45] the FJS scheduling problem with sequence-dependent setup times as well as position-dependent learning and time-dependent deterioration effects is considered. The authors propose a hybrid metaheuristic that combines the genetic algorithm and variable neighborhood search for the purpose of minimizing the makespan. The same problem is considered in [5], where the problem is modeled as a bilevel optimization problem in which both levels have the same objective, namely, minimizing the makespan. The proposed method, called evolutionary bilevel optimization, constructs feasible solutions in a two-stage hierarchical approach that assigns operations to machines in the first stage and schedules operations in the second stage. In [48] a dual resource FJS is considered in which a machine and a worker are required to process an operation. It is also assumed that there is flexibility in the choice of both, and therefore, for each operation there is a set of machines and a set of workers capable of processing it. The problem is described using a mathematical model and a hybrid method that combines genetic algorithms and variable neighborhood search is developed. In [49] an FJS scheduling problem with operation processing time deterioration effect is considered with the objective of minimizing a function that combines makespan and energy consumption. The proposed methodology hybridizes pigeon-inspired optimization and simulated annealing. In [51], the FJS scheduling problem with time-dependent learning effect is considered. The problem is multi-objective and the goal is to minimize the makespan, total carbon emissions, and total workers' cost. A memetic algorithm, based on NSGA-II, associated with a variable neighborhood search is proposed. Four different neighborhoods are proposed and special attention is given to the development of constructive heuristics to develop the initial population. In [44] a dual resource (machine and worker) FJS with position-dependent learning effect is considered. Additionally, a transportation time between machines is also taken into account. The goal is to minimize makespan, energy consumption, and noise. Therefore, the problem is multi-objective. A real-world sand casting problem is the source of all these ingredients. To tackle the problem, a discrete multi-objective imperial competition algorithm is developed in which a local search based on simulated annealing is used. In [37], a multi-objective FJS scheduling problem with position-dependent learning effect, in which makespan and total carbon emissions are minimized, is considered. The problem is modeled as a mixed integer linear multiobjective optimization problem and an improved multiobjective sparrow search algorithm is developed.

The problem considered in the present work was recently considered in [4], where integer linear programming and constraint programming models were introduced and compared. To improve the performance of exact methods applied to small-sized instances of the problem, constructive heuristics were also developed. In the present work we continue that work by developing local search strategies and metaheuristics that can compute good quality solutions to large-sized instances. (It should be noted that the problem is NP-hard since it contains the JS scheduling problem, knowingly NP-hard [25], as a particular case). In [42] it was introduced, for the FJS scheduling problem, a neighborhood reduction that eliminates only neighbors that do not improve the current solution. The reduction is based on the fact that, given a feasible solution, removing and relocating an operation that is not in the “critical path” has no chance of leading to a better solution. This idea was extended to the FJS scheduling problem with nonlinear routes in [39, 40]. In the present paper, we begin by showing that the fundamental principle on which this neighborhood reduction is based does not hold when we consider learning effect. Thus, we propose a new neighborhood reduction strategy that filters out moves with no potential to improve the current solution. We also show that neighborhood cutting based on the critical path associated with the new neighborhood reduction substantially cuts the neighborhood with minimal loss of solution quality. Based on the introduced local search, we then implement four trajectory metaheuristics. With them, we manage to find good quality solutions for large-sized instances and to find solutions very close to the optimal solutions in small-sized instances with known optimal solution.

The rest of this paper is organized as follows. In Section 2, we formally describe the data representing an instance of the FJS and a way to represent a feasible solution of it using a digraph. In Section 3, we introduce the concept of neighborhood and, in the sequel, a local search. We also analyze different possibilities to reduce the number of neighbors of a current solution to be inspected. In Section 4 we describe the considered metaheuristics, namely, iterated local search, greedy randomized adaptive search procedure, tabu search, and simulated annealing. Section 5 is devoted to extensive numerical experiments. Conclusions and lines of future work are stated in Section 6.

2 Problem description and representation of a feasible solution

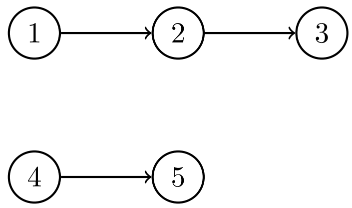
The data of an instance of the FJS with nonlinear routes and position-based learning effect consists of **(i)** a set of operations $\mathcal{O}$ and a set of machines $\mathcal{F}$; **(ii)** for each operation $i \in \mathcal{O}$, a subset $\mathcal{F}_i \subseteq \mathcal{F}$ containing the machines that can process i; **(iii)** for each operation-machine pair (i,k) with $i \in \mathcal{O}$ and $k \in \mathcal{F}_i$, a standard processing time p_{ik}; and **(iv)** a set of arcs $\widehat{A} \subseteq \mathcal{O} \times \mathcal{O}$ representing the precedence relations between the operations. The learning effect is given by a function $\psi_\alpha(p,r)$, where $\alpha > 0$ is a predefined parameter. Given a standard processing time p and a position r, $\psi_\alpha(p,r)$ returns the actual processing time of an operation with standard processing time p when processed at the r-th position of a machine. If necessary, each machine can use a different function and/or a different parameter α. In [11], $\psi_\alpha(p,r) = p/r^\alpha$ was considered. In this expression, r^α is referred to as the learning rate and α is referred to as the learning index. This way of modeling the learning effect assumes that the time required to perform a task decreases with the number of repetitions of the task. The duration of the repetitions has no influence in the learning effect. It also assumes that each machine always performs the same kind of task. According to [11, §2], these are realistic assumptions for many real-world applications, especially for high-technology manufacturing processes. In the current work, following [4] and aiming at integer processing times, we consider $\psi_\alpha(p, r) = \lfloor 100\, p/r^\alpha \rceil$, where $\lfloor \cdot \rceil := \lfloor \cdot + 1/2 \rfloor$. Multiplying by one hundred and rounding corresponds to changing the unit of measures (from second to hundredths of a second, for example) and rounding to the closest integer value. A simple example of an instance is shown in Figure 1.

A feasible solution to an instance of the FJS with nonlinear routes and position-based learning effect can be represented by a DAG $G = (V, A)$ as depicted in Figure 2a. This graph is sometimes referred to as a solution graph in the literature. The vertices of G, represented by the set V, correspond to the operations plus the dummy vertices s and t, i.e. $V = \mathcal{O} \cup \{s, t\}$. The arcs, represented by the set A, correspond to the arcs in $\widehat{A}$ representing the precedence relationships between operations (in black in the picture), arcs going from s to operations that have no predecessors and arcs going to t from operations that do not precede any other operation (in purple in the picture). Arcs that start from s and arcs that reach t are named dummy arcs. Furthermore, dashed arcs represent the assignment of operations to machines and the order in which the operations are processed by each machine. These arcs are named machine arcs. Each node $i \in V \setminus \{s, t\} = \mathcal{O}$, that is, each operation, has a value w_i associated with it that represents its actual processing time, which is calculated with the learning function using the operation's standard processing time and the position that the operation occupies in the machine to which it was assigned. Nodes s and t are associated with the value zero, i.e., $w_s = w_t = 0$. Given a path from s to t, we define its length as the sum of the values of the nodes that constitute the path. The longest path between nodes s and t is called critical path (highlighted in yellow in the figure), and its length corresponds to the makespan of the represented solution. A way to compute a critical path in a graph is described in the next section for completeness.

3 Local search

Given a feasible solution and a DAG $G = (V, A)$ representing it, a new feasible solution can be constructed by removing an operation from the machine to which it was assigned and reinserting it in the same machine but in another position or in another machine. When an operation is removed, the machine arcs adjacent to it must be removed and a new arc going from the operation before the removed one to the one after the removed one (if both exist) must be created. When the operation is reinserted, a similar reverse operation must also be done. When reinserting the operation, it is important to verify that a cycle is not produced in the digraph. Only reinsertions that do not create cycles build a digraph that corresponds to a feasible solution.

Given a feasible solution, we can define its neighborhood as the set of all feasible solutions that can be obtained by removing and reinserting a single operation. When there is no learning effect, it is known [42] that there are chances to build a better feasible solution by removing and relocating operations that are part of the critical path only. If an operation is not part of the critical path, its removal and reinsertion cannot decrease the length of the critical path. It can increase it or create another even longer path. This fact is widely used in the literature to



		Machines	
		1	2
Operations	1	1	1
	2	1	1
	3	1	1
	4	10	10
	5	1	1

Figure 1: On the left, representation of the operations' precedence constraints by a DAG $D = (\mathcal{O}, \widehat{A})$, where $\mathcal{O} = \{1, 2, \ldots, 5\}$ represents the set of operations and $\widehat{A} = \{(1,2), (2,3), (4,5)\}$ is the set of arcs that represents the precedence constraints. This instance has two machines and each of the five operations can be processed in any of the two machines, i.e. $\mathcal{F} = \{1, 2\}$ and $\mathcal{F}_i = \mathcal{F}$ for all $i \in \mathcal{O}$. This means that there is full routing flexibility. The table on the right shows the standard processing times p_{ik} of the five operations on each of the two machines. In this simple example, precedence constraints are given by a linear order, i.e. there are no nonlinear routes. Adding the arc $(5, 3)$ at $\widehat{A}$, which would indicate that the start of operation 3 requires, in addition to the completion of operation 2, the completion of operation 5, would transform the instance into an instance with nonlinear routes.

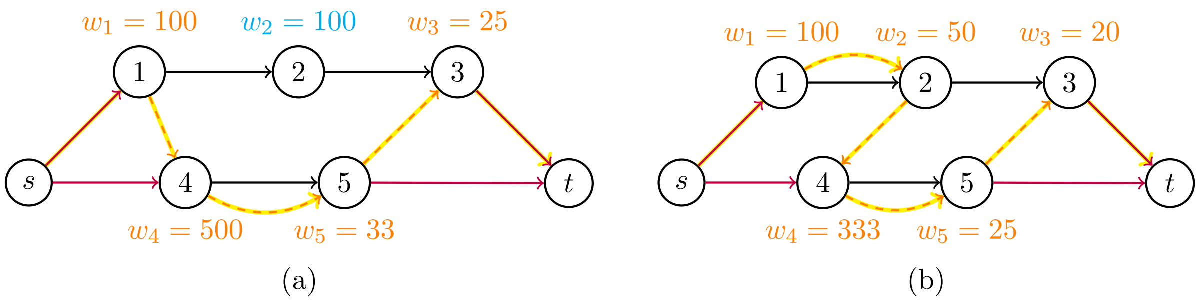


Figure 2: In this figure we consider the instance in Figure 1 with learning index $\alpha = 1$. The digraph on the left (Figure 2a) represents a feasible solution in which machine 1 (associated with the cyan color) processes operation 2 only, while machine 2 (associated with the orange color) processes operations 1, 4, 5, and 3 in this order. The colored numbers represent the actual processing time of the operations, with the influence of the learning effect. The critical path, whose length of 658 corresponds to the makespan, is given by the path $s, 1, 4, 5, 3, t$ (highlighted in yellow in the picture). The digraph on the right (Figure 2b) represents the feasible solution obtained by reallocating operation 2, that was not in the critical path, from machine 1 to machine 2 between operations 1 and 4. The constructed feasible solution has a critical path of length 528 given by $s, 1, 2, 4, 5, 3, t$, i.e. it has a makespan smaller than the original one.

generate just promising neighbors. This is false when considering learning effect. An example is shown in Figure 2b. This shows that, when applied to the problem under consideration in this paper, this neighborhood reduction can result in the loss of neighbors that are better than the current solution. This leads us to analyze whether every removal and reinsertion that does not generate cycles has the potential to generate a neighbor with lower makespan or whether any neighborhood reduction is possible. The main point is to note that when an operation is removed from a machine, the operations that were scheduled to be processed later on that machine have their position decreased by one unit and, consequently, their actual processing time increased. Similarly, in the machine where the operation is inserted, the operations scheduled to be processed after the inserted operation have their position increased by one and, therefore, their processing time is decreased. These modifications may change the makespan whether improving or worsening.

Consider a feasible solution represented by a DAG $G = (V, A)$. Let $\mathcal{P}$ be a critical path in G, with length $C_{\max}$. Let $i \in \mathcal{O}$ be an arbitrary operation. We name f_i the machine to which i is assigned. Let $k \in \mathcal{F}$ be an arbitrary machine. We name $Q_k = i_1, \dots, i_{|Q_k|}$ the ordered list of operations assigned to machine k. If an operation i is assigned to machine f_i and it is in the γ-th position of Q_{f_i}, then its actual processing time is given by $w_i = \psi_\alpha(p_{i,f_i}, \gamma)$. Hereafter we denote by f the set of all f_i for $i \in \mathcal{O}$, by Q the set of the ordered lists Q_k for $k \in \mathcal{F}$, and by w the set of processing times w_i for $i \in \mathcal{O}$. We intend to compute all neighbors of the solution represented by G, f, Q, and w. The neighbors are constructed by considering each $v \in \mathcal{O}$ at a time, removing v and reinserting v at every possible place that does not generate a cycle. We want to determine if there are insertions that can be ignored because we know *a priori* that they will not lead to a makespan reduction.

Let $v \in \mathcal{O}$ be an arbitrary operation. The computation of the neighbors of the current solution (associated with the remotion and reinsertion of v) begins by computing a digraph $G_v^- = (V^-, A^-)$ in which the operation v is removed. Such a graph is sometimes referred to as a reduced graph in the literature. The quantities f^-, Q^-, and w^- associated with G_v^- are also calculated. This digraph with its associated information is an intermediate structure necessary for the computation of the neighbors and, since the operation v is not assigned to any machine, it does not represent a feasible solution. Along with the calculation of G_v^-, it is calculated information that will be relevant for reinserting v into G_v^-. This additional information includes the sets $\mathcal{R}_v^{\leftarrow}$ and $\mathcal{R}_v^{\rightarrow}$ which represent, respectively, the set of vertices from which it is possible to reach v and the set of vertices that can be reached from v in G_v^-. Both sets will be useful for detecting cycles in future possible reinsertions of v. The longest path $\mathcal{P}^-$ in G_v^- and its length ξ are also computed. We do not call this length $C_{\max}^-$ because as G_v^- does not represent a feasible solution, then the length of the path $\mathcal{P}^-$ does not represent a makespan. Along with the computation of $\mathcal{P}^-$, it is also computed, for each machine k, the smallest position τ_k such that, for all $\gamma > \tau_k$, the γ-th operation processed by machine k is not in $\mathcal{P}^-$. If machine k does not process any operation in $\mathcal{P}^-$, then $\tau_k = 0$. This information will be useful to determine whether a reinsertion has chances to decrease the makespan or not.

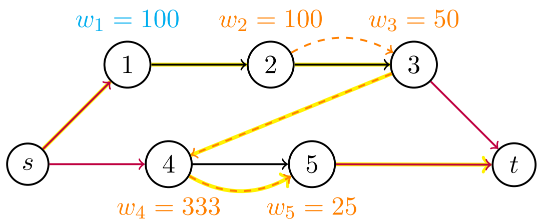
Algorithm 1 describes step by step the construction of G_v^-, f^-, Q^-, w^- and all additional information described in the previous paragraph. In line 2, the assignment of operations to machines is copied for all operations other than v. In line 3, the lists of operations of all machines other than f_v are copied, and, in the list of machine f_v, operation v is eliminated. In line 4, the actual processing times of most of the operations are copied, except for the operations succeeding v in the list Q_{f_v}. The actual processing times of these operations, whose position within the machine was reduced by one, need to be recalculated. The value of w_v^- is set to zero. In line 5, machine arcs adjacent to v are removed (if they exist) and a new machine arc between its predecessor and successor is inserted, unless v is the first or last element in Q_{f_v}. In lines 6 and 7, the sets $\mathcal{R}_v^{\leftarrow}$ and $\mathcal{R}_v^{\rightarrow}$ are calculated. In line 8, the longest path $\mathcal{P}^-$ in the digraph G_v^- and its length ξ are calculated. The tasks in lines 6, 7, and 8 use auxiliary well-known algorithms for topological sorting, depth-first search, and an adaptation [16, §22.2] of the Bellman-Ford algorithm described in Algorithms 2, 3, and 4, respectively, for completeness.

Let G be the digraph, with associated quantities f, Q, and w, representing the current feasible solution; and let $\mathcal{P}$ be its critical path, with length $C_{\max}$. Let v be the operation we removed and wish to reinsert. Let G_v^- be the digraph with v removed and let f^-, Q^-, and w^- be the quantities associated with G_v^-. Let $\mathcal{P}^-$ be the critical path in G_v^-, with length ξ, and, for each machine k, let τ_k be the smallest position in Q_k such that every operation in a position after τ_k is not in $\mathcal{P}^-$. Let κ be a machine and γ be a position in the list Q_κ^- such that inserting v at position γ of Q_κ^- does not generate a cycle. Does such an insertion have a chance of generating a new digraph whose associated feasible solution has a makespan smaller than $C_{\max}$? If $\xi \geq C_{\max}$ and $\gamma > \tau_k$, then the answer is "no". This is because the path $\mathcal{P}^-$ with length ξ not smaller than $C_{\max}$ already exists and the insertion of v in machine κ, at a position γ later than τ_k, will not modify the actual processing time of any operation in $\mathcal{P}^-$. If $\xi < C_{\max}$ or $\xi \geq C_{\max}$ but $\gamma \leq \tau_k$, then chances exist. Figures 3 and 4 illustrate the first and second scenarios, respectively, using as examples feasible solutions of the instance described in Figure 1, with learning index $\alpha = 1$.

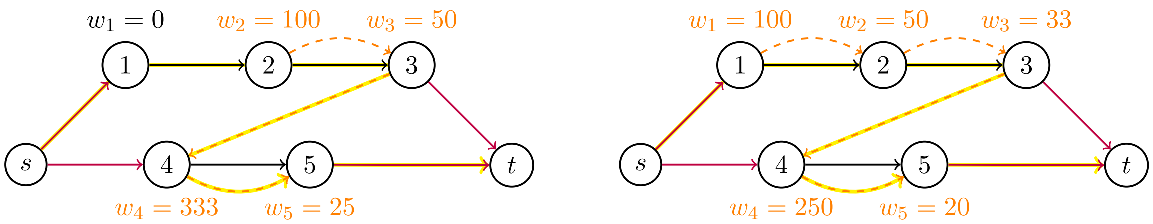
It should be noted that, strictly speaking, the fact of v being in $\mathcal{P}$ or not is not related to the answer to the question above. But already anticipating something that will come later, as the neighborhood reduction driven by the answer to the question may be quite small, we will consider in the experiments, heuristically, $v \in \mathcal{P}$ as equivalent or strongly correlated to $\xi < C_{\max}$. That is, we will consider that removing an operation from the critical path will most likely imply that $\xi < C_{\max}$. This is very plausible for moderate values of the learning index α, with which a possible reduction of one in the machine position of some critical path operations does not cancel out the benefit of removing an operation from the critical path.

The task of reinserting v in G_v^- at position γ of machine κ generates a DAG that we name G_v^+. This task is similar to the removing task. The construction of G_v^+, its associated quantities f^+, Q^+, and w^+, and its critical path $\mathcal{P}^+$ with length $C_{\max}^+$ is described in Algorithm 5. Algorithm 6 implements a best neighbor local search with the neighborhood reduction already discussed. It corresponds to a classical local search with a best neighbor strategy. The only relevant detail that remains to be explained is how to determine whether an insertion generates a cycle or not. A cycle will only be created in G_v^+ if v is inserted in a position that leaves some $u \in \mathcal{R}_v^{\leftarrow}$ to be processed after v in machine κ or some $u \in \mathcal{R}_v^{\rightarrow}$ to be processed before v in machine κ. The limits $\underline{\gamma}$ and $\bar{\gamma}$ such that $\underline{\gamma} + 1 \leq \gamma \leq \bar{\gamma}$ avoids cycles are calculates in lines 7 and 8. A possible reduction of this interval is computed in lines 9 and 10, eliminating the possibility of making insertions after τ_κ if $\xi \geq C_{\max}$, as already discussed.

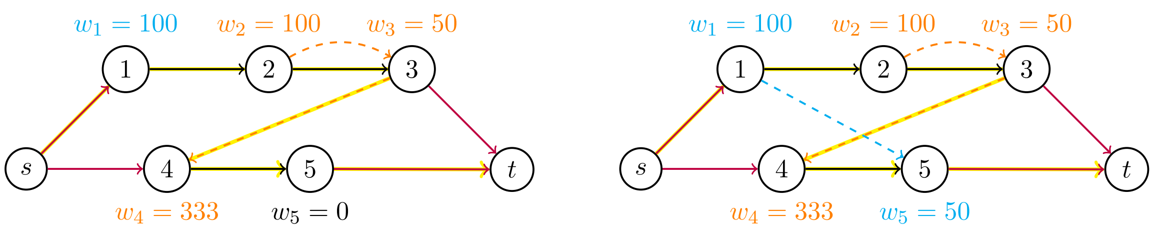
As described in Algorithm 6, the local search uses the best neighbor strategy and makes use of the neighborhood reduction. Therefore, we call this version of "local search with the best neighbor strategy and reduced neighborhood". The neighborhood reduction is implemented in lines 9 and 10. If we remove those two lines we obtain a version that we call "local search with the best neighbor and full neighborhood". The version with reduced neighborhood does not consider neighbors that are guaranteed to be no better than the current solution. Therefore,



(a) Current solution ($C_{\max} = 608$)



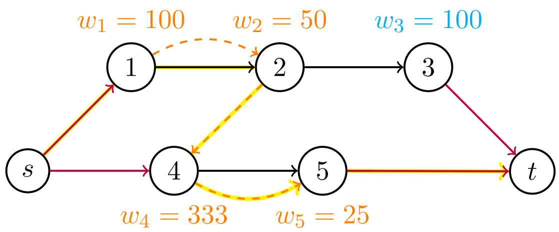
(b) Removal of operation $v = 1$ ($\xi = 508$) and reinsertion in position $\gamma = 1$ of machine 2 ($C_{\max} = 453$)



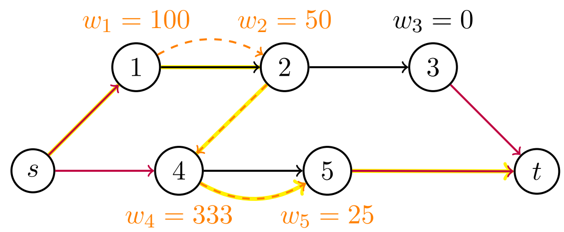
(c) Removal of operation $v = 5$ ($\xi = 583$) and reinsertion in position $\gamma = 2$ of machine 1 ($C_{\max} = 633$)

Figure 3: This figure illustrates the removal and reinsertion of an operation v which results in a digraph G^{-}_{V} whose critical path has length $\xi < C_{\max}$. Figure (a) shows a feasible solution with $C_{\max} = 608$. The case shown in figure (b) corresponds to $v = 1$, while figure (c) corresponds to $v = 5$. In the first case a better neighbor is constructed by the reinsertion of operation v, while in the second case a worse neighbor is constructed.

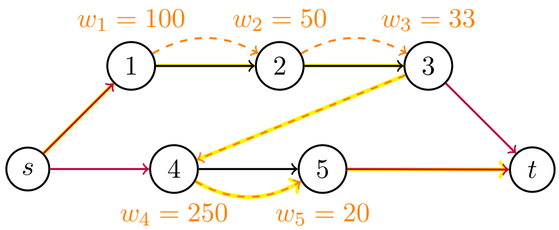
the solution obtained with the reduced neighborhood must be identical to the solution obtained with the full neighborhood. In fact all iterates of the two versions must be identical and not just the final solution. Only a reduction of CPU time is expected. As already mentioned above, we decided to consider yet another version that would exhibit a more drastic reduction in CPU time, albeit with a possible loss of quality in the solution. We call this version of “local search with the best neighbor and cropped neighborhood”. This version consists of changing $v \in \mathcal{O}$ to $v \in \mathcal{P}$ in line 4 of Algorithm 6. That is, only operations on the critical path are reallocated, since there is a greater tendency for these reallocations to generate better quality neighbors. Note that the cropped neighborhood includes the cuts from the reduced neighborhood. We have then three different versions of the local search with the best neighbor strategy that are distinguished by the neighborhood used: full neighborhood, reduced neighborhood, and cropped neighborhood. Each of them corresponds to minimal variations of Algorithm 6 as already described. For further reference, methods that use the full, reduced, or cropped neighborhoods will include the acronyms FN, RN, or CN in their name, respectively. Furthermore, in the



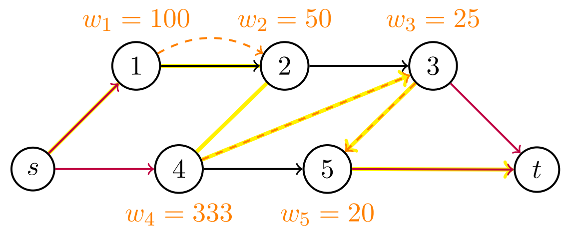
(a) Current solution ($C_{\max} = 508$)



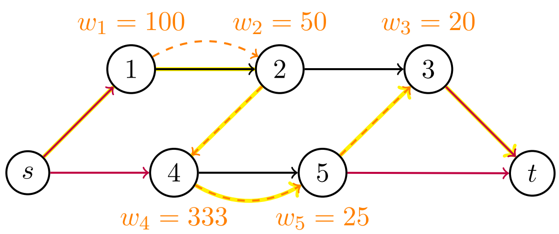
(b) Removal of operation $v = 3$ ($\xi = 508$)



(c) Reinsertion in position $\gamma = 3$ ($C_{\max} = 453$)



(d) Reinsertion in position $\gamma = 4$ ($C_{\max} = 528$)



(e) Reinsertion in position $\gamma = 5$ ($C_{\max} = 528$)

Figure 4: This figure illustrates the removal and reinsertion of operation $v = 3$. Figure (a) shows a solution with makespan $C_{\max} = 508$. Figure (b) shows that if we remove operation v, we get a reduced digraph G_v^- whose critical path has length $508 = \xi \geq C_{\max}$. Therefore, only a reinsertion in a machine k at a position $\gamma \leq \tau_2$ can generate a better neighbor. In this example, operation v was the only one in machine 1 (cyan color), so it only makes sense to reinsert it in machine 2 (orange color). The last operation of machine 2 that belongs to the critical path of G_v^- is operation 5, which is at position 4. Therefore, $\tau_2 = 4$. Figures (c) and (d) show that reinserting operation v at positions $\gamma = 3$ leads to a better solution, while reinserting it at positions $\gamma = 4$ does not. Figure (e) shows that, as already known, reinserting operation v at positions $\gamma = 5$ does not lead to a better solution.

numerical experiments, we will also consider the same three versions but using the strategy of interrupting the inspection of the neighborhood when finding the first neighbor that improves the current solution, i.e. the first-improvement strategy. This change corresponds, in Algorithm 6, to interrupting the loop of line 4 the first time line 16 is executed.

Algorithm 1: Computes $G_v^- = (V_v^-, A_v^-)$, f^-, Q^-, and w^- by removing operation v from the solution graph $G = (V, A)$. Then, in G_v^-, it computes the set $\mathcal{R}_v^{\leftarrow}$ of vertices that reaches v and the set of vertices $\mathcal{R}_v^{\rightarrow}$ that are reached from v, computes the largest path $\mathcal{P}^-$ from s to t and its length ξ, and computes the position τ_k of the last critical operation in each machine $k \in \mathcal{F}$.

Input: $\mathcal{O}$, $\mathcal{F}$, p, v, f, Q, w, $G = (V, A)$
Output: f^-, Q^-, w^-, $G_v^- = (V_v^-, A_v^-)$, $\mathcal{P}^-$, ξ, $\mathcal{R}_v^{\leftarrow}$, $\mathcal{R}_v^{\rightarrow}$, τ

function RemoveOp($\mathcal{O}$, p, v, f, Q, w, G, f^-, Q^-, w^-, G_v^-, $\mathcal{P}^-$, ξ, $\mathcal{R}_v^{\leftarrow}$, $\mathcal{R}_v^{\rightarrow}$, τ)

- Define $f_i^- := f_i$ for all $i \in \mathcal{O} \setminus \{v\}$.
- Let Q_{f_v} be given by the sequence $i_1, \dots, i_{\gamma-1}, i_\gamma, i_{\gamma+1}, \dots, i_{|Q_{f_v}|}$ with $i_\gamma = v$. Define $Q_k^- := Q_k$ for all $k \in \mathcal{F} \setminus \{f_v\}$ and $Q_{f_v}^- := i_1, \dots, i_{\gamma-1}, i_{\gamma+1}, \dots, i_{|Q_{f_v}|}$.
- Define $w_i^- := w_i$ for all $i \in \mathcal{O} \setminus \{v, i_{\gamma+1}, \dots, i_{|Q_{f_v}|}\}$, $w_v^- := 0$, and $w_{i_\ell}^- := \psi_\alpha(p_{i_\ell, f_{i_\ell}}, \ell - 1)$ for $\ell = \gamma + 1, \dots, |Q_{f_v}|$.
- Define the graph $G_v^- = (V_v^-, A_v^-)$ with $V_v^- := V$ and $A_v^- := (A \setminus \{(i_{\gamma-1}, v), (v, i_{\gamma+1})\}) \cup \{(i_{\gamma-1}, i_{\gamma+1})\}$.
- Initialize $\mathcal{V} \leftarrow \emptyset$, $\mathcal{U}$ as an empty list, and $\mathcal{R}_v^{\leftarrow} \leftarrow \{v\}$, and compute in $\mathcal{U}$ and $\mathcal{R}_v^{\leftarrow}$ a topological sort of the vertices in V_v^- and the set of vertices $i \in V_v^-$ such that a path from i to v exists, respectively, by calling TopologicalSort+(G_v^-, $\mathcal{U}$, s, $\mathcal{V}$, v, $\mathcal{R}_v^{\leftarrow}$).
- Initialize $\mathcal{R}_v^{\rightarrow} \leftarrow \emptyset$ and compute in $\mathcal{R}_v^{\rightarrow}$ the set of vertices $i \in V_v^-$ such that a path from v to i exists, by calling DFS(G_v^-, v, $\mathcal{R}_v^{\rightarrow}$).
- Compute the largest path $\mathcal{P}^-$ from s to t, its length ξ, and determine the position τ_k of the last critical operation at each machine k for all $k \in \mathcal{F}$ by calling CriticalPath($\mathcal{F}$, f^-, w^-, Q^-, $G_v^- = (V_v^-, A_v^-)$, $\mathcal{U}$, $\mathcal{P}^-$, ξ, τ).

Algorithm 2: Computes a topological sort $\mathcal{U}$ of the vertices of $G = (V, A)$. In addition, if v and $\mathcal{R}_v^{\leftarrow}$ are present as an input parameter, computes the set $\mathcal{R}_v^{\leftarrow}$ of vertices that reaches v in $G = (V, A)$.

Input: $G = (V, A)$, $\mathcal{U}$, i, $\mathcal{V}$, v, $\mathcal{R}_v^{\leftarrow}$
Output: $\mathcal{U}$, $\mathcal{V}$, $\mathcal{R}_v^{\leftarrow}$

function TopologicalSort+(G, $\mathcal{U}$, i, $\mathcal{V}$, v, $\mathcal{R}_v^{\leftarrow}$)

- Set $\mathcal{V} \leftarrow \mathcal{V} \cup \{i\}$.
- **for** j such that $(i, j) \in A$ **do**
  - **if** $j \notin \mathcal{V}$ **then**
    - TopologicalSort+(G, $\mathcal{U}$, j, $\mathcal{V}$, v, $\mathcal{R}_v^{\leftarrow}$)
  - **if** $i \notin \mathcal{R}_v^{\leftarrow}$ and $j \in \mathcal{R}_v^{\leftarrow}$ **then**
    - set $\mathcal{R}_v^{\leftarrow} \leftarrow \mathcal{R}_v^{\leftarrow} \cup \{i\}$.
- Insert i at the beginning of $\mathcal{U}$.

Algorithm 3: Computes the set of vertices $\mathcal{R}_v^{\rightarrow}$ as the set of vertices that can be reached by v in $G = (V, A)$.

Input: $G = (V, A)$, v, $\mathcal{R}_v^{\rightarrow}$
Output: $\mathcal{R}_v^{\rightarrow}$

function DFS(G, v, $\mathcal{R}_v^{\rightarrow}$)
  Set $\mathcal{R}_v^{\rightarrow} \leftarrow \mathcal{R}_v^{\rightarrow} \cup \{v\}$.
  for j such that $(v, j) \in A$ **do**
    if $j \notin \mathcal{R}_v^{\rightarrow}$ **then**
      DFS(G, j, $\mathcal{R}_v^{\rightarrow}$)

Algorithm 4: Computes a critical path $\mathcal{P}$ and its length ξ for a given graph $G = (V, A)$. In addition, if τ is present as an input parameter, determines the last critical operation in each machine.

Input: $\mathcal{F}$, f, w, Q, $G = (V, A)$, τ
Output: $\mathcal{U}$, $\mathcal{P}$, ξ, τ

function CriticalPath($\mathcal{F}$, f, w, Q, G, $\mathcal{U}$, $\mathcal{P}$, ξ, τ)
  Initialize $d_i \leftarrow -\infty$ for all $i \in V \setminus \{s\}$ and define $d_s := 0$ and $\pi_s := 0$.
  Initialize $\mathcal{V} \leftarrow \emptyset$ and $\mathcal{U}$ as an empty list and compute in $\mathcal{U}$ a topological sort of the vertices in V, by calling TopologicalSort+(G, $\mathcal{U}$, s, $\mathcal{V}$).
  for $\ell = 1, \dots, |V|$ **do**
    Let i be the ℓ-th operation in the topological order given by $\mathcal{U}$.
    for j such that $(i, j) \in A$ **do**
      if $d_j < d_i + w_i$ **then**
        $d_j \leftarrow d_i + w_i$ and $\pi_j \leftarrow i$.
  $\xi := d_t$
  Initialize $i \leftarrow \pi_t$, $\mathcal{P} \leftarrow \emptyset$, and $\tau_k \leftarrow 0$ for all $k \in \mathcal{F}$.
  do
    if $\tau_{f_i} = 0$ **then**
      Let Q_{f_i} be given by the sequence $i_1, \dots, i_{\ell-1}, i, i_{\ell+1}, \dots, i_{|Q_{f_i}|}$. Define $\tau_{f_i} := \ell$.
    $\mathcal{P} \leftarrow \mathcal{P} \cup \{i\}$ and $i \leftarrow \pi_i$.
  while $i \neq s$

4 Metaheuristics

It is known that the direct application of a local search as the one introduced in the previous section suffers from premature convergence to local solutions. Therefore, it is natural to think of using it in connection with metaheuristics. In particular, trajectory metaheuristics are a natural choice since they make direct use of local search strategies. In this paper we consider the well-known metaheuristics iterated local search (ILS), greedy randomized adaptive search procedure

Algorithm 5: Computes $G_v^+ = (V_v^+, A_v^+)$, f^+, Q^+, and w^+ by inserting operation v at position γ of machine κ in the reduced graph G_v^-. Then, in G_v^+, it computes the largest path $\mathcal{P}^+$ from s to t and its length $C_{\max}^+$.

Input: $\mathcal{O}$, $\mathcal{F}$, p, v, γ, κ, f^-, Q^-, w^-, $G_v^- = (V_v^-, A_v^-)$

Output: f^+, Q^+, w^+, $G_v^+ = (V_v^+, A_v^+)$, $\mathcal{P}^+$, $C_{\max}^+$

function InsertOp($\mathcal{O}$, p, v, γ, κ, f^-, Q^-, w^-, G_v^-, f^+, Q^+, w^+, G_v^+, $\mathcal{P}^+$, $C_{\max}^+$)

- Define $f_i^+ := f_i^-$ for all $i \in \mathcal{O} \setminus \{v\}$ and $f_v^+ := \kappa$.
- Let Q_κ^- be given by the sequence $i_1, i_2, \dots, i_{|Q_\kappa^-|}$.
- Define $Q_k^+ := Q_k^-$ for all $k \in \mathcal{F} \setminus \{\kappa\}$ and $Q_\kappa^+ := i_1, \dots, i_{\gamma-1}, v, i_\gamma, \dots, i_{|Q_\kappa^-|}$.
- Define $w_i^+ := w_i^-$ for all $i \in \mathcal{O} \setminus \{v, i_\gamma, \dots, i_{|Q_\kappa|}\}$, and $w_{i_\ell}^+ := \psi(p_{i_\ell,\kappa}, \ell)$ for $\ell = \gamma, \dots, |Q_\kappa^+|$.
- Define the graph $G_v^+ = (V_v^+, A_v^+)$ with $V_v^+ := V_v^-$ and $A_v^+ := (A_v^- \setminus \{(i_{\gamma-1}, i_\gamma)\}) \cup \{(i_{\gamma-1}, v), (v, i_\gamma)\}$.
- Initialize $\mathcal{V} \leftarrow \emptyset$ and $\mathcal{U}$ as an empty list and compute in $\mathcal{U}$ a topological sort of the vertices in V_v^+, by calling TopologicalSort+(G_v^+, $\mathcal{U}$, s, $\mathcal{V}$).
- Compute the critical path $\mathcal{P}^+$ from s to t and its length $C_{\max}$ by calling CriticalPath($\mathcal{F}$, f^+, w^+, Q^+, $G_v^+ = (V_v^+, A_v^+)$, $\mathcal{U}$, $\mathcal{P}^+$, $C_{\max}^+$).

(GRASP), tabu search (TS), and simulated annealing (SA). The basic components of each of them are presented below. In practice, it would be reasonable to terminate metaheuristics when there is a lack of progress in the incumbent solution. For this theoretical study, however, we chose to present the methods without a defined stopping criterion. In the numerical experiments, we use CPU time as the stopping criterion to show how each metaheuristic performs over time.

The four metaheuristics considered require an initial solution. In this work, we construct initial solutions using the constructive heuristics earliest completion time (ECT) and earliest starting time (EST) introduced in [4]. GRASP also considers randomized versions of these constructive heuristics. The two constructive heuristics introduced in [4] are based on the EST rule [8] and the ECT rule [36]. The heuristics schedule one operation at a time. In the EST-based constructive heuristic, the instant $r_{\min}$ which is the earliest instant at which an unscheduled operation could be initiated is computed first. All operation/machine pairs that could start at that instant are considered and the pair with the shortest processing time is selected. Since they all would start at instant $r_{\min}$, selecting the pair with the shortest processing time is the same as selecting the pair that ends earliest. This idea is taken to the extreme in the constructive heuristic based on the ECT rule: without limiting the choice to the operation/machine pairs that could start as soon as possible, the operation/machine pair that will finish earliest is chosen, even if the processing of the operation does not start as soon as possible. For more details on the constructive heuristics EST and ECT see [4, Algorithms 1 and 4].

4.1 Iterated local search

The ILS [38] strategy consists of iteratively running a local search until it converges to a local solution and perturbing the solution found to be used as the initial solution for the next run of

Algorithm 6: Given an instance represented by $\mathcal{O}$, $\mathcal{F}$, p and an initial solution represented by G, f, w, Q, it computes a solution $G^\star$, $f^\star$, $w^\star$, $Q^\star$, with critical path $\mathcal{P}^\star$ and makespan $C_{\max}^\star$ by using the best neighbor local search with reduced neighborhood.

Input: $\mathcal{O}$, $\mathcal{F}$, p, $G = (V, A)$, f, w, Q, $\mathcal{P}$, $C_{\max}$
Output: $G^\star = (V^\star, A^\star)$, $f^\star$, $w^\star$, $Q^\star$, $\mathcal{P}^\star$, $C_{\max}^\star$

function LocalSearch($\mathcal{O}$, $\mathcal{F}$, p, G, f, w, Q, $\mathcal{P}$, $C_{\max}$, $G^\star$, $f^\star$, $w^\star$, $Q^\star$, $\mathcal{P}^\star$, $C_{\max}^\star$)
  do
    $C_{\max}^{\text{bn}} \leftarrow +\infty$
    for $v \in \mathcal{O}$ **do**
      RemoveOp($\mathcal{O}$, p, v, f, Q, w, G, f^-, Q^-, w^-, G_v^-, $\mathcal{P}^-$, ξ, $\mathcal{R}_v^{\leftarrow}$, $\mathcal{R}_v^{\rightarrow}$, τ)
      for $k \in \mathcal{F}_v$ **do**
        Let $\underline{\gamma}$ be the position of the last operation in $Q_k^- = i_1, \dots, i_{|Q_k^-|}$ such that $i_{\underline{\gamma}} \in \mathcal{R}_v^{\leftarrow}$ and let $\underline{\gamma} = 0$ if $i_\ell \notin \mathcal{R}_v^{\leftarrow}$ for $\ell = 1, \dots, |Q_k^-|$.
        Let $\bar{\gamma}$ be the position of the first operation in $Q_k^- = i_1, \dots, i_{|Q_k^-|}$ such that $i_{\bar{\gamma}} \in \mathcal{R}_v^{\rightarrow}$ and let $\bar{\gamma} = |Q_k^-| + 1$ if $i_\ell \notin \mathcal{R}_v^{\rightarrow}$ for $\ell = 1, \dots, |Q_k^-|$.
        if $\xi \geq C_{\max}$ **then**
          $\bar{\gamma} \leftarrow \min\{\bar{\gamma}, \tau_k\}$, where τ_k is such that there is no critical operation after τ_k in Q_k^- ($\tau_k = 0$ if there is no critical operation in Q_k^-).
        for $\gamma = \underline{\gamma} + 1, \dots, \bar{\gamma}$ **do**
          InsertOp($\mathcal{O}$, p, v, γ, k, f^-, Q^-, w^-, G_v^-, f^+, Q^+, w^+, G_v^+, $\mathcal{P}^+$, $C_{\max}^+$)
          if $C_{\max}^+ < C_{\max}^{\text{bn}}$ **then**
            $G^{\text{bn}}, f^{\text{bn}}, w^{\text{bn}}, Q^{\text{bn}}, \mathcal{P}^{\text{bn}}, C_{\max}^{\text{bn}} \leftarrow G_v^+, f^+, w^+, Q^+, \mathcal{P}^+, C_{\max}^+$
    $\delta \leftarrow C_{\max} - C_{\max}^{\text{bn}}$
    if $\delta > 0$ **then**
      $G, f, w, Q, \mathcal{P}, C_{\max} \leftarrow G^{\text{bn}}, f^{\text{bn}}, w^{\text{bn}}, Q^{\text{bn}}, \mathcal{P}^{\text{bn}}, C_{\max}^{\text{bn}}$
  while $\delta > 0$
  $G^\star, f^\star, w^\star, Q^\star, \mathcal{P}^\star, C_{\max}^\star \leftarrow G, f, w, Q, \mathcal{P}, C_{\max}$

the local search. For the first run of the local search, it uses as initial solution the best among the solutions constructed by the constructive heuristics ECT and EST. The perturbation consists of a sequence of ℓ^p modifications, where ℓ^p is a random number between $\ell^p_{\min}$ and $\ell^p_{\max}$. ($\ell^p_{\min}$ and $\ell^p_{\max}$ are the only two parameters of the ILS.) Each modification consists of removing a random operation and relocating it to a random place so that the solution thus constructed is feasible. Algorithms 7 and 8 describe the ILS completely. They make use of Algorithm 6 for the local search and Algorithms 1 and 5 for removing and reinserting an operation, respectively. The random choices at line 12 of Algorithm 7 and lines 2, 4, and 7 of Algorithm 8 follow a discrete uniform distribution within the prescribed range. The same applies to all the methods described below, which make random choices following discrete or continuous (depending on the situation) uniform distributions within prescribed ranges.

Algorithm 7: Given an instance represented by $\mathcal{O}$, $\mathcal{F}$, p, it computes an initial feasible solution using either the ECT or EST constructive heuristics. Then, it applies the iterated local search strategy to obtain an optimal solution approximation represented by $G^\star$, $f^\star$, $w^\star$, $Q^\star$ with a makespan of $C^\star_{\max}$.

Input: $\mathcal{O}$, $\mathcal{F}$, p, $\widehat{A}$, $\ell^p_{\min}$, $\ell^p_{\max}$
Output: $f^\star$, $w^\star$, $Q^\star$, $G^\star$, $C^\star_{\max}$
function ILS($\mathcal{O}$, $\mathcal{F}$, p, $\widehat{A}$, $\ell^p_{\min}$, $\ell^p_{\max}$, $f^\star$, $w^\star$, $Q^\star$, $G^\star$, $C^\star_{\max}$)
  ECT($\mathcal{O}$, $\mathcal{F}$, p, $\widehat{A}$, f^1, w^1, Q^1, G^1, $\mathcal{U}^1$, $\mathcal{P}^1$, $C^1_{\max}$, τ^1)
  EST($\mathcal{O}$, $\mathcal{F}$, p, $\widehat{A}$, f^2, w^2, Q^2, G^2, $\mathcal{U}^2$, $\mathcal{P}^2$, $C^2_{\max}$, τ^2)
  if $C^1_{\max} < C^2_{\max}$ **then**
    $f^\star \leftarrow f^1$, $w^\star \leftarrow w^1$, $Q^\star \leftarrow Q^1$, $G^\star \leftarrow G^1$ and $C^\star_{\max} \leftarrow C^1_{\max}$
  else
    $f^\star \leftarrow f^2$, $w^\star \leftarrow w^2$, $Q^\star \leftarrow Q^2$, $G^\star \leftarrow G^2$ and $C^\star_{\max} \leftarrow C^2_{\max}$
  while *the stopping criterion is not satisfied* **do**
    LocalSearch($\mathcal{O}$, $\mathcal{F}$, p, G, f, w, Q, $\mathcal{P}$, $C_{\max}$, G', f', w', Q', $\mathcal{P}'$, $C'_{\max}$)
    if $C'_{\max} < C^\star_{\max}$ **then**
      $f^\star \leftarrow f'$, $w^\star \leftarrow w'$, $Q^\star \leftarrow Q'$, $G^\star \leftarrow G'$ and $C^\star_{\max} \leftarrow C'_{\max}$
    Set ℓ^p a random number between $\ell^p_{\min}$ and $\ell^p_{\max}$.
    for $i = 1, \ldots, \ell^p$ **do**
      Perturb($\mathcal{O}$, $\mathcal{F}$, p, f', w', Q', G', f, w, Q, G, $\mathcal{P}$, $C_{\max}$)

Algorithm 8: Given an instance represented by $\mathcal{O}$, $\mathcal{F}$, p and a solution represented by G, f, w, Q, it computes a perturbed feasible solution represented by G', f', w', Q', with critical path $\mathcal{P}'$ and makespan $C'_{\max}$.

Input: $\mathcal{O}$, $\mathcal{F}$, p, f, w, Q, $G = (V, A)$
Output: f', w', Q', $G' = (V', A')$, $\mathcal{P}'$, $C'_{\max}$
function Perturb($\mathcal{O}$, $\mathcal{F}$, p, f, w, Q, G, f', w', Q', G', $\mathcal{P}'$, $C'_{\max}$)
  Let $v \in V \setminus \{s, t\}$ be a random operation.
  RemoveOp($\mathcal{O}$, p, v, f, Q, w, G, f^-, Q^-, w^-, G^-_v, $\mathcal{P}^-$, ξ, $\mathcal{R}^{\leftarrow}_v$, $\mathcal{R}^{\rightarrow}_v$, τ)
  Let $k \in \mathcal{F}_v$ be a random machine and let $Q^-_k = i_1, \ldots, i_{|Q^-_k|}$.
  Let $\underline{\gamma}$ be the position of the last operation in Q^-_k such that $i_{\underline{\gamma}} \in \mathcal{R}^{\leftarrow}_v$ and let $\underline{\gamma} = 0$ if $i_\ell \notin \mathcal{R}^{\leftarrow}_v$ for $\ell = 1, \ldots, |Q^-_k|$.
  Let $\bar{\gamma}$ be the position of the first operation in Q^-_k such that $i_{\bar{\gamma}} \in \bar{R}^{\rightarrow}_v$ and let $\bar{\gamma} = |Q^-_k| + 1$ if $i_\ell \notin \mathcal{R}^{\rightarrow}_v$ for $\ell = 1, \ldots, |Q^-_k|$.
  Let $\gamma \in \{\underline{\gamma} + 1, \ldots, \bar{\gamma}\}$ be a random feasible position.
  InsertOp($\mathcal{O}, p, v, \gamma, k, f^-, Q^-, w^-, G^-_v, f', Q', w', G'$, $\mathcal{P}'$, $C'_{\max}$)

4.2 Greedy randomized adaptive procedure

The GRASP [22] consists of iteratively generating an initial solution and running a local search starting from the initial solution just generated. The initial solutions are generated as the best among the solutions generated by randomized versions of the constructive heuristics ECT and EST. In the EST constructive heuristic, the randomization is done as follows. First, the earliest time $r_{\min}$ at which an unscheduled operation can be initiated is calculated. Then we calculate the shortest and the longest processing times a and b associated with the operation/machine pairs that can start at $r_{\min}$. Given $\zeta \in (0, 1]$, a random pair is chosen from those whose processing time is between a and $a + \zeta(b - a)$. In the ECT constructive heuristic, the randomization consists of, from among the unscheduled operations, calculating the lowest and highest completion instants a and b and drawing an operation/machine pair from among those that would terminate between a and $a + \zeta(b - a)$. It is worth noting that the calculation of the effective processing time always takes into consideration the learning effect. The only parameter of the method is ζ. The GRASP strategy is described in Algorithm 9 and the randomized versions of the constructive heuristics EST and ECT are described in Algorithms 10 and 11, respectively. Algorithm 9 uses Algorithm 6 to perform the local search and Algorithms 10 and 11 use Algorithm 4 for the calculation of the critical path associated with the initial solution, which is a necessary input for the local search.

Algorithm 9: Given an instance represented by $\mathcal{O}$, $\mathcal{F}$, p, $\widehat{A}$, it applies the greedy randomized adaptive procedure to obtain an optimal solution approximation represented by $G^\star$, $f^\star$, $w^\star$, $Q^\star$ with a makespan of $C^\star_{\max}$. The procedure employs randomizations of the ECT and EST rules with parameter ζ.

Input: $\mathcal{O}$, $\mathcal{F}$, p, $\widehat{A}$, ζ
Output: $f^\star$, $w^\star$, $Q^\star$, $G^\star$, $C^\star_{\max}$
function GRASP($\mathcal{O}$, $\mathcal{F}$, p, $\widehat{A}$, ζ, $f^\star$, $w^\star$, $Q^\star$, $G^\star$, $C^\star_{\max}$)
  Initialize $C^\star_{\max} \leftarrow +\infty$
  while *the stopping criterion is not satisfied* **do**
    RandomizedECT($\mathcal{O}$, $\mathcal{F}$, p, $\widehat{A}$, ζ, f^1, w^1, Q^1, G^1, $\mathcal{P}^1$, $C^1_{\max}$)
    RandomizedEST($\mathcal{O}$, $\mathcal{F}$, p, $\widehat{A}$, ζ, f^2, w^2, Q^2, G^2, $\mathcal{P}^2$, $C^2_{\max}$)
    if $C^1_{\max} < C^2_{\max}$ **then**
      $f \leftarrow f^1$, $w \leftarrow w^1$, $Q \leftarrow Q^1$, $\mathcal{P} \leftarrow \mathcal{P}^1$, $G \leftarrow G^1$, and $C_{\max} \leftarrow C^1_{\max}$
    else
      $f \leftarrow f^2$, $w \leftarrow w^2$, $Q \leftarrow Q^2$, $\mathcal{P} \leftarrow \mathcal{P}^2$, $G \leftarrow G^2$, and $C_{\max} \leftarrow C^2_{\max}$
    LocalSearch($\mathcal{O}$, $\mathcal{F}$, p, G, f, w, Q, $\mathcal{P}$, $C_{\max}$, G', f', w', Q', $\mathcal{P}'$, $C'_{\max}$)
    if $C'_{\max} < C^\star_{\max}$ **then**
      $f^\star \leftarrow f'$, $w^\star \leftarrow w'$, $Q^\star \leftarrow Q'$, $G^\star \leftarrow G'$ and $C^\star_{\max} \leftarrow C'_{\max}$

Algorithm 10: Given an instance represented by $\mathcal{O}$, $\mathcal{F}$, p, $\widehat{A}$, it computes a feasible solution represented by G, f, w, Q, with critical path $\mathcal{P}$ a makespan of $C_{\max}$, using a randomization of the earliest starting time (EST) rule. The randomization depends on the parameter ζ.

Input: $\mathcal{O}$, $\mathcal{F}$, p, $\widehat{A}$, ζ
Output: f, w, Q, G, $\mathcal{P}$, $C_{\max}$

function RandomizedEST($\mathcal{O}$, $\mathcal{F}$, p, $\widehat{A}$, ζ, f, w, Q, G, $\mathcal{P}$, $C_{\max}$)
- Set $A \leftarrow \widehat{A} \cup \{(s,j) \mid (\cdot, j) \notin \widehat{A}\} \cup \{(i,t) \mid (i,\cdot) \notin \widehat{A}\}$ and define $V := \mathcal{O} \cup \{s,t\}$ and $G = (V, A)$.
- Set $r_v^{\text{op}} \leftarrow +\infty$ and define $r_s^{\text{op}} := 0$, $w_s := w_t := 0$, and $c_s := 0$.
- Set $r_k^{\text{mac}} \leftarrow 0$ and $g_k \leftarrow 1$ for all $k \in \mathcal{F}$.
- Initialize $\Pi \leftarrow V \setminus \{s,t\}$ as the set of non-scheduled operations, and Q_k as an empty list for all $k \in \mathcal{F}$.
- **while** $\Pi \neq \emptyset$ **do**
  - **for** $v \in \Pi$ **do**
    - **if** $\Pi \cap \{i \mid (i,v) \in A\} = \emptyset$ **then**
      - $r_v^{\text{op}} \leftarrow \max\{c_i \mid i \in V \setminus \Pi \text{ such that } (i,v) \in A\}$
  - Set $r_{\min} = \min_{v \in \Pi, k \in \mathcal{F}_v} \{\max(r_v^{\text{op}}, r_k^{\text{mac}})\}$ and let E be the set of pairs (v,k) with $v \in \Pi$ and $k \in \mathcal{F}_v$ such that $\max(r_v^{\text{op}}, r_k^{\text{mac}}) = r_{\min}$.
  - Compute $\underline{\psi} \leftarrow \min_{(v,k) \in E}\{\psi(p_{v,k}, g_k)\}$ and $\overline{\psi} \leftarrow \max_{(v,k) \in E}\{\psi(p_{v,k}, g_k)\}$, set $\mathcal{L} \leftarrow \{(v,k) \in E \mid \psi(p_{v,k}, g_k) \le \underline{\psi} + \zeta(\overline{\psi} - \underline{\psi})\}$ and choose $(\hat{v}, \hat{k}) \in \mathcal{L}$ randomly.
  - Define $w_{\hat{v}} := \psi(p_{\hat{v},\hat{k}}, g_{\hat{k}})$, $f_{\hat{v}} := \hat{k}$, $c_{\hat{v}} := c_{\hat{v},\hat{k}}$ and set $r_{\hat{k}}^{\text{mac}} \leftarrow c_{\hat{v}}$ and $g_{\hat{k}} \leftarrow g_{\hat{k}} + 1$.
  - **if** $|Q_{\hat{k}}| \neq 0$ **then**
    - Set $A \leftarrow A \cup \{(i_{|Q_{\hat{k}}|}, \hat{v})\}$, where $Q_{\hat{k}} = i_1, \dots, i_{|Q_{\hat{k}}|}$.
  - Insert $\hat{v}$ at the end of $Q_{\hat{k}}$ and set $\Pi \leftarrow \Pi \setminus \{\hat{v}\}$.
- CriticalPath($\mathcal{F}$, f, w, Q, G, $\mathcal{U}$, $\mathcal{P}$, $C_{\max}$, τ).

4.3 Tabu search

The TS [26, 27] considered consists basically of a local search with a modification in the acceptance criteria of a neighbor as a new solution. The acceptance depends on a list of tabu moves. The size $t_{\max}$ of this list is the only parameter of the method. In a given iteration, the best neighbor is computed, disregarding neighbors constructed with a tabu move unless they correspond to the best solution already constructed. The way of constructing the neighborhood follows exactly the same scheme as the local search described in Algorithm 6. The move that transforms the solution of the current iteration into the solution of the next iteration consists in relocating a certain operation v at the γ position of a machine $k \in \mathcal{F}_v$. We keep in the tabu list the operation/machine pair (v,k). The complete method is described in Algorithm 12.

Algorithm 11: Given an instance represented by $\mathcal{O}$, $\mathcal{F}$, p, $\widehat{A}$, it computes a feasible solution represented by G, f, w, Q, with critical path $\mathcal{P}$ a makespan of $C_{\max}$, using a randomization of the earliest completion time (ECT) rule. The randomization depends on the parameter ζ.

Input: $\mathcal{O}$, $\mathcal{F}$, p, $\widehat{A}$, ζ
Output: f, w, Q, G, $\mathcal{P}$, $C_{\max}$
function RandomizedECT($\mathcal{O}$, $\mathcal{F}$, p, $\widehat{A}$, ζ, f, w, Q, G, $\mathcal{P}$, $C_{\max}$)
  Set $A \leftarrow \widehat{A} \cup \{(s,j) \mid (\cdot,j) \notin \widehat{A}\} \cup \{(i,t) \mid (i,\cdot) \notin \widehat{A}\}$ and define $V := \mathcal{O} \cup \{s,t\}$ and $G = (V,A)$.
  Set $r_v^{\text{op}} \leftarrow +\infty$ and define $r_s^{\text{op}} := 0$, $w_s := w_t := 0$, and $c_s := 0$.
  Set $r_k^{\text{mac}} \leftarrow 0$ and $g_k \leftarrow 1$ for all $k \in \mathcal{F}$.
  Initialize $\Pi \leftarrow V \setminus \{s,t\}$ as the set of non-scheduled operations, and Q_k as an empty list for all $k \in \mathcal{F}$.
  while $\Pi \neq \emptyset$ **do**
    for $v \in \Pi$ **do**
      if $\Pi \cap \{i \mid (i,v) \in A\} = \emptyset$ **then**
        $r_v^{\text{op}} \leftarrow \max\{c_i \mid i \in V \setminus \Pi \text{ such that } (i,v) \in A\}$
    for $v \in \Pi$ **do**
      for $k \in \mathcal{F}_v$ **do**
        $c_{v,k} \leftarrow \max(r_v^{\text{op}}, r_k^{\text{mac}}) + \psi(p_{v,k}, g_k)$
    Set $c^{\min} \leftarrow \min\{c_{v,k} \mid v \in \Pi, k \in \mathcal{F}_v\}$, $c^{\max} \leftarrow \max\{c_{v,k} \mid v \in \Pi, k \in \mathcal{F}_v\}$ and RCL $\leftarrow \{(v,k) \mid c_{v,k} \leq c^{\min} + \zeta(c^{\max} - c^{\min}), v \in \Pi, k \in \mathcal{F}_v\}$.
    Choose $(\hat{v}, \hat{k})$ randomly in RCL.
    Define $w_{\hat{v}} := \psi(p_{\hat{v},\hat{k}}, g_{\hat{k}})$, $f_{\hat{v}} := \hat{k}$, $c_{\hat{v}} := c_{\hat{v},\hat{k}}$ and set $r_{\hat{k}}^{\text{mac}} \leftarrow c_{\hat{v}}$ and $g_{\hat{k}} \leftarrow g_{\hat{k}} + 1$.
    if $|Q_{\hat{k}}| \neq 0$ **then**
      Let $Q_{\hat{k}} = i_1, \ldots, i_{|Q_{\hat{k}}|}$. Set $A \leftarrow A \cup \{(i_{|Q_{\hat{k}}|}, \hat{v})\}$.
    Insert $\hat{v}$ at the end of $Q_{\hat{k}}$ and set $\Pi \leftarrow \Pi \setminus \{\hat{v}\}$.
  CriticalPath($\mathcal{F}$, f, w, Q, G, $\mathcal{U}$, $\mathcal{P}$, $C_{\max}$, τ).

4.4 Simulated annealing

In the SA [35], as in the other metaheuristics, the initial solution is given by the best solution among those provided by the constructive heuristics EST and ECT. The simulated annealing advances from one solution to another by making perturbations of the solution consisting of a random movement of removal and insertion (described in Algorithm 8). The main feature is that, for a new solution to be accepted, it need not be better than the current solution. On the other hand, the probability of accepting a solution that is worse than the current solution decreases as the method progresses, i.e. as the temperature decreases. The method has as parameters the initial and final temperatures T_0 and T_f. The temperature decreases as the method progresses being multiplied by a parameter $\delta \in (0,1)$. The amount of perturbations for a fixed temperature

Algorithm 12: Given an instance represented by $\mathcal{O}$, $\mathcal{F}$, p, $\widehat{A}$, it applies the tabu search strategy to obtain an optimal solution approximation represented by $G^\star$, $f^\star$, $w^\star$, $Q^\star$ with a makespan of $C^\star_{\max}$. Parameter $t_{\max}$ corresponds to the maximum size of the tabu list.

Input: $\mathcal{O}$, $\mathcal{F}$, p, $\widehat{A}$, $t_{\max}$
Output: $f^\star$, $w^\star$, $Q^\star$, $G^\star$, $C^\star_{\max}$

function TS($\mathcal{O}$, $\mathcal{F}$, p, $\widehat{A}$, $t_{\max}$, $f^\star$, $w^\star$, $Q^\star$, $G^\star$, $C^\star_{\max}$)
- ECT($\mathcal{O}$, $\mathcal{F}$, p, $\widehat{A}$, f^1, w^1, Q^1, G^1, $\mathcal{U}^1$, $\mathcal{P}^1$, $C^1_{\max}$, τ^1)
- EST($\mathcal{O}$, $\mathcal{F}$, p, $\widehat{A}$, f^2, w^2, Q^2, G^2, $\mathcal{U}^2$, $\mathcal{P}^2$, $C^2_{\max}$, τ^2)
- **if** $C^1_{\max} < C^2_{\max}$ **then**
  - $f^\star \leftarrow f^1$, $w^\star \leftarrow w^1$, $Q^\star \leftarrow Q^1$, $G^\star \leftarrow G^1$ and $C^\star_{\max} \leftarrow C^1_{\max}$
- **else**
  - $f^\star \leftarrow f^2$, $w^\star \leftarrow w^2$, $Q^\star \leftarrow Q^2$, $G^\star \leftarrow G^2$ and $C^\star_{\max} \leftarrow C^2_{\max}$
- Initialize $\mathcal{T}$ as an empty list.
- **while** *the stopping criterion is not satisfied* **do**
  - $C'_{\max} \leftarrow +\infty$
  - **for** $v \in \mathcal{O}$ **do**
    - RemoveOp($\mathcal{O}$, p, v, f, Q, w, G, f^-, Q^-, w^-, G^-_v, $\mathcal{P}^-$, ξ, $\mathcal{R}^{\leftarrow}_v$, $\mathcal{R}^{\rightarrow}_v$, τ)
    - **for** $k \in \mathcal{F}_v$ **do**
      - Let $\underline{\gamma}$ be the position of the last operation in $Q^-_k = i_1, \dots, i_{|Q^-_k|}$ such that $i_{\underline{\gamma}} \in \mathcal{R}^{\leftarrow}_v$ and let $\underline{\gamma} = 0$ if $i_\ell \notin \mathcal{R}^{\leftarrow}_v$ for $\ell = 1, \dots, |Q^-_k|$.
      - Let $\bar{\gamma}$ be the position of the first operation in $Q^-_k = i_1, \dots, i_{|Q^-_k|}$ such that $i_{\bar{\gamma}} \in \bar{R}^{\rightarrow}_v$ and let $\bar{\gamma} = |Q^-_k| + 1$ if $i_\ell \notin \mathcal{R}^{\rightarrow}_v$ for $\ell = 1, \dots, |Q^-_k|$.
      - **if** $\xi \geq C_{\max}$ **then**
        - $\bar{\gamma} \leftarrow \min\{\bar{\gamma}, \tau_k\}$, where τ_k is such that there is no critical operation after τ_k in Q^-_k ($\tau_k = 0$ if there is no critical operation in Q^-_k).
      - **for** $\gamma = \underline{\gamma} + 1, \dots, \bar{\gamma}$ **do**
        - InsertOp($\mathcal{O}$, p, v, γ, κ, f^-, Q^-, w^-, G^-_v, f^+, Q^+, w^+, G^+_v, $\mathcal{P}^+$, $C^+_{\max}$)
        - **if** ($C^+_{\max} < C'_{\max}$ and $(v, k) \notin \mathcal{T}$) or $C^+_{\max} < \min\{C'_{\max}, C^\star_{\max}\}$ **then**
          - $G' \leftarrow G^+_v$, $f' \leftarrow f^+$, $w' \leftarrow w^+$, $\mathcal{P}' \leftarrow \mathcal{P}^+$ and $C'_{\max} \leftarrow C^+_{\max}$
          - $v' \leftarrow v$, $k' \leftarrow k$
  - If $(v', k') \in \mathcal{T}$ then remove it from $\mathcal{T}$. Anyhow, insert (v', k') at the end of $\mathcal{T}$.
  - **if** $|\mathcal{T}| > t_{\max}$ **then**
    - Remove the first element of $\mathcal{T}$
  - $G \leftarrow G'_v$, $f \leftarrow f'$, $w \leftarrow w'$,$\mathcal{P} \leftarrow \mathcal{P}'$ and $C_{\max} \leftarrow C'_{\max}$
  - **if** $C_{\max} < C^\star_{\max}$ **then**
    - $G^\star \leftarrow G_v$, $f^\star \leftarrow f$, $w^\star \leftarrow w'$, $\mathcal{P}^\star \leftarrow \mathcal{P}$ and $C^\star_{\max} \leftarrow C_{\max}$

is given by a parameter $\bar{\ell} \in \mathbb{N}$, $\bar{\ell} \geq 1$. The method as a whole is described in Algorithm 14.

Algorithm 14: Given an instance represented by $\mathcal{O}$, $\mathcal{F}$, p, $\widehat{A}$, it applies the simulated annealing strategy to obtain an optimal solution approximation represented by $G^\star$, $f^\star$, $w^\star$, $Q^\star$ with a makespan of $C^\star_{\max}$. The parameters T_0 and T_f correspond to the initial and final temperatures, respectively, while the parameters ℓ and δ are related to how the temperature updates progress over iterations.

Input: $\mathcal{O}$, $\mathcal{F}$, p, $\widehat{A}$, $\bar{\ell}$, δ, T_0, T_f

Output: $f^\star$, $w^\star$, $Q^\star$, $G^\star$, $C^\star_{\max}$

function SA($\mathcal{O}$, $\mathcal{F}$, p, $\widehat{A}$, $\bar{\ell}$, δ, T_0, T_f, $f^\star$, $w^\star$, $Q^\star$, $G^\star$, $C^\star_{\max}$)

    ECT($\mathcal{O}$, $\mathcal{F}$, p, $\widehat{A}$, f^1, w^1, Q^1, G^1, $\mathcal{U}^1$, $\mathcal{P}^1$, $C^1_{\max}$, τ^1)

    EST($\mathcal{O}$, $\mathcal{F}$, p, $\widehat{A}$, f^2, w^2, Q^2, G^2, $\mathcal{U}^2$, $\mathcal{P}^2$, $C^2_{\max}$, τ^2)

    if $C^1_{\max} < C^2_{\max}$ **then**

        $f \leftarrow f^1$, $w \leftarrow w^1$, $Q \leftarrow Q^1$, $G \leftarrow G^1$ and $C_{\max} \leftarrow C^1_{\max}$

    else

        $f \leftarrow f^2$, $w \leftarrow w^2$, $Q \leftarrow Q^2$, $G \leftarrow G^2$ and $C_{\max} \leftarrow C^2_{\max}$

    $f^\star \leftarrow f$, $w^\star \leftarrow w$, $Q^\star \leftarrow Q$, $G^\star \leftarrow G$ and $C^\star_{\max} \leftarrow C_{\max}$

    Set $T \leftarrow T_0$.

    while *the stopping criterion is not satisfied* **do**

        for $\ell = 1, \dots, \bar{\ell}$ **do**

            Perturb($\mathcal{O}$, $\mathcal{F}$, p, f, w, Q, G, f', w', Q', G', $\mathcal{P}'$, $C'_{\max}$)

            Set $\Delta C \leftarrow (C'_{\max} - C_{\max})/C_{\max}$ and let $r \in [0,1]$ be a random number.

            if $e^{-\Delta C/T} \geq r$ **then**

                Set $f \leftarrow f'$, $w \leftarrow w'$, $Q \leftarrow Q'$, $G \leftarrow G'$ and $C_{\max} \leftarrow C'_{\max}$.

                if $C'_{\max} < C^\star_{\max}$ **then**

                    Set $f^\star \leftarrow f'$, $w^\star \leftarrow w'$, $Q^\star \leftarrow Q'$, $G^\star \leftarrow G'$ and $C^\star_{\max} \leftarrow C'_{\max}$.

        $T \leftarrow \max(\delta T, T_f)$

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5 Numerical experiments

In this section, we present numerical experiments with the proposed local search and the considered metaheuristics. In all experiments, we consider the 60 small-sized instances introduced in [4] and the 50 large-sized instances introduced in [8], with learning index $\alpha \in \{0.1, 0.2, 0.3\}$, totaling 330 instances. Small-sized instances have between 5 and 7 machines and between 9 and 24 operations. The operations are divided into up to 6 tasks, which correspond to connected components of the DAG of precedence relations, and the DAGs have up to 21 edges. The MILP models [4, p.7, Eq.(1–4)] of the small-sized instances have up to almost 1,000 binary variables and 12,000 constraints. Large-sized instances have between 5 and 26 machines and between 25 and 289 operations. The operations are divided into up to 17 tasks and the DAGs representing the precedence relations have up to 272 edges. Their MILP models have up to 73,000 binary variables and 3,800,000 constraints. Instance names always begin with Y or DA. Instances whose names start with Y correspond to instances with assembling operations only, and thus the DAGs representing the precedence of the operations are Y-shaped. By Y-shaped, we mean that nodes can have multiple incoming arcs but only one outgoing arc. Instances whose names start with DA correspond to instances with assembling and disassembling operations, and thus the DAGs representing the precedence of the operations are arbitrary DAGs. The former are called instances of type Y, and the latter are called instances of type DA. Curious readers can refer to Figures 1(b) and 1(a) in [8] for graphical representations of Y and DA-type instances, respectively. Instances of type DA have a higher degree of flexibility than instances of type Y, and thus generating a full neighborhood of a solution is more expensive. It is worth recalling that the considered problem has two types of flexibility: routing flexibility and nonlinear routes. The first refers to the fact that an operation can be processed by a machine within a set of machines instead of a single machine and corresponds to the "flexible" of the FJS. The second corresponds to the fact that the operations of the same task have their precedences represented by an arbitrary DAG instead of obeying a linear order. The sum of these two flexibilities causes the problem to have a large search space and makes it difficult to find a proven optimal solution even in instances that may initially appear to be simple. For a more detailed description of the instances and their characteristics, see [4].

The local search and the metaheuristics were implemented in C++ programming language. The code was compiled using g++ 10.2.1. To ensure full reproducibility of the results presented in the present work, as well as future comparisons, the instances, code and solutions found are available at `https://github.com/kennedy94/FJS`. The experiments were carried out in an Intel i9-12900K (12th Gen) processor operating at 5.200GHz and 128 GB of RAM.

5.1 Experiments with local search variations

In this section we evaluate six variations of the local search described in Algorithm 6, namely local searches that combine the first- and best-improvement strategies with the full, reduced, and cropped neighborhoods. In [4] the two constructive heuristics ECT and EST are introduced for the same problem being considered in this paper. The constructive heuristics are different and, while one constructs, in general, better solutions for instances of type Y, the other constructs better solutions for instances of type DA; see [4] for details. Either way, the two constructive heuristics take negligible time and, for this reason, in the present work we use, as the initial solution for the local search, the best among the solutions constructed by the two constructive heuristics ECT and EST.

The results of the six variations of the local search applied to the 330 considered instances are shown in Tables A1–A4 in the Appendix. The tables do not show anything related to the full neighborhood. What should be said about the use of the full neighborhood is that, in all instances, as expected, the solution obtained was identical to the solution obtained with the reduced neighborhood, the number of iterations was also the same, and the reduced neighborhood promoted a reduction of 52.51% in the CPU time.

When we compare the first-improvement and best-improvement strategies, the results are quite similar, but the best-improvement strategy always finds better quality solutions using less

CPU time. Specifically, the best improvement strategy returns solutions that are on average 1.02% and 0.70% better than the solutions returned by the first-improvement strategy, when we consider the reduced and cropped neighborhoods, respectively. Therefore, from now on, we focus on evaluating the reduced neighborhood and the cropped neighborhood associated with the best-improvement strategy.

On average, the local search using the cropped neighborhood performs 58.58% of the iterations performed by the local search using the reduced neighborhood. Local searches stop when they find a solution with no better neighbors. Therefore, it is natural that the search stops earlier when the neighborhood is smaller. Additionally, generating the cropped neighborhood costs, on average, 21.67% of what it costs to generate the reduced neighborhood. Combining these results, we find that the local search using the cropped neighborhood should take an average of 12.69% of the time used by the local search using the reduced neighborhood. Subject to measurement errors, the measured CPU times show that this number is, in practice, 13.87%. All in all, the local search using the cropped neighborhood takes about one-tenth the CPU time of the local search using the reduced neighborhood. However, adopting the cropped neighborhood can lead to a loss of quality in the final solution obtained by the local search method. On average, when compared to the local search with the reduced neighborhood, the local search with the cropped neighborhood finds solution with a makespan 0.69% worse. When we compare the final solution with the initial solution, the local search using the reduced neighborhood improves the initial solution by, on average, 6.88%, while the local search using the cropped neighborhood improves the initial solution by 6.11%. In conclusion, the local search with the cropped neighborhood is significantly faster than the local search with the reduced neighborhood and finds solutions that are only slightly worse than the solutions found by the latter.

5.2 Experiments with the metaheuristics in the large-sized instances

In this section we report experiments to evaluate the behavior of the metaheuristics on the 50 large-sized instances, totaling 150 instances as we vary $\alpha \in \{0.1, 0.2, 0.3\}$. The ILS and GRASP metaheuristics (Algorithms 7 and 9, respectively) make use of the local search (Algorithm 6). We analyzed these two metaheuristics in connection with the local search using the reduced neighborhood (RN) and in connection with the local search using the cropped neighborhood (CN). We call these versions of ILS-RN, ILS-CN, GRASP-RN and GRASP-CN, respectively, hereafter. The tabu search metaheuristic was already conceived using the reduced neighborhood (see lines 16 and 17 of Algorithm 12), and applying to it the concept of cropped neighborhood corresponds simply to changing in line 11 of Algorithm 12, $v \in \mathcal{O}$ by $v \in \mathcal{P}$. Hereafter we refer to these two versions as TS-RN and TS-CN, respectively. The simulated annealing metaheuristic does not use local search and therefore we considered only a single version of it, which we denote SA hereafter. In all metaheuristics we used the total CPU time as a stopping criterion and considered a maximum limit of 5 minutes. Additionally, and only during the parameters calibration phase, we placed an additional stopping criterion, also based on CPU time, which is satisfied if 5 seconds elapse without improvement of the incumbent solution.

The ILS metaheuristic has as parameters $\ell^p_{\min}$ and $\ell^p_{\max} \in \mathbb{N}$ which must satisfy $1 \leq \ell^p_{\min} \leq \ell^p_{\max}$. In the calibration experiments, we evaluated $\ell^p_{\min}, \ell^p_{\max} \in \{1, 2, 3, 4, 5\}$. The GRASP metaheuristic has as a single parameter $\zeta \in \mathbb{R}$ which must satisfy $0 \leq \zeta \leq 1$. In the calibration experiments we evaluated $\zeta \in \{0.10, 0.11, \ldots, 0.99\}$. The TS metaheuristic has as a single parameter $t_{\max} \in \mathbb{N}$. In the calibration experiments, we evaluated $t_{\max} = \lceil (|\mathcal{O}| + |\mathcal{F}|) \times t \rceil$ with $t \in \{0.1, 0.2, \ldots, 5.0\}$. In the SA metaheuristic, the parameters are $\bar{\ell} \in \mathbb{N}$ and $T_0, T_f, \delta \in \mathbb{R}$ and must satisfy $1 \leq \bar{\ell}$, $T_0 \geq T_f > 0$ and $\delta \in (0, 1)$. In the calibration experiments, we evaluated $\bar{\ell} \in \{1{,}000, 2{,}000, \ldots, 5{,}000\}$, $T_0 = -T_0^m / \ln(T_0^p)$ with $T_0^m \in \{0.10, 0.11, \ldots, 0.99\}$ and $T_0^p \in \{0.10, 0.11, \ldots, 0.99\}$, $T_f \in \{10^{-1}, 10^{-3}, 10^{-5}\}$, and $\delta \in \{0.80, 0.81, \ldots, 0.99\}$. (See [31] for details on how to choose the initial temperature in SA). We calibrated the seven methods independently using the irace package [41]. In irace all default parameters were considered, except for maxExperiments = 10,000. We considered in the calibration only the 50 large-sized instances with learning index $\alpha = 0.2$. We separated these instances into two sets of 25 instances each, one for training and the other for testing. As a result, irace returned the following parameters: ILS-RN: $\ell^p_{\min} = 2$ and $\ell^p_{\max} = 4$, ILS-CN: $\ell^p_{\min} = 1$ and $\ell^p_{\max} = 3$, GRASP-RN: $\zeta = 0.38$, GRASP-CN: $\zeta = 0.59$, TS-RN: $t_{\max} = \lceil (|\mathcal{O} + |\mathcal{F}|) \times t \rceil$ with $t = 0.9$, TS-CN: $t_{\max} = \lceil (|\mathcal{O} + |\mathcal{F}|) \times t \rceil$ with $t = 0.5$, SA: $\bar{\ell} = 3$, $T_0^m = 0.79$, $T_0^p = 0.78$, $T_f = 10^{-3}$, and $\delta = 0.82$.

With the selected parameters, we ran the metaheuristics on the 50 large instances by varying the learning index $\alpha \in \{0.1, 0.2, 0.3\}$. Since methods ILS-RN, ILS-CN, GRASP-RN, GRASP-CN, and SA have random components, these methods were applied 5 times to each instance. The same does not apply for methods TS-RN and TS-CN, which were run only once for each instance. Tables A5, A6, and A7 in the Appendix show the results associated with the learning index $\alpha \in \{0.1, 0.2, 0.3\}$, respectively. Consolidation statistics are shown at the top of Table 6. The line named $\bar{C}_{\max}$ shows the average of the makespans found by each method (for each instance, the average considers the best over the five runs, when applicable). The line named $\overline{CV}$ shows the average of the coefficients of variation of the makespan found in the five runs of the same instance. The line named #best shows, for each method, the number of instances in which the method found the best solution among the solutions found by the seven methods being considered. The line named gap(%), shows, for each method, the average gap when, for each instance, the solution found is compared with the best solution found by the seven methods. If the seven methods applied to the same instance found makespans $c_1, \ldots, c_7$, the gap of method i in the considered instance corresponds to $100\% \times (c_i - c_{\min})/c_{\min}$, where $c_{\min} = \min\{c_1, \ldots, c_7\}$. The gap reported in line gap(%) for each method corresponds to the average over all instances in the table. In the previous sentence, if a method i is run five times per instance, the value c_i make reference to the best over the five runs. So the value reported in line gap(%) does not refer to what happened in "the other four runs". In these other cases, does each method find something close to its best makespan or something much worse? To answer this question, let us now consider that the values $\bar{c}_1, \ldots, \bar{c}_7$ correspond to the average of the makespans found in the five runs. The line $\overline{\text{gap}}(\%)$ shows the same as line gap(%) but considering $\bar{c}_1, \ldots, \bar{c}_7$ instead of $c_1, \ldots, c_7$. Both lines coincide for the case of TS-RN and TS-CN which have no random components and are therefore only executed once per instance. Thus, when TS-RN or TS-CN is compared with other method, it is the value of $\overline{\text{gap}}(\%)$ that must be compared. Otherwise, we would be comparing a method that is run only once against a method that is run five times and keeps the best solution found only.

Figure 5 shows, for each method, the average value of $\bar{C}_{\max}$ as a function of the CPU time. As mentioned above, the considered average is the average of the best makespan over the five runs for each method/instance pair. Using this value in the comparison is harmful for the TS-RN

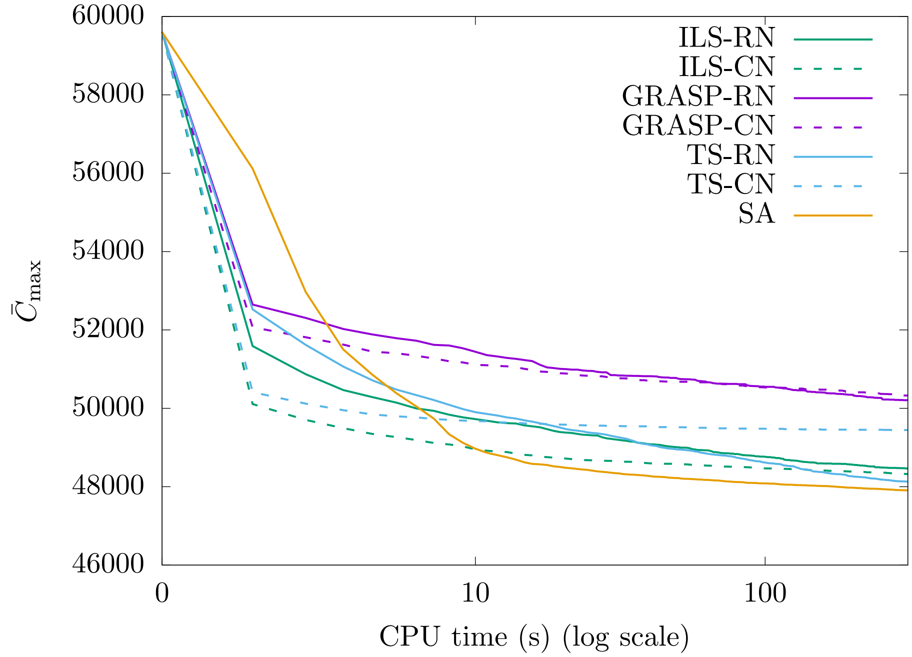

Figure 5: This figure shows the value of $\bar{C}_{\max}$ as a function of the CPU time for each of the seven methods applied to the large-sized instances. The average makespan shown in the graphic is the average of the different values shown in Tables A5, A6, and A7, that separate the instances by DA type, Y type and learning index $\alpha \in \{0.1, 0.2, 0.3\}$.

and TS-CN methods. Note that if, we consider five minutes of CPU time, the TN-RN method is the second best, even using this unfair measure in the comparison. Figure 6 shows the same thing except that, for methods that solve each instance five times, instead of considering the best of the five makespan, we consider the average of the five makespan. With this measure, and considering that we have five minutes of CPU time available, the TS-RN method appears as the best. The ranking of all the other methods, considering the five-minutes CPU time budget, remains the same.

Figures 5 and 6 show that the ranking of the methods in relation to the average quality of the solutions found is affected by the time limit used in the stopping criterion, since shorter times would yield a different ranking. In general, Figures 5 and 6 show that (a) with more time available, metaheuristics that use RN find slightly better solutions on average, and (b) with little time available, using CN is preferable, as it promotes a faster decrease in the incumbent. Moreover, the figures also show that longer times would have little effect on the comparison. (Note that the time axis in the figures is in logarithmic scale and that the methods almost stabilize at five minutes).

Figures 5 and 6 and Table 6 clearly show that GRASP-RN, GRASP-CN and TS-CN were the worst performing methods. There seems to be an explanation for that: lack of diversification. In the case of TS-CN, the difference in efficacy is remarkable when compared to TS-RN. It is worth noting that the cropped neighborhood is radically smaller than the reduced neighborhood

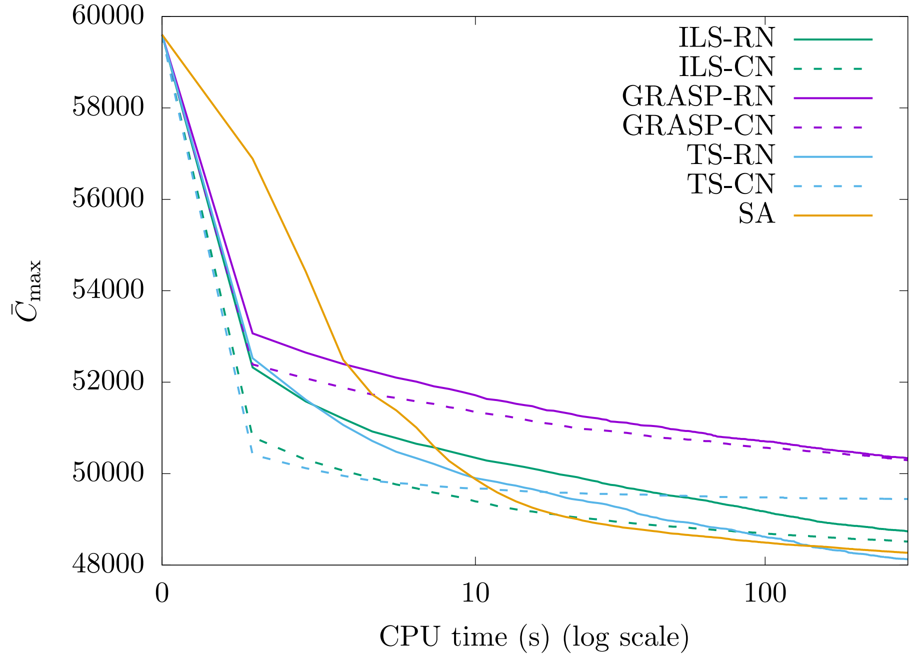

Figure 6: This figure is similar to Figure 5. The difference is that, for methods that solve each instance five times, instead of considering the best of the five makespan, we consider the average of the five makespan. This measure is more fair when in the comparison there are methods that do not posses a random ingredient and, therefore, are run only one time per instance.

and this fact limits the ability of the tabu search to escape from local solutions. Something similar occurs in the two versions of GRASP, in which local searches are initiated from solutions constructed by a randomized version of the constructive heuristics ECT and EST. Apparently, the randomization of these constructive heuristics was not able to generate sufficiently diverse solutions and repeated runs of the local search ended up repeatedly converging to the same local solutions. On the other side of the spectrum, TS-RN and SA performed outstandingly well, followed closely by the two versions of ILS. The fact that SA (see Figures 5 and 6) takes a little longer to get good quality solutions is entirely expected, since it is the method that less explores the neighborhood of the current solution in the search for a new solution. Regardless of anything else, it is important to note that all metaheuristics substantially improved the solutions delivered by the constructive heuristics and found by the local search. The constructive heuristics EST and ECT deliver solutions that are, on average, 32.36% and 33.01% worse than the best solutions found, respectively. The local search with the reduced neighborhood, which considered as a stand-alone method finds solutions of better quality than the local search with the cropped neighborhood, provides solutions that are on average 16.18% worse than the best solutions found. On the other hand, all the metaheuristics deliver solutions that are, on average, at no more than 6% of the best solutions found. If we discard the three worst performing methods, this number drops to approximately 2%. The highlight is for TS-RN and SA, which deliver solutions, on average, at approximately 1% and 1.5% of the best solutions found. The two versions of ILS deliver solutions that at approximately 2% of the best solutions found, in average. SA also stands out for the number of best solutions obtained, 71 out of a total of 150, followed closely by ILS-RN and ILS-CN, with 59 and 58, respectively. TS-RN finds 55 best solutions. On the one hand, TS-RN has the advantage of running only once for each instance. On the other hand, the other metaheuristics are run five times and the best of the five is considered in this statistic. However, the coefficients of variation are small on average. This indicates that running one of the metaheuristics with a random component only once would have a very small degradation in its performance.

In order to strengthen the comparison between the methods, we applied the Wilcoxon [47] test for each pair of methods, with a significance level of $\boldsymbol{\alpha} = 0.05$, to accept or reject the null hypothesis "the samples of the two methods come from the same distribution" or, equivalently, "the difference between the samples of the two methods follows a symmetrical distribution around zero". In this test we left out the GRASP-RN and GRASP-CN methods, which clearly performed worse than all the others. Table 1 shows the results. Suppose that two methods M_1 and M_2 are compared and that these methods, when applied to a set of N instances, find solutions with makespan c_{1i} and c_{2i}, respectively, for $i = 1, \dots, N$. Instances i with $c_{2i} = c_{1i}$ are eliminated from the test. For simplicity, let us assume that N already represents the number of instances in which the methods found different makespans. The test assigns each instance i a weight R_i which corresponds to the position of the instance when they are ordered from lowest to highest by the value of $|c_{2i} - c_{1i}|$. In the case of ties, R_i corresponds to the average of the positions of the tied values. With the values of R_i for each instance, we calculate the statistic

$$W = R^+ - R^-, \text{ where } R^+ = \sum_{\{i \mid c_{2i} > c_{1i}\}} R_i \text{ and } R^- = \sum_{\{i \mid c_{1i} > c_{2i}\}} R_i.$$

Under the null hypothesis and for large values of N, the value of W is divided by its variance $N(N+1)(2N+1)/6$ to obtain z. In a two-tailed test, the null hypothesis is rejected if $|z| > z_{\text{critical}}(\boldsymbol{\alpha}/2)$ or, equivalently, if $p < \boldsymbol{\alpha}$. The table shows, for each pair of methods compared, the values of R^+, R^-, and the p-value of z. The comparison considering a CPU time limit of 10 seconds shows that ILS-RN is worse than the other four methods compared and that the other four methods are equivalent to each other. This coincides with Figure 6, which uses the average makespan as a measure of comparison. The comparison considering a CPU time limit of 5 minutes shows that all the methods are different. Furthermore, the two-by-two comparison makes it possible to construct a ranking between the methods in which TS-RN is the best of all, followed by SA, ILS-CN, ILS-RN and TS-CN in that order. This ranking also coincides with that shown on the right-hand side of Figure 6, which uses the average makespan as a criterion.

We close this section by analyzing the behavior of the seven methods in the seven large-sized instances for which proved optimal solutions were reported in [4]. Table 2 shows the results. The table shows the instance name and instance learning index for the seven instances for which the optimal value $C^\star_{\max}$ is known. Then, for each method, it shows, as in the previous tables, when applicable, the best makespan found over the five runs in each of these instances. It can be seen that ILS-RN, ILS-CN and SA found the optimal solutions in all the seven instances, followed by TS-RN that found six out of the seven optimal solutions. Independently of finding the optimal solutions, appart from TS-CN, the methods found solutions with an average gap to the optimal solution smaller than 1%. It must be highlighted that ILS-RN and ILS-CN found

comparison	10 seconds of CPU time limit			5 minutes of CPU time limit		
	R^+	R^-	p-value	R^+	R^-	p-value
ILS-RN versus ILS-CN	804	7,974	0.000	1,402	5,385	0.000
ILS-RN versus TS-RN	2,544	6,636	0.000	1,692	6,437	0.000
ILS-RN versus TS-CN	3,844	6,887	0.003	6,311	4,274	0.044
ILS-RN versus SA	3,579	7,006	0.001	3,221	7,075	0.000
ILS-CN versus TS-RN	5,036	4,009	0.254	1,996	5,879	0.000
ILS-CN versus TS-CN	5,296	5,289	0.994	6,777	3,808	0.003
ILS-CN versus SA	5,624	4,961	0.513	4,102	6,051	0.047
TS-RN versus TS-CN	5,015	5,570	0.584	9,533	1,052	0.000
TS-RN versus SA	5,623	4,962	0.514	7,142	3,443	0.000
TS-CN versus SA	6,452	4,724	0.102	1,939	9,236	0.000

Table 1: Details of the Wilcoxon test comparing each pair of methods, when applied to the large-sized instances, to accept or reject the null hypothesis "the difference between the two methods follows a symmetrical distribution around zero".

the optimal solutions in all the five runs of each instance. An evidence of this is the line "$\overline{\text{gap}}$ to optimal" that exhibits a 0.0% gap.

instance	α	$C^\star_{\max}$	ILS-RN	ILS-CN	GRASP-RN	GRASP-CN	TS-RN	TS-CN	SA
YFJS03	0.1	32,538	32,538	32,538	32,538	32,538	32,538	33,174	32,538
YFJS04	0.1	35,883	35,883	35,883	36,946	35,883	35,883	37,340	35,883
YFJS08	0.1	32,573	32,573	32,573	32,623	32,655	32,573	34,452	32,573
YFJS03	0.2	30,073	30,073	30,073	30,073	30,073	30,073	30,073	30,073
YFJS08	0.2	30,184	30,184	30,184	30,267	30,384	30,267	32,595	30,184
YFJS03	0.3	27,686	27,686	27,686	27,686	27,686	27,686	28,772	27,686
YFJS08	0.3	28,192	28,192	28,192	28,597	28,355	28,192	28,638	28,192
#optimal			7	7	3	4	6	1	7
gap to optimal (%)			0.00	0.00	0.69	0.21	0.04	3.61	0.00
$\overline{\text{gap}}$ to optimal (%)			0.00	0.00	0.73	0.31	0.04	3.61	0.24

Table 2: Performance of the metaheuristics in the seven large-sized instances for which proved optimal solutions are known.

5.3 Experiments with the metaheuristics in the small-sized instances

In this section we analyze the behavior of the metaheuristics on the small-sized instances. It is important to note that for all metaheuristics we considered their parameters exactly as defined in the previous section, i.e., with the calibration done for the large-sized instances. Tables A8, A9, and A10 in the Appendix show the results for the instances with learning index $\alpha \in \{0.1, 0.2, 0.3\}$, respectively. As in the case of the large-sized instances, a summary consolidating the data of Tables A8, A9, and A10 can be seen in the middle of Table 6. Unlike in the large-sized instances, the ILS-RN, ILS-CN, GRASP-RN, GRASP-CN, GRASP-CN, TS-RN and SA methods stand out in terms of performance, while only TS-CN underperforms. Of particular interest is the comparison of the performance of the metaheuristics in the 173 small-sized instances, out of a total of 180, for which optimal solutions were reported in [4]. Table 3 shows this comparison. In the table, for each method, we show the number of instances in which the method found an optimal solution (#optimal). We also show the average gap in relation to the optimal solution when, for each method and instance, we consider the best solution of those found in the five runs of that method/instance pair (line "gap to optimal"). Finally, in another line of the table we show the gap in relation to the optimal solution when, for a method/instance pair, we consider the average makespan of the five runs of that pair (line "$\overline{\text{gap}}$ to optimal"). The numbers in the table show that ILS-RN and ILS-CN found optimal solutions in *all* 173 instances and, moreover, in all five runs of each method in each instance. Leaving aside TS-CN, the other methods also perform well, finding a large number of optimal solutions and with average gaps relative to the optimal solution of less than 1%.

			ILS-RN	ILS-CN	GRASP-RN	GRASP-CN	TS-RN	TS-CN	SA
$\alpha = 0.1$	DA-type	#optimal	30	30	28	29	22	7	27
		gap to optimal (%)	0.00	0.00	0.14	0.13	0.95	2.61	0.10
		$\overline{\text{gap}}$ to optimal (%)	0.00	0.00	0.18	0.17	0.95	2.61	0.35
	Y-type	#optimal	30	30	28	29	25	10	29
		gap to optimal (%)	0.00	0.00	0.03	0.02	0.67	2.92	0.02
		$\overline{\text{gap}}$ to optimal (%)	0.00	0.00	0.39	0.06	0.67	2.92	0.18
$\alpha = 0.2$	DA-type	#optimal	28	28	26	27	20	6	26
		gap to optimal (%)	0.00	0.00	0.04	0.03	1.13	2.63	0.02
		$\overline{\text{gap}}$ to optimal (%)	0.00	0.00	0.13	0.16	1.13	2.63	0.31
	Y-type	#optimal	30	30	29	28	24	7	30
		gap to optimal (%)	0.00	0.00	0.15	0.05	0.54	3.36	0.00
		$\overline{\text{gap}}$ to optimal (%)	0.00	0.00	0.43	0.17	0.54	3.36	0.24
$\alpha = 0.3$	DA-type	#optimal	27	27	27	27	19	5	26
		gap to optimal (%)	0.00	0.00	0.00	0.00	0.81	2.99	0.03
		$\overline{\text{gap}}$ to optimal (%)	0.00	0.00	0.13	0.11	0.81	2.99	0.48
	Y-type	#optimal	28	28	25	28	25	4	28
		gap to optimal (%)	0.00	0.00	0.11	0.00	0.37	3.58	0.00
		$\overline{\text{gap}}$ to optimal (%)	0.00	0.00	0.29	0.14	0.37	3.58	0.13
Summary		#optimal	173	173	163	168	135	39	166
		gap to optimal (%)	0.00	0.00	0.08	0.04	0.74	3.01	0.03
		$\overline{\text{gap}}$ to optimal (%)	0.00	0.00	0.26	0.13	0.74	3.01	0.28

Table 3: Behavior of the seven metaheuristics on the 173 small-sized instances for which a proven optimal solution is known.

5.4 Experiments with classical instances with linear routes

To the authors' knowledge, no other method has been developed so far that applies to the exact problem considered in this paper. For this reason, in order to be able to relate the developed methods with those existing in the literature, we decided to apply them to a problem that is a particular case of the problem considered. We are referring to the FJS with position-based learning effect but with linear routes only. A method that also applies to this problem and minimizes the makespan was developed in [45]. The developed method is a hybrid metaheuristic that mixes genetic algorithms with variable neighborhood search with affinity function, thus named GVNSWAF. This method was chosen because, its careful and detailed description allowed us to reimplement it in C++, the same language in which the introduced methods were implemented. In [45], for lack of a better option, random instances were considered, and the Taguchi's robust design method was used to determine the best method's parameters. In our experiments, we considered the parameters determined in [45]. In that respect, the comparison is fair, since neither method was specifically calibrated for the instances that were considered in this section. Our implementation of GVNSWAF is also available for download at `https://github.com/kennedy94/FJS`.

In the experiments of this section, we consider 35 instances of the classic FJS from the literature [13, 21], with linear routes only, to which we added the position-based learning effect. As we considered learning index $\alpha \in \{0.1, 0.2, 0.3\}$, we have in total 105 instances. On this set of instances, we compared the methods introduced in the present work with the method introduced in [45], which we denote GVNSWAF hereafter. Since the method has random components, it was run five times on each instance, as well as the methods introduced in the present work that contain random components as well. Tables A11, A12, and A13 in the Appendix show the results. A summary of the results from these tables can be seen at the bottom of Table 6. Table 6 shows that the methods that stand out are ILS-RN and ILS-CN. It also shows that the ILS-RN, ILS-CN and SA methods have superior performance to the GVNSWAF method. The GVNSWAF method has a performance similar to the performance of GRASP-RN, GRASP-CN and TS-RN methods and only outperforms the TS-CN methods which already showed the worst performance among all the considered methods. The Wilcoxon test, details of which are shown in the Table 4 shows that, with a CPU time limit of 10 seconds, the performance of GVNSWAF is equal to the performance of TS-CN, and GVNSWAF is outperformed by all the other six methods. With the CPU time limit of 5 minutes, GVNSWAF outperforms only TS-CN and is equal to GRASP-RN, but is outperformed by all the other five methods.

As a final experiment, we tried to solve with an exact commercial solver models of these classical instances to obtain proven optimal solutions. We were able to obtain proven optimal solutions for 47 instances, out of the total of 105 instances. Table 5 compares the solutions obtained by the eight methods in relation to the known optima on that set of 47 instances. Again, the ILS-RN and ILS-CN methods found all optimal solutions in all five runs of each method/instance pair. The GVNSWAF method also achieved the same result. Apart from TS-CN, all others found solutions with average gap less than 1%.

In addition to the tests described above, we tested 168 additional instances of the classical FJS from the literature [7, 18, 29] which, considering $\alpha \in \{0.1, 0.2, 0.3\}$, amounted to 504. The results essentially confirmed what we have already shown. For more details, see [3].

comparison	10 seconds of CPU time limit			5 minutes of CPU time limit		
	R^+	R^-	p-value	R^+	R^-	p-value
ILS-RN versus GVNSWAF	2,262	153	0.000	2,004	12	0.000
ILS-CN versus GVNSWAF	2,415	0	0.000	2,016	0	0.000
GRASP-RN versus GVNSWAF	1,792	623	0.000	1,016	1,064	0.872
GRASP-CN versus GVNSWAF	2,193	222	0.000	1,327	689	0.029
TS-RN versus GVNSWAF	2,132	718	0.000	1,628	787	0.012
TS-CN versus GVNSWAF	1,991	1,412	0.181	1,081	2,079	0.015
SA versus GVNSWAF	2,187	228	0.000	1,953	532	0.000

Table 4: Details of the Wilcoxon test comparing each introduced method versus GVNSWAF, when applied to classical instances of the FJS with learning effect but with linear routes only, to accept or reject the null hypothesis "the difference between the two methods follows a symmetrical distribution around zero".

6 Conclusions

In this paper we introduced a local search and four trajectory metaheuristics for the flexible job shop scheduling problem with nonlinear routes and position-based learning effect. For the local search, we showed that, in the presence of learning effect, the classical approach of considering only reallocations of operations in the critical path fails to consider potentially better neighbors than the current solution. Consequently, we proposed a new neighborhood reduction that does not eliminate potentially better neighbors and cuts the neighborhood by approximately 50%. We further proposed a neighborhood cutoff that reduces the neighborhood size significantly (by about an order of magnitude) and finds solutions that are at most 1% worse. The introduced local searches and/or neighborhoods were used in the development of four trajectory metaheuristics. We performed extensive numerical experiments and showed that variants of ILS, TS, and SA stands out for its effectiveness and efficiency. Furthermore, in connection with the metaheuristics, we noted that the cropped neighborhood promotes a rapid decay of the incumbent, making it the best option when time is limited. As a whole, we build a test suite that can be used in the development of future work. The methods introduced and the solutions found are freely available.

As future work, we intend to consider different learning effects, which do not depend only on the position of the operation in the machine to which it was attributed. We also intend to consider objective functions that take into account the energy consumption (green scheduling).

Conflict of interest statement: On behalf of all authors, the corresponding author states that there is no conflict of interest.

Author contributions: All authors contributed equally to all phases of the development of this work.

Data availability: The datasets generated during and/or analysed during the current study are available in the GitHub repository, `https://github.com/kennedy94/FJS`.

Instance	α	$C^{\star}_{\max}$	ILS-RN	ILS-CN	GRASP-RN	GRASP-CN	TS-RN	TS-CN	SA	GVNSWAF
mfjs01	0.10	45,306	45,306	45,306	45,306	45,306	46,264	46,264	45,306	45,306
mfjs02	0.10	42,986	42,986	42,986	42,986	42,986	42,986	42,986	42,986	42,986
mfjs03	0.10	45,331	45,331	45,331	45,331	45,331	45,331	45,331	45,331	45,331
mfjs04	0.10	52,012	52,012	52,012	52,480	52,012	52,012	54,075	52,630	52,012
mfjs05	0.10	47,630	47,630	47,630	47,630	47,630	47,630	47,630	49,988	47,630
mfjs06	0.10	59,523	59,523	59,523	59,523	59,523	59,523	60,854	60,402	59,523
sfjs01	0.10	6,459	6,459	6,459	6,459	6,459	6,459	6,459	6,459	6,459
sfjs02	0.10	10,271	10,271	10,271	10,271	10,271	10,271	10,271	10,271	10,271
sfjs03	0.10	20,623	20,623	20,623	20,623	20,623	20,623	21,716	20,623	20,623
sfjs04	0.10	33,429	33,429	33,429	33,429	33,429	33,429	34,483	33,429	33,429
sfjs05	0.10	11,006	11,006	11,006	11,006	11,006	11,006	12,107	11,006	11,006
sfjs06	0.10	29,926	29,926	29,926	29,926	29,926	31,835	32,057	29,926	29,926
sfjs07	0.10	37,824	37,824	37,824	37,824	37,824	37,824	37,824	37,824	37,824
sfjs08	0.10	23,842	23,842	23,842	23,842	23,842	23,842	23,842	23,842	23,842
sfjs09	0.10	19,406	19,406	19,406	19,406	19,406	19,406	19,406	19,406	19,406
sfjs10	0.10	49,368	49,368	49,368	49,368	49,368	49,368	49,368	49,368	49,368
#optimal			16	16	15	16	14	9	13	16
gap to optimal (%)			0.00	0.00	0.06	0.00	0.53	2.12	0.48	0.00
$\overline{\text{gap}}$ to optimal (%)			0.00	0.00	0.32	0.18	0.53	2.12	0.69	0.00

Instance	α	$C^{\star}_{\max}$	ILS-RN	ILS-CN	GRASP-RN	GRASP-CN	TS-RN	TS-CN	SA	GVNSWAF
mfjs01	0.20	43,208	43,208	43,208	43,208	43,208	43,208	44,880	43,208	43,208
mfjs02	0.20	41,273	41,273	41,273	41,273	41,273	41,273	45,208	41,273	41,273
mfjs03	0.20	43,412	43,412	43,412	43,412	43,412	43,412	43,412	43,412	43,412
mfjs04	0.20	47,717	47,717	47,717	49,024	49,024	47,717	47,717	49,921	47,717
mfjs05	0.20	45,670	45,670	45,670	45,670	45,670	45,670	45,670	45,670	45,670
mfjs06	0.20	56,743	56,743	56,743	56,743	57,005	57,093	57,889	57,055	56,743
sfjs01	0.20	6,328	6,328	6,328	6,328	6,328	6,328	6,328	6,328	6,328
sfjs02	0.20	9,872	9,872	9,872	9,872	9,872	9,872	9,872	9,872	9,872
sfjs03	0.20	19,281	19,281	19,281	19,281	19,281	19,281	20,027	19,281	19,281
sfjs04	0.20	31,553	31,553	31,553	31,553	31,553	32,472	32,472	31,553	31,553
sfjs05	0.20	10,198	10,198	10,198	10,198	10,198	10,198	11,209	10,198	10,198
sfjs06	0.20	28,024	28,024	28,024	28,024	28,024	28,024	28,024	28,024	28,024
sfjs07	0.20	36,075	36,075	36,075	36,075	36,075	36,075	36,075	36,075	36,075
sfjs08	0.20	22,515	22,515	22,515	22,515	22,515	22,515	22,515	22,515	22,515
sfjs09	0.20	17,552	17,552	17,552	17,552	17,552	17,552	17,552	17,552	17,552
sfjs10	0.20	47,323	47,323	47,323	47,323	47,323	47,323	47,323	47,323	47,323
#optimal			16	16	15	14	14	10	14	16
gap to optimal (%)			0.00	0.00	0.17	0.20	0.22	2.01	0.32	0.00
$\overline{\text{gap}}$ to optimal (%)			0.00	0.00	0.28	0.21	0.22	2.01	0.74	0.00

Instance	α	$C^{\star}_{\max}$	ILS-RN	ILS-CN	GRASP-RN	GRASP-CN	TS-RN	TS-CN	SA	GVNSWAF
mfjs01	0.30	40,508	40,508	40,508	40,508	40,508	42,562	41,785	40,508	40,508
mfjs02	0.30	38,996	38,996	38,996	38,996	38,996	38,996	39,834	38,996	38,996
mfjs03	0.30	41,318	41,318	41,318	41,318	41,318	41,318	44,254	41,318	41,318
mfjs04	0.30	44,869	44,869	44,869	44,869	46,048	46,048	46,558	46,048	44,869
mfjs05	0.30	44,376	44,376	44,376	44,376	44,738	44,376	44,738	44,376	44,376
sfjs01	0.30	6,206	6,206	6,206	6,206	6,206	6,206	6,206	6,206	6,206
sfjs02	0.30	9,498	9,498	9,498	9,498	9,498	9,498	9,498	9,498	9,498
sfjs03	0.30	18,062	18,062	18,062	18,062	18,062	18,062	18,062	18,062	18,062
sfjs04	0.30	29,852	29,852	29,852	29,852	29,852	30,648	30,648	29,852	29,852
sfjs05	0.30	9,465	9,465	9,465	9,465	9,465	9,465	10,288	9,465	9,465
sfjs06	0.30	26,281	26,281	26,281	26,281	26,281	26,281	26,281	26,281	26,281
sfjs07	0.30	34,443	34,443	34,443	34,443	34,443	34,443	34,443	34,443	34,443
sfjs08	0.30	21,309	21,309	21,309	21,309	21,309	21,715	21,309	21,309	21,309
sfjs09	0.30	15,973	15,973	15,973	15,973	15,973	15,973	15,973	15,973	15,973
sfjs10	0.30	45,450	45,450	45,450	45,450	45,450	45,450	45,450	45,450	45,450
#optimal			15	15	15	13	11	8	14	15
gap to optimal (%)			0.00	0.00	0.00	0.23	0.82	1.89	0.18	0.00
$\overline{\text{gap}}$ to optimal (%)			0.00	0.00	0.18	0.26	0.82	1.89	0.33	0.00

#optimal	47	47	45	43	39	27	41	47
gap to optimal (%)	0.00	0.00	0.08	0.14	0.52	2.01	0.33	0.00
$\overline{\text{gap}}$ to optimal (%)	0.00	0.00	0.26	0.21	0.52	2.01	0.59	0.00

Table 5: Performance of the metaheuristics and GVNSWAF in the 47 classical instances for which proved optimal solutions are known.

instance group	metric	ILS-RN	ILS-CN	GRASP-RN	GRASP-CN	TS-RN	TS-CN	SA	GVNSWAF
large-sized	$\bar{C}_{\max}$	48,458.93	48,319.72	50,206.70	50,327.07	48,125.00	49,446.14	47,907.31	–
	$\overline{CV}$	0.0040	0.0030	0.0058	0.0045	–	–	0.0075	–
	#best	59	58	9	7	55	10	71	–
	gap(%)	1.80	1.55	4.96	5.10	1.02	3.39	0.66	–
	$\overline{\text{gap}}$(%)	2.33	1.95	5.80	5.73	1.02	3.39	1.51	–
small-sized	$\bar{C}_{\max}$	26,756.48	26,756.48	26,783.73	26,778.00	26,977.45	27,570.54	26,768.89	–
	$\overline{CV}$	0.0000	0.0000	0.0017	0.0011	–	–	0.0029	–
	#best	180	180	168	170	140	39	169	–
	gap(%)	0.00	0.00	0.11	0.09	0.75	2.99	0.05	–
	$\overline{\text{gap}}$(%)	0.00	0.00	0.30	0.19	0.75	2.99	0.32	–
classical	$\bar{C}_{\max}$	31,989.65	31,915.34	32,565.66	32,480.55	32,362.34	32,975.85	32,122.78	32,288.28
	$\overline{CV}$	0.0030	0.0013	0.0039	0.0026	–	–	0.0048	0.0113
	#best	64	88	49	46	52	30	63	53
	gap(%)	0.40	0.15	2.10	1.88	1.81	3.78	0.54	1.78
	$\overline{\text{gap}}$(%)	0.74	0.31	2.63	2.26	1.81	3.78	1.06	2.97

Table 6: Summary of the performance of the seven methods analyzed on the 150 large-sized instances, the 180 small-sized instances, and the 105 classical instances from the literature, the latter with linear routes only.

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Appendix

This appendix contains 13 tables detailing the experiments reported in the paper. Tables A1–A4 show the results of the six variations of the local search applied to the 330 considered instances. Tables A1 and A2 correspond to the first-improvement strategy while Tables A3 and A4 correspond to the best-improvement strategy. In the tables, we show the makespan of the obtained solution, the number of iterations that the local search made until finding an iterate that is better than all its neighbors (this is the stopping criterion as described in Algorithm 6), and the CPU time in seconds. In the case of the small-sized instances introduced in [4], the tables do not show the CPU time because it was always less than 1 millisecond. Tables A5, A6, and A7 show the results of applying the seven metaheuristics on the 50 large instances. In the tables, for each method/instance pair, when applicable, we show the best makespan (among the five runs) and the CPU time the method needed to achieve it. If a method reached its best makespan more than once, the indicated CPU time corresponds in fact to the average CPU time among all the times the method reached the indicated makespan. In the tables, statistics as the ones shown in Table 6 are individualy shown for the DA-type and Y-type instances and the learning index $\alpha \in \{0.1, 0.2, 0.3\}$. Tables A8, A9, and A10 show the results of applying the seven metaheuristics on the 60 small-sized instances. Tables A11, A12, and A13 show the results of applying the seven metaheuristics and the method GVNSWAF [45] on the considered 35 instances of the classic FJS from the literature [13, 21].

instance	$\alpha = 0.1$				$\alpha = 0.2$				$\alpha = 0.3$			
	Cropped		Reduced		Cropped		Reduced		Cropped		Reduced	
	$C_{\max}$	#it	$C_{\max}$	#it	$C_{\max}$	#it	$C_{\max}$	#it	$C_{\max}$	#it	$C_{\max}$	#it
miniDAFJS01	23,264	1	23,019	2	22,099	3	21,741	4	20,007	4	19,767	7
miniDAFJS02	23,242	1	22,927	4	22,161	1	22,046	2	21,136	2	20,987	2
miniDAFJS03	18,363	1	18,363	1	17,972	1	17,972	1	17,619	2	17,619	2
miniDAFJS04	21,690	1	21,151	4	20,757	2	19,602	5	19,867	2	18,800	5
miniDAFJS05	22,598	1	22,418	2	20,826	1	20,523	2	19,253	1	18,878	2
miniDAFJS06	23,370	1	23,370	1	20,783	2	20,783	2	18,712	3	18,712	3
miniDAFJS07	28,644	1	28,644	1	25,088	1	24,715	2	24,636	2	24,256	4
miniDAFJS08	19,878	1	19,878	1	18,857	1	18,857	1	17,900	2	17,900	2
miniDAFJS09	25,425	1	25,425	1	23,830	1	23,715	2	22,398	1	22,258	2
miniDAFJS10	21,563	1	20,359	4	21,021	1	18,823	6	19,723	2	17,466	6
miniDAFJS11	33,689	2	33,686	3	30,941	2	30,550	5	28,510	2	27,907	6
miniDAFJS12	20,342	1	20,342	1	19,624	1	19,624	1	19,094	1	19,094	1
miniDAFJS13	17,091	2	16,313	3	16,616	2	15,143	3	15,278	2	15,053	2
miniDAFJS14	23,248	4	23,140	5	21,990	5	21,817	5	20,711	5	20,620	5
miniDAFJS15	22,472	1	22,472	1	20,653	4	20,620	5	18,628	3	20,328	2
miniDAFJS16	25,691	1	25,426	2	24,593	1	24,114	2	22,939	3	22,939	3
miniDAFJS17	21,070	1	20,155	3	20,183	2	20,183	2	19,139	2	19,139	2
miniDAFJS18	18,512	3	18,135	7	18,829	1	18,201	4	17,784	1	17,295	2
miniDAFJS19	21,293	1	20,945	3	20,107	1	19,642	3	19,030	1	18,474	3
miniDAFJS20	23,443	2	23,212	3	21,286	1	21,286	1	19,587	1	19,587	1
miniDAFJS21	24,404	3	24,274	4	22,215	2	22,151	6	21,182	3	20,368	5
miniDAFJS22	25,923	4	25,923	4	24,273	4	24,273	4	22,767	3	22,767	3
miniDAFJS23	25,839	2	25,788	3	24,253	2	24,164	3	21,772	5	21,561	6
miniDAFJS24	26,932	1	26,610	4	24,500	1	23,709	3	22,524	1	21,714	4
miniDAFJS25	23,370	1	22,982	2	22,613	1	21,946	2	19,900	3	19,900	3
miniDAFJS26	22,306	6	22,230	9	20,724	5	20,724	7	19,333	5	19,333	7
miniDAFJS27	27,086	3	27,086	3	23,865	6	23,145	9	22,513	3	22,334	4
miniDAFJS28	23,841	10	23,841	11	22,745	7	22,745	7	20,020	2	20,020	2
miniDAFJS29	21,151	1	21,057	2	19,789	1	19,789	1	18,560	1	18,414	3
miniDAFJS30	26,428	2	26,126	5	23,736	2	22,973	5	21,356	2	20,760	5
mean	23,405.60	2.03	23,176.57	3.30	21,897.63	2.17	21,519.20	3.50	20,395.93	2.33	20,141.67	3.47
wins	12		30		10		30		12		19	

instance	$\alpha = 0.1$				$\alpha = 0.2$				$\alpha = 0.3$			
	Cropped		Reduced		Cropped		Reduced		Cropped		Reduced	
	$C_{\max}$	#it	$C_{\max}$	#it	$C_{\max}$	#it	$C_{\max}$	#it	$C_{\max}$	#it	$C_{\max}$	#it
miniYFJS01	35,243	1	35,046	2	34,443	1	33,132	4	33,697	1	32,199	2
miniYFJS02	28,688	1	28,688	1	27,557	1	27,557	1	25,969	1	25,969	1
miniYFJS03	48,141	3	47,391	6	44,195	3	42,896	6	39,880	8	38,935	11
miniYFJS04	25,394	1	25,394	1	24,669	1	24,485	2	24,017	1	23,774	2
miniYFJS05	26,371	2	25,750	3	25,635	3	24,235	3	24,743	1	24,743	1
miniYFJS06	30,952	1	30,952	1	29,080	1	29,080	1	27,366	1	27,366	1
miniYFJS07	46,782	2	45,705	6	43,886	3	42,578	5	41,487	3	39,694	5
miniYFJS08	33,954	2	33,883	7	31,992	2	31,773	7	30,276	2	29,990	6
miniYFJS09	37,049	2	37,049	2	36,098	2	36,098	2	34,357	1	34,357	1
miniYFJS10	29,416	6	29,416	6	30,290	3	29,858	2	27,547	2	27,425	2
miniYFJS11	51,212	1	51,129	2	47,079	1	46,939	2	43,254	1	43,254	1
miniYFJS12	36,343	4	36,343	6	33,404	5	33,404	5	29,989	5	29,989	5
miniYFJS13	32,915	7	32,792	8	30,219	5	30,219	7	25,881	3	25,619	4
miniYFJS14	31,826	7	31,826	9	31,284	4	31,284	4	28,131	6	28,131	6
miniYFJS15	45,901	4	45,442	6	42,828	4	45,903	5	39,862	4	46,387	2
miniYFJS16	33,965	3	33,791	7	32,481	3	32,165	7	30,483	2	30,483	2
miniYFJS17	44,181	5	52,936	3	41,316	5	48,475	4	38,739	5	44,650	3
miniYFJS18	34,133	4	34,044	9	29,883	3	30,201	5	27,944	3	29,698	2
miniYFJS19	39,165	4	36,706	14	33,965	6	33,805	12	31,743	6	31,743	9
miniYFJS20	36,071	9	36,071	11	35,100	4	34,689	8	32,603	4	32,718	5
miniYFJS21	40,994	7	39,978	10	37,414	5	35,570	16	33,355	5	33,355	7
miniYFJS22	35,319	7	34,282	15	32,485	1	32,337	2	30,212	1	29,512	4
miniYFJS23	45,713	9	45,725	10	42,016	3	42,016	7	38,777	4	37,963	9
miniYFJS24	40,291	6	36,367	14	38,144	2	38,144	4	30,173	11	31,054	14
miniYFJS25	43,592	3	43,555	4	40,933	1	40,425	3	34,388	6	34,388	5
miniYFJS26	54,739	2	54,675	3	49,412	1	49,412	1	44,766	8	43,452	18
miniYFJS27	36,899	6	36,765	18	34,359	7	34,943	7	32,539	3	31,749	7
miniYFJS28	41,951	2	41,838	3	38,640	2	33,096	8	35,729	2	35,729	7
miniYFJS29	44,930	9	47,807	4	38,993	3	36,561	9	35,726	3	38,122	5
miniYFJS30	44,038	3	43,725	8	41,290	3	41,640	5	32,238	6	33,215	13
mean	38,538.93	4.10	38,502.37	6.63	35,969.67	2.93	35,764.00	5.13	32,862.37	3.63	33,188.77	5.33
wins	11		27		14		25		19		13	

Table A1: Local search results with the first-improvement strategy using the reduced neighborhood and the cropped neighborhood, applied to the small-sized instances with learning index $\alpha \in \{0.1, 0.2, 0.3\}$.

instance	$\alpha = 0.1$						$\alpha = 0.2$						$\alpha = 0.3$					
	Cropped			Reduced			Cropped			Reduced			Cropped			Reduced		
	$C_{\max}$	#it	Time	$C_{\max}$	#it	Time	$C_{\max}$	#it	Time	$C_{\max}$	#it	Time	$C_{\max}$	#it	Time	$C_{\max}$	#it	Time
DAFJS01	28,036	4	0.001	26,794	14	0.004	26,371	3	0.000	25,970	19	0.001	21,685	3	0.000	21,439	5	0.000
DAFJS02	28,098	9	0.002	28,098	10	0.001	28,141	2	0.000	27,860	5	0.000	23,672	8	0.000	23,808	7	0.000
DAFJS03	53,688	1	0.001	51,579	15	0.005	47,879	3	0.000	45,458	37	0.011	43,509	3	0.000	42,867	13	0.005
DAFJS04	54,082	1	0.001	52,563	15	0.001	47,778	5	0.000	46,577	8	0.001	41,899	2	0.000	41,697	3	0.000
DAFJS05	40,195	25	0.004	41,604	24	0.002	39,771	16	0.001	37,797	36	0.006	35,610	9	0.000	31,254	47	0.009
DAFJS06	41,271	37	0.006	50,586	10	0.002	38,087	20	0.001	36,252	69	0.014	32,168	29	0.002	31,122	36	0.006
DAFJS07	54,989	7	0.004	54,802	36	0.026	45,737	34	0.006	48,959	29	0.021	40,830	6	0.001	39,662	24	0.024
DAFJS08	58,930	13	0.006	60,359	27	0.026	52,816	5	0.001	50,928	28	0.022	44,590	4	0.001	42,777	43	0.038
DAFJS09	45,139	11	0.001	44,917	17	0.003	36,871	29	0.002	36,735	39	0.009	34,185	19	0.002	34,025	39	0.011
DAFJS10	53,977	6	0.001	51,565	50	0.011	45,971	4	0.000	45,560	12	0.003	36,279	8	0.001	34,289	42	0.010
DAFJS11	66,872	6	0.003	63,878	55	0.052	51,746	37	0.016	54,661	30	0.049	45,657	17	0.006	44,809	46	0.068
DAFJS12	62,198	31	0.014	59,612	157	0.155	53,738	9	0.002	52,467	38	0.040	44,490	8	0.004	43,717	28	0.043
DAFJS13	57,874	16	0.005	57,310	36	0.021	48,847	22	0.004	48,816	29	0.016	41,717	4	0.001	41,960	11	0.006
DAFJS14	70,169	39	0.008	65,321	59	0.029	56,639	35	0.006	55,881	44	0.027	46,392	26	0.006	44,707	54	0.035
DAFJS15	61,882	62	0.032	63,408	124	0.166	53,807	6	0.003	53,686	14	0.031	47,692	10	0.004	47,474	33	0.067
DAFJS16	70,727	40	0.017	70,838	84	0.125	63,277	11	0.005	58,436	107	0.183	55,112	12	0.006	53,095	78	0.172
DAFJS17	68,446	50	0.018	70,306	26	0.016	57,893	41	0.009	57,518	54	0.031	46,340	80	0.026	47,331	96	0.087
DAFJS18	71,871	31	0.006	74,631	21	0.015	58,533	19	0.004	58,907	29	0.019	50,818	7	0.002	48,672	26	0.015
DAFJS19	63,999	12	0.002	63,314	26	0.019	52,824	24	0.006	57,082	10	0.014	41,426	28	0.004	41,865	56	0.029
DAFJS20	64,005	75	0.020	63,841	225	0.207	61,871	5	0.002	57,378	99	0.115	46,868	53	0.018	48,600	40	0.051
DAFJS21	72,142	57	0.020	76,433	31	0.032	61,483	12	0.004	62,677	9	0.009	50,318	15	0.005	49,422	39	0.040
DAFJS22	60,081	132	0.043	60,143	315	0.448	53,248	44	0.014	54,318	42	0.097	43,566	17	0.011	41,615	94	0.235
DAFJS23	48,338	9	0.001	48,054	18	0.012	41,433	17	0.003	44,473	31	0.018	39,105	3	0.000	38,334	12	0.006
DAFJS24	55,507	9	0.004	55,353	33	0.032	47,277	14	0.004	47,656	9	0.007	40,921	17	0.004	42,473	27	0.019
DAFJS25	79,286	26	0.011	76,253	92	0.227	59,672	68	0.033	61,805	92	0.184	48,469	25	0.013	49,390	39	0.098
DAFJS26	73,969	54	0.020	75,329	96	0.190	70,211	8	0.003	65,524	109	0.373	53,944	39	0.023	55,269	39	0.117
DAFJS27	79,234	13	0.004	79,064	23	0.053	62,437	21	0.010	62,596	29	0.079	53,719	46	0.023	55,569	46	0.134
DAFJS28	59,201	18	0.003	54,866	109	0.103	48,414	21	0.004	47,884	46	0.039	41,408	11	0.003	40,097	34	0.050
DAFJS29	68,684	21	0.006	66,929	80	0.106	57,356	16	0.005	57,179	34	0.044	49,819	7	0.005	49,519	24	0.050
DAFJS30	57,350	17	0.005	60,016	13	0.013	51,283	14	0.004	50,961	45	0.052	43,426	3	0.001	42,592	6	0.008
mean	59,008.00	27.73	0.009	58,925.53	61.37	0.070	50,713.70	18.83	0.005	50,400.03	39.40	0.051	42,854.47	17.30	0.006	42,315.00	36.23	0.048
wins	12			19			10			20			9			12		

instance	$\alpha = 0.1$						$\alpha = 0.2$						$\alpha = 0.3$					
	Cropped			Reduced			Cropped			Reduced			Cropped			Reduced		
	$C_{\max}$	#it	Time	$C_{\max}$	#it	Time	$C_{\max}$	#it	Time	$C_{\max}$	#it	Time	$C_{\max}$	#it	Time	$C_{\max}$	#it	Time
YFJS01	79,662	10	0.000	79,737	21	0.003	74,547	9	0.000	70,624	31	0.004	66,246	9	0.000	67,588	10	0.001
YFJS02	76,529	3	0.000	76,813	4	0.000	66,853	1	0.000	66,853	1	0.000	59,211	3	0.000	59,001	4	0.001
YFJS03	38,424	3	0.000	35,936	14	0.000	33,273	5	0.000	33,273	5	0.000	30,995	3	0.000	31,407	2	0.000
YFJS04	45,683	3	0.000	45,533	4	0.000	42,553	4	0.000	42,553	4	0.000	38,092	5	0.000	38,092	5	0.000
YFJS05	45,421	4	0.000	45,421	4	0.000	40,738	2	0.000	40,738	2	0.000	38,324	5	0.000	38,324	9	0.000
YFJS06	44,848	13	0.000	47,384	12	0.001	43,384	15	0.000	42,207	21	0.002	45,370	2	0.000	45,370	2	0.000
YFJS07	48,980	7	0.000	45,722	21	0.001	43,925	12	0.000	45,971	11	0.000	38,453	5	0.000	38,453	5	0.000
YFJS08	43,073	7	0.000	41,658	19	0.001	42,901	3	0.000	41,573	10	0.000	35,429	2	0.000	33,432	17	0.001
YFJS09	25,067	3	0.000	24,350	4	0.000	23,059	3	0.000	22,211	5	0.000	20,916	7	0.000	20,916	7	0.000
YFJS10	42,583	1	0.000	42,278	2	0.000	39,126	5	0.000	38,683	7	0.000	34,487	8	0.000	36,415	7	0.000
YFJS11	55,765	4	0.000	55,416	7	0.001	49,781	2	0.000	49,160	6	0.001	45,051	1	0.000	44,455	4	0.000
YFJS12	56,674	24	0.001	54,852	59	0.007	55,174	11	0.000	53,329	21	0.003	47,092	3	0.000	44,873	12	0.001
YFJS13	44,083	16	0.000	45,844	16	0.001	42,442	10	0.000	39,720	27	0.003	35,776	16	0.000	35,524	19	0.003
YFJS14	119,977	27	0.016	124,518	60	0.141	104,506	19	0.010	105,770	58	0.181	89,687	12	0.007	87,368	61	0.182
YFJS15	126,713	33	0.016	127,073	171	0.481	106,511	28	0.016	108,412	58	0.192	93,075	14	0.007	86,141	166	0.529
YFJS16	116,201	37	0.018	121,141	73	0.196	101,164	13	0.005	99,801	65	0.129	87,351	10	0.005	87,648	48	0.136
YFJS17	104,688	21	0.026	102,614	159	1.134	83,355	18	0.014	82,211	87	0.621	71,201	26	0.036	71,075	65	0.493
YFJS18	126,647	34	0.032	128,529	87	0.752	97,157	15	0.022	94,689	179	2.000	84,518	9	0.015	82,967	77	0.834
YFJS19	93,306	96	0.100	100,855	209	1.353	86,398	21	0.026	85,028	128	1.115	68,792	53	0.054	68,955	135	0.984
YFJS20	96,248	17	0.019	92,240	196	1.205	79,408	85	0.088	80,875	294	1.923	71,676	8	0.009	68,562	123	0.993
mean	71,528.60	18.15	0.011	71,895.70	57.10	0.264	62,812.75	14.05	0.009	62,184.05	51.00	0.309	55,087.10	10.05	0.007	54,328.30	38.90	0.208
wins	10			11			8			16			10			15		

Table A2: Local search results with the first-improvement strategy using the reduced neighborhood and the cropped neighborhood, applied to the large-sized instances proposed in [8] with learning index $\alpha \in \{0.1, 0.2, 0.3\}$.

instance	$\alpha = 0.1$				$\alpha = 0.2$				$\alpha = 0.3$			
	Cropped		Reduced		Cropped		Reduced		Cropped		Reduced	
	$C_{\max}$	#it	$C_{\max}$	#it	$C_{\max}$	#it	$C_{\max}$	#it	$C_{\max}$	#it	$C_{\max}$	#it
miniDAFJS01	23,264	1	23,019	2	21,588	3	21,546	4	20,007	4	19,848	4
miniDAFJS02	23,242	1	22,927	3	22,161	1	22,046	2	21,136	2	20,987	2
miniDAFJS03	18,363	1	18,363	1	17,972	1	17,972	1	17,619	2	17,619	2
miniDAFJS04	21,690	1	21,151	4	20,691	2	19,602	4	19,867	2	18,800	4
miniDAFJS05	22,598	1	22,418	2	20,826	1	20,523	2	19,253	1	18,878	2
miniDAFJS06	23,370	1	23,370	1	20,783	2	20,783	2	19,605	2	19,605	2
miniDAFJS07	28,644	1	28,644	1	25,088	1	24,715	2	24,636	2	24,256	2
miniDAFJS08	19,878	1	19,878	1	18,857	1	18,857	1	17,900	2	17,900	2
miniDAFJS09	25,425	1	25,425	1	23,830	1	23,715	2	22,398	1	22,258	2
miniDAFJS10	21,563	1	20,359	4	21,021	1	18,823	5	19,723	2	17,466	5
miniDAFJS11	33,689	2	33,585	4	30,941	2	30,550	5	28,510	2	28,342	2
miniDAFJS12	20,342	1	20,342	1	19,624	1	19,624	1	19,094	1	19,094	1
miniDAFJS13	17,091	2	16,313	3	16,616	2	15,143	3	15,278	2	15,053	2
miniDAFJS14	23,248	4	23,140	5	21,990	5	21,817	5	20,711	5	20,620	5
miniDAFJS15	22,472	1	22,472	1	20,620	3	20,620	3	18,628	3	18,628	3
miniDAFJS16	25,691	1	25,426	2	24,593	1	24,114	2	22,939	3	22,939	3
miniDAFJS17	21,070	1	20,155	3	20,183	2	20,183	2	19,139	2	19,139	2
miniDAFJS18	18,445	2	18,135	4	18,829	1	18,201	4	17,784	1	17,295	2
miniDAFJS19	21,293	1	20,945	3	20,107	1	19,642	3	19,030	1	18,474	3
miniDAFJS20	23,443	2	23,212	3	21,286	1	21,286	1	19,587	1	19,587	1
miniDAFJS21	24,404	3	24,274	4	22,215	2	22,151	4	21,182	3	20,368	4
miniDAFJS22	25,923	3	25,923	3	24,273	3	24,273	3	22,767	3	22,767	3
miniDAFJS23	25,839	2	25,788	3	24,253	2	24,164	3	21,772	5	22,710	3
miniDAFJS24	26,932	1	26,610	4	24,500	1	23,709	3	22,524	1	21,714	4
miniDAFJS25	23,370	1	22,982	2	22,613	1	21,946	2	18,472	6	20,834	3
miniDAFJS26	22,306	3	22,230	4	20,273	4	20,273	4	18,921	4	18,921	4
miniDAFJS27	27,086	3	27,086	3	23,145	5	23,145	5	22,513	3	22,334	4
miniDAFJS28	23,841	6	23,841	6	21,927	7	21,927	7	20,020	2	20,020	2
miniDAFJS29	21,151	1	21,057	2	19,789	1	19,789	1	18,560	1	18,414	2
miniDAFJS30	26,428	2	26,126	4	23,736	2	22,973	4	21,356	2	20,760	3
mean	23,403.37	1.73	23,173.20	2.80	21,811.00	2.03	21,470.40	3.00	20,364.37	2.37	20,187.67	2.77
wins	12		30		12		30		13		18	

instance	$\alpha = 0.1$				$\alpha = 0.2$				$\alpha = 0.3$			
	Cropped		Reduced		Cropped		Reduced		Cropped		Reduced	
	$C_{\max}$	#it	$C_{\max}$	#it	$C_{\max}$	#it	$C_{\max}$	#it	$C_{\max}$	#it	$C_{\max}$	#it
miniYFJS01	35,243	1	35,046	2	34,443	1	33,132	4	33,697	1	32,199	2
miniYFJS02	28,688	1	28,688	1	27,557	1	27,557	1	25,969	1	25,969	1
miniYFJS03	53,381	2	52,098	3	49,111	2	46,806	3	42,787	3	42,787	3
miniYFJS04	25,394	1	25,394	1	24,669	1	24,485	2	24,017	1	23,774	2
miniYFJS05	26,371	2	25,750	3	25,635	3	24,235	3	24,743	1	24,743	1
miniYFJS06	30,952	1	30,952	1	29,080	1	29,080	1	27,366	1	27,366	1
miniYFJS07	46,782	2	46,089	4	44,137	2	42,578	5	41,737	2	39,229	5
miniYFJS08	33,954	2	33,883	3	31,992	2	31,597	4	30,276	2	29,631	4
miniYFJS09	37,049	2	37,049	2	36,098	2	36,098	2	34,357	1	34,357	1
miniYFJS10	29,416	5	29,416	5	27,604	4	29,858	2	27,547	2	27,425	2
miniYFJS11	51,212	1	51,129	2	47,079	1	46,939	2	43,254	1	43,254	1
miniYFJS12	36,343	4	36,343	4	33,404	5	33,404	5	29,989	5	29,989	5
miniYFJS13	32,915	5	32,792	5	30,219	4	30,219	4	25,881	3	25,619	4
miniYFJS14	31,826	6	31,826	6	31,284	4	31,284	4	28,131	6	28,131	6
miniYFJS15	45,597	3	45,442	4	42,306	3	41,893	4	39,187	3	38,619	4
miniYFJS16	33,965	3	33,791	7	32,481	3	32,165	6	30,483	2	30,483	2
miniYFJS17	44,181	5	44,044	7	41,316	5	41,199	5	38,739	5	38,584	5
miniYFJS18	34,133	3	34,044	4	29,883	2	29,883	2	27,944	2	27,944	2
miniYFJS19	39,165	3	39,046	4	33,965	6	33,805	7	31,743	6	31,082	8
miniYFJS20	30,837	6	30,837	6	31,304	6	31,304	6	29,599	4	29,599	4
miniYFJS21	40,523	6	40,523	6	35,570	9	35,570	9	33,355	5	33,355	5
miniYFJS22	35,019	4	35,019	4	32,485	1	31,946	2	30,212	1	29,498	2
miniYFJS23	45,725	4	45,725	4	42,446	2	42,016	3	38,777	4	37,963	5
miniYFJS24	40,291	5	40,291	6	38,144	2	38,144	2	30,173	7	31,572	9
miniYFJS25	43,592	3	43,555	3	40,933	1	40,425	3	34,388	3	34,388	3
miniYFJS26	54,739	2	54,675	3	49,412	1	49,412	1	44,633	3	44,207	6
miniYFJS27	36,765	4	36,765	4	34,125	4	34,225	6	31,749	4	31,571	6
miniYFJS28	41,951	2	41,838	3	38,640	2	38,450	3	35,729	2	35,729	2
miniYFJS29	44,930	5	44,930	5	38,993	3	38,586	4	34,330	4	34,604	3
miniYFJS30	44,038	3	43,725	4	41,290	3	40,744	4	31,093	7	31,093	7
mean	38,499.23	3.20	38,356.83	3.87	35,853.50	2.87	35,567.97	3.63	32,729.50	3.07	32,492.13	3.70
wins	14		30		13		28		17		18	

Table A3: Local search results with the best-improvement strategy using the reduced neighborhood and the cropped neighborhood, applied to the small-sized instances with learning index $\alpha \in \{0.1, 0.2, 0.3\}$.

instance	$\alpha = 0.1$						$\alpha = 0.2$						$\alpha = 0.3$					
	Cropped			Reduced			Cropped			Reduced			Cropped			Reduced		
	$C_{\max}$	#it	Time	$C_{\max}$	#it	Time	$C_{\max}$	#it	Time	$C_{\max}$	#it	Time	$C_{\max}$	#it	Time	$C_{\max}$	#it	Time
DAFJS01	26,465	4	0.000	26,203	7	0.004	26,371	3	0.000	24,993	8	0.001	21,685	3	0.000	21,685	3	0.000
DAFJS02	28,098	3	0.000	28,098	3	0.001	27,949	2	0.000	27,727	4	0.000	23,021	3	0.000	23,021	3	0.000
DAFJS03	53,688	1	0.000	51,580	14	0.009	47,879	2	0.000	45,916	15	0.011	43,509	2	0.000	42,258	8	0.006
DAFJS04	54,082	1	0.000	52,494	11	0.004	47,571	4	0.000	46,502	7	0.003	41,899	2	0.000	41,533	4	0.001
DAFJS05	40,723	10	0.006	40,723	12	0.005	37,118	15	0.002	38,484	12	0.005	33,331	12	0.001	33,813	7	0.003
DAFJS06	45,234	11	0.007	44,877	15	0.009	41,995	6	0.001	41,909	9	0.005	33,956	5	0.001	30,556	20	0.011
DAFJS07	54,937	5	0.006	53,651	15	0.032	44,414	16	0.009	44,147	21	0.046	40,442	5	0.002	40,057	13	0.027
DAFJS08	58,941	10	0.009	56,561	14	0.031	52,803	3	0.001	51,128	16	0.038	44,590	4	0.001	43,737	16	0.034
DAFJS09	43,032	9	0.003	45,796	5	0.002	38,062	12	0.002	37,876	12	0.007	36,488	5	0.001	36,303	10	0.008
DAFJS10	54,858	5	0.002	51,277	24	0.025	45,971	4	0.001	45,868	3	0.002	35,273	8	0.002	35,023	9	0.009
DAFJS11	63,795	8	0.010	64,796	20	0.066	53,770	7	0.007	52,845	17	0.061	44,701	7	0.005	44,599	11	0.035
DAFJS12	62,433	15	0.024	61,533	28	0.128	54,073	5	0.007	52,355	17	0.080	44,051	5	0.004	43,449	11	0.045
DAFJS13	58,902	6	0.004	56,453	22	0.027	48,838	13	0.007	49,179	10	0.011	41,717	4	0.001	41,717	4	0.005
DAFJS14	62,637	42	0.029	70,357	8	0.014	55,503	19	0.012	54,609	25	0.040	44,777	16	0.008	44,646	23	0.042
DAFJS15	64,527	21	0.032	62,030	39	0.205	52,115	9	0.011	53,677	7	0.035	46,921	12	0.012	47,268	15	0.071
DAFJS16	69,300	21	0.029	66,153	71	0.358	61,676	8	0.011	60,010	20	0.093	54,072	15	0.015	49,031	46	0.260
DAFJS17	70,803	9	0.011	69,975	22	0.054	55,294	6	0.006	54,456	18	0.045	45,667	17	0.016	47,550	28	0.103
DAFJS18	67,984	27	0.022	71,052	10	0.031	57,965	16	0.012	57,274	25	0.042	48,351	7	0.004	49,753	5	0.013
DAFJS19	57,786	16	0.014	57,471	22	0.046	51,065	8	0.004	51,065	8	0.019	43,998	9	0.003	40,678	26	0.053
DAFJS20	67,045	22	0.021	66,674	30	0.115	57,271	27	0.027	55,355	47	0.180	45,160	23	0.024	49,481	14	0.055
DAFJS21	72,979	17	0.025	71,913	23	0.103	58,044	14	0.018	57,787	20	0.070	48,693	16	0.023	48,912	16	0.064
DAFJS22	60,238	46	0.068	62,189	29	0.196	54,147	11	0.017	53,728	15	0.088	41,480	28	0.040	42,035	25	0.156
DAFJS23	47,719	9	0.004	47,520	15	0.022	42,480	8	0.003	42,943	11	0.014	39,105	3	0.001	38,185	8	0.011
DAFJS24	54,427	13	0.009	55,115	12	0.033	46,797	15	0.012	46,985	12	0.024	41,832	4	0.005	39,979	21	0.063
DAFJS25	74,359	37	0.049	73,353	48	0.301	58,767	31	0.043	61,465	29	0.165	49,919	9	0.013	49,527	14	0.059
DAFJS26	68,577	50	0.076	76,328	12	0.080	63,243	36	0.049	68,469	18	0.142	51,544	16	0.021	50,829	31	0.219
DAFJS27	79,254	2	0.004	78,967	6	0.036	62,520	12	0.020	62,359	18	0.120	51,382	29	0.041	53,010	23	0.168
DAFJS28	58,564	4	0.004	53,505	41	0.100	48,593	12	0.012	48,941	9	0.029	41,206	4	0.004	40,923	7	0.019
DAFJS29	70,450	6	0.005	65,758	36	0.120	56,387	20	0.014	57,708	14	0.054	49,906	5	0.005	48,412	18	0.074
DAFJS30	57,326	6	0.004	56,839	13	0.032	49,649	17	0.012	49,312	30	0.092	42,819	2	0.002	42,592	4	0.011
mean	58,305.43	14.53	0.016	57,974.70	20.90	0.073	49,944.33	12.03	0.011	49,835.73	15.90	0.051	42,383.17	9.33	0.009	42,018.73	14.77	0.054
wins	9			23			10			21			11			13		

instance	$\alpha = 0.1$						$\alpha = 0.2$						$\alpha = 0.3$					
	Cropped			Reduced			Cropped			Reduced			Cropped			Reduced		
	$C_{\max}$	#it	Time	$C_{\max}$	#it	Time	$C_{\max}$	#it	Time	$C_{\max}$	#it	Time	$C_{\max}$	#it	Time	$C_{\max}$	#it	Time
YFJS01	80,167	6	0.000	79,657	8	0.002	71,592	9	0.001	68,822	14	0.004	66,246	6	0.000	60,208	15	0.005
YFJS02	76,529	3	0.000	76,529	3	0.000	66,853	1	0.000	66,853	1	0.000	59,001	2	0.000	59,001	2	0.000
YFJS03	34,166	5	0.000	36,409	4	0.000	32,864	5	0.000	32,864	5	0.000	30,995	3	0.000	31,407	2	0.000
YFJS04	45,683	3	0.000	45,533	4	0.000	42,553	4	0.000	42,553	4	0.000	38,092	3	0.000	38,092	3	0.000
YFJS05	45,421	3	0.000	45,421	3	0.000	40,738	2	0.000	40,738	2	0.000	38,324	4	0.000	38,324	8	0.001
YFJS06	44,936	10	0.001	44,936	10	0.002	40,272	9	0.000	43,744	6	0.001	43,123	6	0.000	43,099	7	0.001
YFJS07	48,491	9	0.000	48,355	10	0.002	40,941	12	0.001	40,941	12	0.002	38,453	2	0.000	38,453	2	0.000
YFJS08	47,429	3	0.000	41,754	9	0.001	41,994	3	0.000	37,681	10	0.001	35,429	2	0.000	33,392	5	0.000
YFJS09	24,413	2	0.000	23,944	4	0.000	23,059	2	0.000	22,211	4	0.000	20,916	7	0.000	20,916	7	0.001
YFJS10	42,583	1	0.000	42,278	2	0.000	39,126	4	0.000	39,361	4	0.000	34,487	7	0.000	34,487	10	0.001
YFJS11	52,815	4	0.000	52,196	8	0.002	49,781	2	0.000	49,206	4	0.001	45,051	1	0.000	44,729	2	0.000
YFJS12	53,094	11	0.001	52,650	12	0.004	56,022	7	0.000	54,186	12	0.004	46,471	5	0.000	43,150	9	0.003
YFJS13	47,064	4	0.000	46,578	9	0.003	41,618	5	0.000	41,498	6	0.002	36,962	4	0.000	35,800	11	0.003
YFJS14	119,792	13	0.016	118,731	27	0.202	97,201	18	0.022	100,177	23	0.161	89,371	6	0.008	87,518	21	0.162
YFJS15	121,960	14	0.020	125,754	66	0.553	110,858	8	0.011	109,314	33	0.284	90,041	15	0.019	88,664	28	0.220
YFJS16	116,033	19	0.024	115,312	31	0.204	101,073	7	0.008	98,976	34	0.238	88,521	5	0.006	85,252	27	0.176
YFJS17	104,862	9	0.025	104,466	18	0.396	83,240	14	0.042	82,167	42	0.895	71,114	9	0.027	70,851	46	1.004
YFJS18	124,738	13	0.043	119,224	84	1.872	97,366	9	0.027	94,472	45	1.123	83,681	9	0.030	82,795	27	0.639
YFJS19	98,795	24	0.072	100,158	38	0.847	86,289	9	0.031	83,530	48	1.159	70,454	22	0.061	68,592	69	1.435
YFJS20	95,573	10	0.030	94,201	42	0.764	79,653	24	0.062	79,638	35	0.630	70,928	9	0.028	70,253	19	0.365
mean	71,227.20	8.30	0.012	70,704.30	19.60	0.243	62,154.65	7.70	0.010	61,446.60	17.20	0.225	54,883.00	6.35	0.009	53,749.15	16.00	0.201
wins	6			17			8			17			7			19		

Table A4: Local search results with the best-improvement strategy using the reduced neighborhood and the cropped neighborhood, applied to the large-sized instances proposed in [8] with learning index $\alpha \in \{0.1, 0.2, 0.3\}$.

instance	ILS-RN		ILS-CN		GRASP-RN		GRASP-CN		TS-RN		TS-CN		SA	
	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time
DAFJS01	**23,460**	12.29	**23,460**	3.39	24,172	90.48	23,961	131.52	24,424	0.05	24,661	0.01	23,992	46.48
DAFJS02	**26,432**	60.95	**26,432**	25.84	26,989	0.01	26,869	0.28	**26,432**	0.32	27,620	0.00	27,577	0.55
DAFJS03	**50,343**	164.37	50,462	34.55	50,424	241.44	51,028	156.99	**50,343**	7.20	51,612	0.26	50,458	261.30
DAFJS04	**51,872**	62.24	51,992	181.28	51,927	30.24	52,358	275.80	52,291	0.11	53,348	0.00	51,927	151.36
DAFJS05	**34,660**	167.91	**34,660**	45.10	37,081	35.04	36,883	80.14	35,278	1.96	36,009	0.26	36,141	139.08
DAFJS06	35,435	111.76	**35,302**	176.69	37,023	122.84	36,338	98.55	35,636	15.04	37,854	0.05	35,697	256.36
DAFJS07	45,805	240.68	45,306	72.33	46,624	107.20	46,791	57.95	44,281	158.67	45,060	297.79	**44,219**	241.82
DAFJS08	53,202	52.07	53,463	41.54	53,575	174.57	54,129	129.70	**53,002**	261.88	54,018	0.69	53,292	276.00
DAFJS09	40,573	145.06	**40,537**	166.88	41,066	119.76	40,759	184.35	41,123	4.34	42,241	0.09	41,265	4.00
DAFJS10	44,523	99.32	**44,058**	254.61	44,917	217.73	44,735	277.52	45,820	10.28	47,438	0.32	44,851	70.25
DAFJS11	55,180	121.71	55,393	151.06	58,257	219.22	58,527	228.89	**54,351**	260.24	55,337	158.73	54,708	264.50
DAFJS12	54,035	227.92	53,175	20.54	56,670	237.70	56,495	79.61	51,432	238.43	51,794	224.88	**51,067**	234.61
DAFJS13	54,654	35.35	54,371	0.05	54,392	277.63	54,583	107.84	**53,340**	233.35	53,792	0.10	54,502	93.52
DAFJS14	61,500	133.79	60,882	169.24	61,487	185.43	61,641	265.85	59,668	196.33	61,524	0.74	**59,573**	254.00
DAFJS15	57,552	111.67	57,554	216.92	59,362	201.70	59,997	220.71	**55,086**	253.37	55,381	115.31	55,164	197.01
DAFJS16	58,828	213.11	58,327	98.92	63,365	189.53	62,494	242.24	55,927	242.72	56,758	115.48	**55,868**	208.74
DAFJS17	65,252	138.93	65,121	1.17	64,804	269.21	65,643	74.78	**64,262**	283.76	65,525	1.33	65,602	55.29
DAFJS18	66,429	288.91	65,889	271.51	66,633	179.15	66,419	214.60	64,916	134.40	67,040	0.23	**64,820**	219.02
DAFJS19	45,416	136.34	44,744	95.28	46,757	147.62	46,301	76.22	44,633	121.09	45,602	1.30	**44,086**	144.96
DAFJS20	59,190	298.29	58,137	241.93	59,670	43.48	59,323	79.25	**56,432**	155.97	57,141	9.73	57,245	290.79
DAFJS21	65,432	83.75	66,006	217.22	66,547	87.71	66,752	277.45	64,563	264.94	64,902	13.37	**64,059**	124.80
DAFJS22	58,025	2.64	57,426	227.59	59,477	62.55	59,086	30.34	55,246	269.80	**55,038**	24.67	55,624	283.70
DAFJS23	42,818	226.75	42,255	149.85	43,565	66.68	43,426	150.96	41,941	57.87	42,426	1.56	**41,286**	227.89
DAFJS24	48,266	207.07	48,911	123.21	51,313	127.05	51,532	43.92	47,443	139.28	47,365	281.95	**46,945**	217.47
DAFJS25	62,813	127.68	63,039	16.33	65,321	147.01	65,866	65.19	60,824	205.95	60,308	248.14	**60,187**	270.84
DAFJS26	62,389	131.50	61,645	158.57	65,982	299.75	65,951	196.11	58,995	185.68	**58,749**	67.28	59,450	297.29
DAFJS27	69,386	73.63	68,795	254.71	70,375	231.60	70,762	6.13	66,539	245.31	**66,003**	148.50	66,070	204.00
DAFJS28	48,091	133.90	47,738	187.43	48,669	164.03	49,309	294.79	47,428	164.05	47,551	75.18	**46,569**	268.66
DAFJS29	57,824	58.01	57,384	259.73	59,825	173.11	59,564	234.87	55,820	10.14	56,122	37.45	**54,686**	296.94
DAFJS30	48,572	292.30	48,329	137.76	50,395	59.47	50,593	18.06	47,140	237.35	47,307	76.44	**46,344**	232.86
$\overline{C}_{\max}$	51,598.57		51,359.77		52,888.80		52,937.17		50,487.20		51,184.20		50,442.47	
$\overline{CV}$	0.0046		0.0044		0.0057		0.0046		0.0000		0.0000		0.0083	
#best	5		6		0		0		8		3		13	
gap(%)	2.79		2.33		5.46		5.48		0.91		2.45		0.84	
$\overline{\text{gap}}$(%)	3.43		2.94		6.23		6.14		0.91		2.45		1.87	

instance	ILS-RN		ILS-CN		GRASP-RN		GRASP-CN		TS-RN		TS-CN		SA	
	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time
YFJS01	**68,714**	0.51	**68,714**	0.80	**68,714**	0.12	69,189	0.02	**68,714**	0.34	75,595	0.03	**68,714**	0.81
YFJS02	**72,465**	1.40	**72,465**	2.82	73,053	0.12	73,149	0.44	**72,465**	0.02	73,053	0.04	**72,465**	1.79
YFJS03	**32,538**	0.03	**32,538**	0.02	**32,538**	254.14	**32,538**	0.08	**32,538**	0.10	33,174	0.00	**32,538**	0.47
YFJS04	**35,883**	1.73	**35,883**	0.17	36,946	0.15	**35,883**	0.10	**35,883**	1.25	37,340	0.02	**35,883**	0.59
YFJS05	**40,186**	9.99	**40,186**	1.76	41,034	160.74	41,034	0.52	**40,186**	4.42	41,034	0.03	41,034	0.74
YFJS06	**40,441**	218.75	**40,441**	122.67	41,123	3.28	41,174	59.04	40,522	13.18	41,123	0.61	40,784	0.76
YFJS07	**40,887**	131.61	**40,887**	37.21	41,625	54.84	41,354	11.63	**40,887**	38.15	41,394	0.17	**40,887**	16.55
YFJS08	**32,573**	0.69	**32,573**	0.88	32,623	0.09	32,655	29.50	**32,573**	0.91	34,452	0.00	**32,573**	63.60
YFJS09	**22,681**	2.68	**22,681**	0.54	**22,681**	6.30	**22,681**	56.94	**22,681**	0.01	**22,681**	0.24	**22,681**	2.03
YFJS10	**37,372**	11.63	**37,372**	19.65	37,685	1.92	37,685	0.62	39,621	0.03	37,482	0.00	**37,372**	3.04
YFJS11	**47,800**	12.50	**47,800**	94.06	49,493	174.55	49,208	41.96	48,195	4.95	49,721	0.04	**47,800**	85.90
YFJS12	**46,728**	26.81	**46,728**	15.79	49,411	13.84	48,774	92.81	**46,728**	149.42	48,608	0.02	**46,728**	63.09
YFJS13	**36,911**	12.30	**36,911**	13.38	38,042	2.35	37,361	108.86	36,926	33.80	39,207	0.02	**36,911**	76.79
YFJS14	**110,185**	216.19	110,558	269.22	112,062	221.06	113,009	59.06	110,625	9.33	117,667	0.03	111,282	147.69
YFJS15	105,935	192.42	106,221	189.75	108,296	17.04	111,374	226.73	**105,837**	259.22	116,525	0.19	106,892	149.18
YFJS16	**105,003**	56.50	105,423	119.50	111,439	107.53	111,432	284.88	105,905	197.45	115,417	0.07	105,112	56.87
YFJS17	**93,469**	198.07	94,558	174.10	99,912	159.01	101,069	107.68	96,737	226.68	98,329	0.99	94,950	112.29
YFJS18	**101,013**	215.24	101,517	63.58	109,763	247.62	107,336	230.38	103,359	239.97	115,332	0.48	102,477	184.95
YFJS19	82,879	221.94	**81,172**	115.47	94,020	109.95	92,172	67.18	86,500	242.82	88,400	2.14	81,946	93.62
YFJS20	**83,689**	233.90	83,731	259.71	94,228	280.03	93,689	13.39	85,867	282.07	88,516	3.63	84,707	200.84
$\overline{C}_{\max}$	61,867.60		61,917.95		64,734.40		64,638.30		62,637.45		65,752.50		62,186.80	
$\overline{CV}$	0.0022		0.0008		0.0052		0.0040		0.0000		0.0000		0.0052	
#best	18		14		3		3		10		1		11	
gap(%)	0.11		0.14		3.84		3.51		1.17		5.15		0.51	
$\overline{\text{gap}}$(%)	0.40		0.23		4.59		4.02		1.17		5.15		0.97	

Table A5: Results of applying the metaheuristics to the large-sized instances with learning index $\alpha = 0.1$.

instance	ILS-RN		ILS-CN		GRASP-RN		GRASP-CN		TS-RN		TS-CN		SA	
	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time
DAFJS01	**21,683**	17.45	**21,683**	7.46	21,861	8.41	21,861	136.04	22,286	0.03	22,703	0.01	21,936	0.38
DAFJS02	**24,235**	48.48	**24,235**	55.62	24,618	0.05	24,571	4.00	24,875	0.14	24,477	0.00	25,141	0.51
DAFJS03	**44,266**	213.30	44,453	208.60	44,481	211.71	45,079	169.49	44,419	12.80	45,275	0.37	**44,266**	147.45
DAFJS04	**44,713**	54.08	44,938	53.17	**44,713**	298.64	45,644	29.88	44,821	1.41	46,807	0.28	44,816	239.37
DAFJS05	30,904	28.28	**30,773**	144.29	31,848	54.88	32,065	22.88	32,028	0.92	31,772	0.66	31,295	61.32
DAFJS06	30,602	65.94	**30,553**	16.14	31,994	90.56	31,768	145.27	31,521	5.17	31,039	0.52	31,216	44.70
DAFJS07	40,343	46.23	39,788	223.54	40,321	121.98	40,891	84.05	**38,731**	37.49	39,488	223.99	38,833	112.43
DAFJS08	45,636	273.46	45,784	218.76	45,955	9.16	46,589	7.49	**45,162**	133.36	46,129	119.13	45,302	128.92
DAFJS09	**35,098**	25.22	35,244	74.17	35,759	18.62	35,507	224.49	35,161	5.16	36,004	0.20	35,418	193.84
DAFJS10	37,326	238.28	37,172	198.27	37,736	257.38	37,856	76.13	**36,791**	293.02	38,453	0.75	37,183	11.88
DAFJS11	47,417	203.56	47,221	248.85	49,271	182.94	49,049	30.18	45,861	300.00	46,470	169.13	**45,822**	242.59
DAFJS12	45,613	256.81	45,394	45.36	48,195	14.33	47,896	195.90	44,167	232.32	43,932	259.97	**43,227**	261.22
DAFJS13	45,230	112.83	45,187	255.42	45,154	195.56	45,422	200.48	**44,180**	128.30	46,329	0.17	45,309	154.25
DAFJS14	50,958	250.28	50,897	103.42	51,082	235.59	50,954	267.23	**49,611**	171.05	51,800	0.36	50,555	271.75
DAFJS15	49,326	277.73	49,120	158.41	50,696	137.86	50,910	95.17	47,211	221.78	47,414	152.93	**47,151**	282.47
DAFJS16	50,300	40.01	49,896	227.89	53,242	121.42	54,087	243.46	48,972	297.39	48,312	169.85	**47,661**	169.18
DAFJS17	53,388	122.05	52,783	157.04	52,905	171.59	53,513	237.27	**51,715**	55.57	52,459	2.64	52,081	94.05
DAFJS18	54,313	275.00	54,237	103.14	54,569	292.99	55,154	147.54	**52,624**	164.38	54,008	1.39	53,942	183.68
DAFJS19	39,480	94.81	39,312	77.07	41,046	124.48	40,931	162.08	38,740	130.01	38,941	6.77	**38,012**	150.83
DAFJS20	49,099	259.17	49,226	160.00	49,290	184.19	50,041	197.61	48,088	230.79	49,274	0.55	**47,680**	282.84
DAFJS21	54,311	5.09	54,163	118.65	54,851	216.71	55,002	20.65	**52,681**	264.76	53,996	9.95	53,196	291.79
DAFJS22	46,679	213.56	46,468	161.65	48,169	285.94	47,938	42.82	46,088	288.61	**44,851**	59.63	45,011	251.83
DAFJS23	37,459	248.79	36,733	262.51	38,106	230.28	38,487	243.86	**36,137**	166.72	36,190	13.53	36,373	49.89
DAFJS24	42,129	124.24	41,596	275.85	43,617	113.82	43,693	295.60	**40,079**	216.52	41,307	26.26	40,453	297.11
DAFJS25	53,096	203.63	52,727	274.34	54,913	107.96	55,011	220.93	52,604	264.38	50,965	248.46	**50,887**	116.74
DAFJS26	52,492	211.10	52,147	218.93	54,563	21.44	54,439	241.66	**50,006**	140.87	50,465	74.69	50,062	110.48
DAFJS27	57,249	244.03	56,428	299.00	58,674	259.45	59,078	30.84	55,599	293.44	55,398	29.41	**55,291**	270.23
DAFJS28	41,928	91.85	42,013	119.00	42,847	90.69	43,197	208.33	40,686	26.31	41,780	30.78	**40,431**	212.68
DAFJS29	49,863	282.73	49,538	162.49	51,391	213.00	51,881	188.89	48,953	155.28	48,942	2.58	**47,091**	293.59
DAFJS30	42,001	145.84	42,138	130.38	43,276	119.87	43,617	4.51	40,839	246.80	40,746	63.14	**40,325**	137.56
$\overline{C_{\max}}$	43,904.57		43,728.23		44,838.10		45,071.03		43,021.20		43,524.20		42,865.53	
$\overline{CV}$	0.0042		0.0043		0.0057		0.0051		0.0000		0.0000		0.0079	
#best	5		4		1		0		11		1		12	
gap(%)	2.91		2.52		5.12		5.62		1.13		2.27		0.74	
$\overline{\text{gap}}$(%)	3.49		3.15		5.99		6.30		1.13		2.27		1.69	

instance	ILS-RN		ILS-CN		GRASP-RN		GRASP-CN		TS-RN		TS-CN		SA	
	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time
YFJS01	**61,606**	1.10	**61,606**	1.12	64,456	0.19	62,327	9.05	**61,606**	3.43	62,366	0.05	62,124	2.24
YFJS02	**64,172**	0.59	**64,172**	1.35	65,967	0.99	66,394	0.87	**64,172**	1.65	64,805	0.16	**64,172**	28.72
YFJS03	**30,073**	0.36	**30,073**	0.26	**30,073**	0.08	**30,073**	0.02	**30,073**	0.01	**30,073**	0.00	**30,073**	0.59
YFJS04	**32,670**	0.51	**32,670**	0.37	33,302	226.25	32,802	166.10	**32,670**	1.53	33,302	0.01	**32,670**	0.65
YFJS05	**37,182**	7.66	**37,182**	6.16	37,670	0.33	37,255	0.79	**37,182**	4.32	38,682	0.01	37,682	0.68
YFJS06	**36,749**	68.53	**36,749**	11.18	37,293	94.00	36,812	17.99	**36,749**	137.03	37,943	0.17	37,107	1.01
YFJS07	**36,611**	102.75	**36,611**	57.41	37,159	3.75	37,159	8.11	**36,611**	77.85	37,159	0.34	**36,611**	1.07
YFJS08	**30,184**	0.23	**30,184**	0.51	30,267	0.46	30,384	227.80	30,267	1.93	32,595	0.00	**30,184**	1.10
YFJS09	**21,363**	12.05	**21,363**	1.63	**21,363**	130.79	**21,363**	141.45	**21,363**	10.87	**21,363**	0.89	21,450	2.64
YFJS10	**34,364**	10.57	**34,364**	8.30	35,389	0.03	34,689	11.28	35,251	0.14	35,116	0.00	**34,364**	38.75
YFJS11	**43,454**	158.07	43,544	164.27	45,613	12.83	45,649	33.90	44,094	0.53	46,983	0.00	**43,454**	81.93
YFJS12	**42,232**	177.44	42,397	131.72	45,207	91.71	44,084	69.04	42,597	221.30	46,047	0.01	**42,232**	4.28
YFJS13	**33,875**	34.01	**33,875**	18.97	34,401	4.99	34,240	50.77	**33,875**	34.33	34,335	0.20	**33,875**	3.48
YFJS14	**92,522**	58.55	93,435	164.41	96,665	218.81	96,415	81.18	93,409	43.69	97,155	0.11	93,812	117.48
YFJS15	91,555	188.91	91,006	203.20	96,697	210.91	97,126	71.71	**90,801**	216.11	95,840	0.40	91,216	253.51
YFJS16	90,957	133.94	90,591	138.68	95,294	289.35	96,871	283.70	91,357	103.88	100,478	0.11	**90,266**	150.82
YFJS17	78,747	203.06	**78,585**	294.38	85,736	0.00	85,736	0.00	78,982	276.75	83,631	0.18	79,177	283.88
YFJS18	85,415	255.16	**85,191**	56.76	92,382	86.94	94,410	189.36	92,710	70.33	97,864	0.04	86,119	263.05
YFJS19	70,527	299.05	**68,718**	122.25	78,364	245.26	79,427	14.98	74,284	184.84	71,701	8.45	69,662	57.52
YFJS20	72,991	221.56	**70,286**	170.65	80,074	29.02	81,808	36.17	71,131	268.24	74,652	0.97	73,215	138.10
$\overline{C_{\max}}$	54,362.45		54,130.10		57,168.60		57,251.20		54,959.20		57,104.50		54,473.25	
$\overline{CV}$	0.0031		0.0012		0.0069		0.0035		0.0000		0.0000		0.0066	
#best	14		15		2		2		10		2		10	
gap(%)	0.43		0.11		4.63		4.47		1.30		4.77		0.64	
$\overline{\text{gap}}$(%)	0.80		0.26		5.67		4.97		1.30		4.77		1.36	

Table A6: Results of applying the metaheuristics to the large-sized instances with learning index $\alpha = 0.2$.

instance	ILS-RN		ILS-CN		GRASP-RN		GRASP-CN		TS-RN		TS-CN		SA	
	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time
DAFJS01	**19,716**	42.15	**19,716**	93.26	19,969	181.01	19,994	1.14	20,137	0.35	21,082	0.01	20,092	0.59
DAFJS02	**22,147**	44.81	**22,147**	33.01	22,404	1.20	22,236	0.04	22,191	0.44	22,429	0.00	22,695	25.70
DAFJS03	38,916	229.98	38,994	282.80	39,280	94.82	40,454	111.31	38,928	105.79	40,724	7.17	**38,853**	42.33
DAFJS04	**39,117**	275.95	39,492	39.94	39,358	60.46	39,943	283.63	39,347	1.53	40,425	0.53	39,285	246.66
DAFJS05	**27,192**	175.00	27,221	266.61	27,438	55.27	28,152	92.66	27,401	0.83	28,767	0.14	27,453	0.86
DAFJS06	26,565	224.96	26,552	138.78	27,706	107.01	27,111	240.56	**26,535**	10.26	28,134	0.06	26,763	23.70
DAFJS07	35,287	15.78	35,262	252.14	36,318	168.90	36,309	197.95	35,735	8.84	35,074	74.05	**34,493**	271.96
DAFJS08	39,120	225.81	39,413	106.90	39,394	121.44	40,345	135.78	38,629	192.08	39,620	201.57	**38,605**	53.16
DAFJS09	30,627	93.31	30,945	4.12	31,421	14.02	31,004	255.55	**30,353**	79.14	32,242	0.11	30,833	46.92
DAFJS10	**30,880**	131.80	31,191	202.19	31,544	192.89	31,802	279.43	30,983	182.18	31,853	1.87	31,252	197.99
DAFJS11	40,212	68.69	40,549	187.16	41,870	242.15	42,027	84.21	39,227	253.69	39,751	124.91	**38,537**	252.92
DAFJS12	38,128	83.41	38,671	61.78	40,625	271.43	40,937	93.63	37,351	174.33	37,394	263.18	**37,030**	286.43
DAFJS13	37,672	48.20	37,699	163.35	37,806	244.56	38,010	273.30	37,324	59.41	38,811	0.32	**37,197**	242.77
DAFJS14	42,469	275.82	42,260	88.40	42,350	198.57	42,507	278.76	**41,203**	164.66	41,969	0.43	41,854	234.65
DAFJS15	42,125	156.71	42,172	190.76	43,215	186.52	43,776	40.58	**40,092**	252.92	40,345	40.70	40,465	180.67
DAFJS16	43,378	39.90	42,572	191.37	45,354	201.36	45,439	214.16	41,337	298.16	41,201	18.64	**40,487**	102.62
DAFJS17	43,350	289.76	43,143	193.23	43,698	248.64	43,121	173.83	**42,367**	156.85	42,622	6.69	43,083	219.02
DAFJS18	44,635	0.79	45,043	140.76	45,611	263.32	45,211	76.37	**43,899**	105.90	43,981	0.84	44,417	250.00
DAFJS19	34,218	89.09	34,104	52.36	35,401	198.41	35,425	94.86	33,501	43.82	34,187	0.66	**33,250**	238.49
DAFJS20	41,035	31.13	41,242	196.95	41,550	118.66	42,268	165.12	40,059	214.87	40,245	8.86	**39,812**	262.00
DAFJS21	45,000	50.80	44,519	215.72	45,409	141.46	44,883	249.15	44,190	297.58	43,840	25.11	**43,684**	173.57
DAFJS22	37,634	290.20	37,610	55.93	38,705	135.46	39,392	124.86	37,083	120.04	**36,276**	131.71	36,866	124.16
DAFJS23	33,077	95.58	32,960	10.78	33,756	107.08	33,543	44.93	**31,860**	39.67	32,412	15.90	32,017	188.80
DAFJS24	36,252	239.59	36,121	68.70	37,838	92.47	37,964	29.84	35,122	299.58	35,273	54.63	**34,422**	268.69
DAFJS25	44,698	215.09	45,061	277.98	46,361	44.37	47,007	1.62	43,654	198.69	43,176	115.77	**43,050**	244.18
DAFJS26	44,105	263.42	43,877	72.88	47,208	44.13	46,799	79.24	43,025	189.92	42,082	249.03	**42,080**	211.51
DAFJS27	47,643	81.38	47,648	269.30	48,748	188.12	49,400	275.14	**46,192**	284.20	46,887	17.37	46,510	204.55
DAFJS28	36,496	246.88	36,494	299.25	37,703	157.63	37,808	192.62	35,694	31.02	36,285	31.11	**35,637**	191.18
DAFJS29	42,923	148.58	43,334	244.96	45,174	110.20	44,735	40.40	42,218	267.02	42,648	22.61	**41,410**	281.11
DAFJS30	37,051	148.59	37,006	190.53	38,067	194.66	37,421	258.42	35,609	250.85	35,593	14.62	**35,242**	283.70
$\overline{C}_{\max}$	37,388.93		37,433.93		38,376.03		38,500.77		36,708.20		37,177.60		36,579.13	
$\overline{CV}$	0.0052		0.0034		0.0057		0.0051		0.0000		0.0000		0.0084	
#best	5		2		0		0		8		1		16	
gap(%)	2.58		2.71		5.26		5.57		0.89		2.38		0.59	
$\overline{\text{gap}}$(%)	3.31		3.17		6.04		6.23		0.89		2.38		1.53	

instance	ILS-RN		ILS-CN		GRASP-RN		GRASP-CN		TS-RN		TS-CN		SA	
	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time
YFJS01	**55,128**	1.60	**55,128**	1.29	55,458	0.18	56,790	2.84	55,693	0.92	57,900	0.02	**55,128**	0.73
YFJS02	**57,187**	1.20	**57,187**	2.25	59,000	0.32	58,470	0.21	**57,187**	1.60	57,634	0.18	**57,187**	10.48
YFJS03	**27,686**	0.03	**27,686**	0.03	**27,686**	0.00	**27,686**	0.01	**27,686**	0.02	28,772	0.00	**27,686**	0.47
YFJS04	**29,692**	0.29	**29,692**	1.66	**29,692**	23.14	30,683	160.12	**29,692**	0.20	**29,692**	0.05	**29,692**	0.49
YFJS05	**33,736**	2.48	**33,736**	3.45	34,430	2.41	33,856	182.52	**33,736**	12.59	35,577	0.01	**33,736**	0.90
YFJS06	**33,276**	35.79	**33,276**	111.43	33,557	23.84	33,557	6.56	**33,276**	32.04	33,892	0.09	33,892	0.78
YFJS07	**33,011**	91.58	**33,011**	82.50	33,352	30.98	33,059	126.97	33,092	4.76	33,317	0.01	33,092	0.55
YFJS08	**28,192**	0.35	**28,192**	0.62	28,597	0.07	28,355	0.14	**28,192**	0.09	28,638	0.00	**28,192**	0.82
YFJS09	**20,077**	3.69	**20,077**	2.38	**20,077**	33.66	**20,077**	22.83	**20,077**	9.58	**20,077**	0.23	20,121	165.05
YFJS10	**31,559**	33.20	**31,559**	119.22	32,582	0.72	32,159	0.57	31,998	0.77	33,111	0.00	**31,559**	74.40
YFJS11	40,204	102.14	**40,103**	105.15	41,079	136.76	40,619	20.45	40,350	6.53	41,547	0.00	40,319	2.58
YFJS12	**38,003**	173.04	**38,003**	154.12	39,993	21.82	38,618	35.05	38,392	268.63	41,620	0.02	38,392	103.93
YFJS13	**30,711**	88.73	**30,711**	89.29	31,823	1.59	31,518	55.89	30,714	180.10	33,449	0.01	30,874	1.20
YFJS14	79,051	186.80	79,386	200.32	81,558	137.18	84,481	176.28	**78,762**	142.02	83,816	0.18	79,486	38.18
YFJS15	79,374	139.96	78,660	198.52	84,181	133.33	84,277	230.45	79,092	191.45	87,555	0.10	**78,545**	69.16
YFJS16	78,426	106.57	78,333	198.20	83,753	2.48	85,453	284.11	78,491	112.32	83,317	0.43	**78,004**	83.92
YFJS17	66,188	295.79	**66,029**	82.47	73,682	0.00	73,682	0.00	66,802	275.82	71,161	0.04	66,388	158.14
YFJS18	73,732	298.52	**71,791**	115.35	80,279	246.11	82,201	17.58	76,031	298.20	77,670	1.43	73,078	243.24
YFJS19	61,283	149.66	**58,933**	123.03	70,115	118.17	68,545	185.31	60,919	297.73	61,788	4.36	59,633	299.10
YFJS20	60,960	148.73	**59,846**	186.72	68,963	197.07	71,916	244.37	60,137	237.53	62,668	1.26	61,277	170.47
$\overline{C}_{\max}$	47,873.80		47,566.95		50,492.85		50,800.10		48,015.95		50,160.05		47,814.05	
$\overline{CV}$	0.0035		0.0019		0.0056		0.0044		0.0000		0.0000		0.0079	
#best	12		17		3		2		8		2		9	
gap(%)	0.55		0.07		4.97		5.23		0.83		4.84		0.56	
$\overline{\text{gap}}$(%)	0.94		0.25		5.87		5.98		0.83		4.84		1.39	

Table A7: Results of applying the metaheuristics to the large-sized instances with learning index $\alpha = 0.3$.

instance	ILS-RN		ILS-CN		GRASP-RN		GRASP-CN		TS-RN		TS-CN		SA	
	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time
miniDAFJS01	**22,875**	0.01	**22,875**	0.05	**22,875**	8.09	**22,875**	16.21	**22,875**	0.00	23,264	0.00	**22,875**	0.40
miniDAFJS02	**22,708**	0.00	**22,708**	0.00	**22,708**	0.03	**22,708**	0.04	**22,708**	0.00	23,242	0.00	**22,708**	0.14
miniDAFJS03	**18,363**	0.00	**18,363**	0.00	**18,363**	0.00	**18,363**	0.00	**18,363**	0.00	**18,363**	0.00	**18,363**	0.00
miniDAFJS04	**20,498**	0.00	**20,498**	0.00	**20,498**	0.00	**20,498**	0.00	**20,498**	0.00	21,172	0.00	**20,498**	0.10
miniDAFJS05	**20,593**	0.01	**20,593**	0.00	**20,593**	2.01	**20,593**	6.26	**20,593**	0.01	21,324	0.00	**20,593**	0.32
miniDAFJS06	**22,867**	0.01	**22,867**	0.00	**22,867**	0.69	**22,867**	1.04	23,370	0.00	23,370	0.00	**22,867**	0.33
miniDAFJS07	**25,715**	0.01	**25,715**	0.01	**25,715**	0.15	**25,715**	0.31	**25,715**	0.00	26,978	0.00	**25,715**	0.18
miniDAFJS08	**19,878**	0.00	**19,878**	0.00	**19,878**	0.00	**19,878**	0.00	**19,878**	0.00	**19,878**	0.00	**19,878**	0.00
miniDAFJS09	**24,267**	0.03	**24,267**	0.16	**24,267**	1.30	**24,267**	8.19	**24,267**	0.00	24,304	0.00	**24,267**	0.41
miniDAFJS10	**20,336**	0.00	**20,336**	0.00	**20,336**	0.00	**20,336**	0.00	20,359	0.00	20,873	0.00	**20,336**	0.24
miniDAFJS11	**29,968**	0.01	**29,968**	0.02	**29,968**	1.11	**29,968**	0.00	33,263	0.00	33,689	0.00	**29,968**	0.42
miniDAFJS12	**18,670**	0.01	**18,670**	0.01	**18,670**	0.00	**18,670**	2.87	20,222	0.00	20,342	0.00	**18,670**	0.26
miniDAFJS13	**16,313**	0.00	**16,313**	0.00	**16,313**	0.00	**16,313**	0.00	**16,313**	0.00	17,091	0.00	**16,313**	0.01
miniDAFJS14	**23,140**	0.00	**23,140**	0.00	**23,140**	0.00	**23,140**	3.36	**23,140**	0.00	**23,140**	0.00	**23,140**	0.30
miniDAFJS15	**21,715**	0.00	**21,715**	0.00	**21,715**	0.00	**21,715**	0.24	**21,715**	0.00	21,882	0.00	**21,715**	0.16
miniDAFJS16	**25,426**	0.00	**25,426**	0.00	**25,426**	0.00	**25,426**	0.00	**25,426**	0.00	25,691	0.00	**25,426**	0.26
miniDAFJS17	**20,155**	0.00	**20,155**	0.00	**20,155**	0.00	**20,155**	0.00	**20,155**	0.00	**20,155**	0.00	**20,155**	0.19
miniDAFJS18	**18,135**	0.00	**18,135**	0.00	**18,135**	0.00	**18,135**	0.00	**18,135**	0.00	18,445	0.00	**18,135**	0.25
miniDAFJS19	**20,945**	0.00	**20,945**	0.00	**20,945**	0.00	**20,945**	0.00	**20,945**	0.00	21,293	0.00	**20,945**	0.18
miniDAFJS20	**21,838**	0.02	**21,838**	0.01	**21,838**	12.83	**21,838**	0.60	**21,838**	0.00	22,675	0.00	**21,838**	0.30
miniDAFJS21	**23,344**	0.60	**23,344**	0.14	**23,344**	50.28	**23,344**	74.78	23,456	0.02	24,554	0.00	23,563	35.44
miniDAFJS22	**25,923**	0.00	**25,923**	0.00	**25,923**	0.00	**25,923**	0.03	**25,923**	0.00	**25,923**	0.00	**25,923**	0.41
miniDAFJS23	**24,038**	3.23	**24,038**	0.85	24,998	0.03	24,998	0.01	24,585	0.11	24,438	0.00	24,438	0.49
miniDAFJS24	**24,579**	0.04	**24,579**	0.01	**24,579**	0.01	**24,579**	0.02	25,521	0.01	25,694	0.00	**24,579**	0.43
miniDAFJS25	**21,143**	0.01	**21,143**	0.00	**21,143**	23.97	**21,143**	0.00	**21,143**	0.00	22,682	0.00	**21,143**	0.33
miniDAFJS26	**21,120**	0.16	**21,120**	0.11	**21,120**	42.71	**21,120**	43.64	**21,120**	0.00	21,468	0.00	**21,120**	0.57
miniDAFJS27	**22,050**	0.14	**22,050**	0.20	22,106	0.12	**22,050**	0.24	22,106	0.03	22,106	0.00	22,106	0.41
miniDAFJS28	**22,708**	0.03	**22,708**	0.02	**22,708**	0.00	**22,708**	0.00	**22,708**	0.00	23,120	0.00	**22,708**	0.33
miniDAFJS29	**20,278**	0.01	**20,278**	0.01	**20,278**	0.01	**20,278**	0.00	**20,278**	0.00	**20,278**	0.00	**20,278**	0.33
miniDAFJS30	**23,558**	0.05	**23,558**	0.03	**23,558**	33.60	**23,558**	48.64	**23,558**	0.09	**23,558**	0.01	**23,558**	0.44
$\overline{C}_{\max}$	22,104.87		22,104.87		22,138.73		22,136.87		22,339.20		22,699.73		22,127.37	
$\overline{CV}$	0.0000		0.0000		0.0007		0.0003		0.0000		0.0000		0.0029	
#best	30		30		28		29		22		7		27	
gap(%)	0.00		0.00		0.14		0.13		0.95		2.61		0.10	
$\overline{\text{gap}}$(%)	0.00		0.00		0.18		0.17		0.95		2.61		0.35	

instance	ILS-RN		ILS-CN		GRASP-RN		GRASP-CN		TS-RN		TS-CN		SA	
	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time
miniYFJS01	**35,046**	0.00	**35,046**	0.00	**35,046**	0.00	**35,046**	0.00	**35,046**	0.00	35,243	0.00	**35,046**	0.20
miniYFJS02	**24,359**	0.01	**24,359**	0.01	**24,359**	0.00	**24,359**	0.00	**24,359**	0.00	25,654	0.00	**24,359**	0.29
miniYFJS03	**47,391**	0.00	**47,391**	0.01	**47,391**	0.00	**47,391**	0.14	52,098	0.00	53,381	0.00	**47,391**	0.35
miniYFJS04	**25,394**	0.00	**25,394**	0.00	**25,394**	0.00	**25,394**	0.00	**25,394**	0.00	**25,394**	0.00	**25,394**	0.00
miniYFJS05	**23,985**	0.00	**23,985**	0.00	**23,985**	0.00	**23,985**	0.00	**23,985**	0.00	26,371	0.00	**23,985**	0.24
miniYFJS06	**29,469**	0.01	**29,469**	0.01	**29,469**	0.35	**29,469**	0.25	30,508	0.00	30,952	0.00	**29,469**	0.30
miniYFJS07	**45,705**	0.01	**45,705**	0.05	**45,705**	0.01	**45,705**	14.81	**45,705**	0.01	47,524	0.00	**45,705**	0.43
miniYFJS08	**33,829**	0.01	**33,829**	0.03	**33,829**	0.00	**33,829**	0.16	33,883	0.00	34,338	0.00	**33,829**	0.38
miniYFJS09	**37,049**	0.00	**37,049**	0.00	**37,049**	0.00	**37,049**	0.00	**37,049**	0.00	37,354	0.00	**37,049**	0.30
miniYFJS10	**27,310**	0.01	**27,310**	0.00	**27,310**	0.01	**27,310**	0.00	**27,310**	0.00	**27,310**	0.00	**27,310**	0.33
miniYFJS11	**41,300**	0.00	**41,300**	0.01	**41,300**	0.00	**41,300**	0.05	**41,300**	0.00	41,423	0.00	**41,300**	0.39
miniYFJS12	**30,145**	0.11	**30,145**	0.06	**30,145**	55.26	**30,145**	64.48	**30,145**	0.03	33,329	0.00	**30,145**	0.54
miniYFJS13	**30,962**	0.01	**30,962**	0.00	**30,962**	0.00	**30,962**	0.01	**30,962**	0.01	**30,962**	0.00	**30,962**	0.35
miniYFJS14	**31,398**	0.01	**31,398**	0.01	**31,398**	0.00	**31,398**	0.00	**31,398**	0.00	**31,398**	0.00	**31,398**	0.38
miniYFJS15	**45,442**	0.00	**45,442**	0.00	**45,442**	0.00	**45,442**	0.00	**45,442**	0.00	**45,442**	0.00	**45,442**	0.39
miniYFJS16	**33,791**	0.00	**33,791**	0.02	**33,791**	0.00	**33,791**	19.85	**33,791**	0.00	**33,791**	0.00	**33,791**	0.41
miniYFJS17	**42,838**	0.02	**42,838**	0.02	**42,838**	0.00	**42,838**	13.56	**42,838**	0.00	44,181	0.00	**42,838**	0.46
miniYFJS18	**28,247**	0.01	**28,247**	0.01	**28,247**	0.01	**28,247**	0.00	**28,247**	0.00	**28,247**	0.00	**28,247**	0.41
miniYFJS19	**33,601**	0.04	**33,601**	0.05	**33,601**	0.00	**33,601**	0.01	**33,601**	0.01	36,706	0.00	**33,601**	0.45
miniYFJS20	**30,837**	0.00	**30,837**	0.00	**30,837**	0.00	**30,837**	0.00	**30,837**	0.02	33,385	0.00	**30,837**	0.41
miniYFJS21	**37,096**	0.74	**37,096**	0.34	37,308	0.03	37,275	0.23	**37,096**	0.41	37,275	0.02	37,308	70.41
miniYFJS22	**34,282**	0.03	**34,282**	0.03	**34,282**	0.01	**34,282**	0.08	**34,282**	0.00	34,917	0.00	**34,282**	0.48
miniYFJS23	**42,079**	0.04	**42,079**	0.02	**42,079**	0.07	**42,079**	38.95	**42,079**	0.01	**42,079**	0.00	**42,079**	0.51
miniYFJS24	**30,905**	0.11	**30,905**	0.08	30,965	69.07	**30,905**	53.76	**30,905**	0.14	33,144	0.00	**30,905**	0.56
miniYFJS25	**36,170**	0.10	**36,170**	0.13	**36,170**	76.58	**36,170**	1.25	**36,170**	0.11	37,251	0.00	**36,170**	0.94
miniYFJS26	**51,466**	0.01	**51,466**	0.02	**51,466**	0.83	**51,466**	1.23	54,534	0.03	51,793	0.00	**51,466**	0.49
miniYFJS27	**36,719**	0.01	**36,719**	0.01	**36,719**	0.00	**36,719**	0.02	**36,719**	0.01	36,823	0.00	**36,719**	0.47
miniYFJS28	**34,509**	0.04	**34,509**	0.03	**34,509**	87.68	**34,509**	52.00	**34,509**	0.04	**34,509**	0.02	**34,509**	31.22
miniYFJS29	**39,798**	0.06	**39,798**	0.14	**39,798**	0.07	**39,798**	0.05	39,949	0.01	40,891	0.00	**39,798**	0.63
miniYFJS30	**33,974**	0.03	**33,974**	0.06	**33,974**	0.05	**33,974**	0.08	**33,974**	0.12	**33,974**	0.00	**33,974**	0.52
$\overline{C}_{\max}$	35,169.87		35,169.87		35,178.93		35,175.83		35,470.50		36,168.03		35,176.93	
$\overline{CV}$	0.0000		0.0000		0.0023		0.0004		0.0000		0.0000		0.0028	
#best	30		30		28		29		25		10		29	
gap(%)	0.00		0.00		0.03		0.02		0.67		2.92		0.02	
$\overline{\text{gap}}$(%)	0.00		0.00		0.39		0.06		0.67		2.92		0.18	

Table A8: Results of applying the metaheuristics to the small-sized instances with learning index $\alpha = 0.1$.

instance	ILS-RN		ILS-CN		GRASP-RN		GRASP-CN		TS-RN		TS-CN		SA	
	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time
miniDAFJS01	**21,327**	0.02	**21,327**	0.03	**21,327**	0.03	**21,327**	28.07	21,546	0.00	21,546	0.00	**21,327**	0.33
miniDAFJS02	**20,635**	0.01	**20,635**	0.01	**20,635**	0.05	**20,635**	0.04	**20,635**	0.00	22,161	0.00	**20,635**	0.19
miniDAFJS03	**17,972**	0.00	**17,972**	0.00	**17,972**	0.00	**17,972**	0.00	**17,972**	0.00	**17,972**	0.00	**17,972**	0.00
miniDAFJS04	**19,602**	0.00	**19,602**	0.00	**19,602**	0.00	**19,602**	0.00	**19,602**	0.00	20,691	0.00	**19,602**	0.11
miniDAFJS05	**18,803**	0.00	**18,803**	0.01	**18,803**	3.57	**18,803**	4.21	**18,803**	0.01	**18,803**	0.00	**18,803**	0.28
miniDAFJS06	**20,568**	0.04	**20,568**	0.04	**20,568**	0.01	**20,568**	0.01	20,783	0.00	20,783	0.00	**20,568**	0.31
miniDAFJS07	**24,715**	0.00	**24,715**	0.01	**24,715**	0.10	**24,715**	0.63	**24,715**	0.00	**24,715**	0.00	**24,715**	0.22
miniDAFJS08	**18,857**	0.00	**18,857**	0.00	**18,857**	0.00	**18,857**	0.00	**18,857**	0.00	**18,857**	0.00	**18,857**	0.00
miniDAFJS09	**22,660**	0.02	**22,660**	0.05	**22,660**	4.57	**22,660**	2.76	22,730	0.01	22,730	0.00	**22,660**	0.31
miniDAFJS10	**18,823**	0.00	**18,823**	0.00	**18,823**	0.00	**18,823**	0.00	**18,823**	0.00	20,180	0.00	**18,823**	0.18
miniDAFJS11	**27,455**	0.02	**27,455**	0.01	**27,455**	2.05	**27,455**	0.00	30,550	0.00	30,941	0.00	**27,455**	0.35
miniDAFJS12	**17,874**	0.01	**17,874**	0.01	**17,874**	0.92	**17,874**	1.47	19,624	0.00	18,974	0.00	**17,874**	0.24
miniDAFJS13	**15,143**	0.00	**15,143**	0.00	**15,143**	0.00	**15,143**	0.00	**15,143**	0.00	16,593	0.00	**15,143**	0.01
miniDAFJS14	**21,817**	0.00	**21,817**	0.00	**21,817**	0.00	**21,817**	3.31	**21,817**	0.00	21,990	0.00	**21,817**	0.27
miniDAFJS15	**20,236**	0.00	**20,236**	0.00	**20,236**	0.00	**20,236**	0.00	**20,236**	0.00	**20,236**	0.00	**20,236**	0.14
miniDAFJS16	**24,114**	0.00	**24,114**	0.00	**24,114**	0.00	**24,114**	0.01	**24,114**	0.00	24,593	0.00	**24,114**	0.27
miniDAFJS17	**19,145**	0.00	**19,145**	0.00	**19,145**	0.00	**19,145**	0.00	**19,145**	0.00	**19,145**	0.00	**19,145**	0.15
miniDAFJS18^{+}	**17,270**	0.00	**17,270**	0.00	**17,270**	0.00	**17,270**	0.00	**17,270**	0.00	17,966	0.00	**17,270**	0.25
miniDAFJS19	**19,642**	0.00	**19,642**	0.00	**19,642**	0.00	**19,642**	0.00	**19,642**	0.00	20,107	0.00	**19,642**	0.16
miniDAFJS20	**20,086**	0.04	**20,086**	0.01	**20,086**	24.76	**20,086**	1.87	21,286	0.00	20,646	0.00	**20,086**	0.28
miniDAFJS21	**21,352**	0.21	**21,352**	0.92	21,433	43.93	21,536	93.99	**21,352**	0.04	21,783	0.01	21,433	1.51
miniDAFJS22	**23,852**	0.01	**23,852**	0.02	**23,852**	38.91	**23,852**	24.30	24,273	0.00	24,273	0.00	**23,852**	0.41
miniDAFJS23^{+}	**22,390**	2.60	**22,390**	0.82	23,228	0.02	23,300	0.02	**22,390**	0.31	22,491	0.02	23,120	0.49
miniDAFJS24	**22,521**	0.07	**22,521**	0.07	**22,521**	19.38	**22,521**	44.36	**22,521**	0.03	22,551	0.00	**22,521**	0.40
miniDAFJS25	**19,809**	0.01	**19,809**	0.03	**19,809**	38.66	**19,809**	0.00	19,913	0.01	19,913	0.00	**19,809**	0.35
miniDAFJS26	**19,724**	0.20	**19,724**	0.23	**19,724**	24.84	**19,724**	48.07	**19,724**	0.00	19,994	0.00	19,773	0.33
miniDAFJS27	**20,245**	0.13	**20,245**	0.03	**20,245**	0.07	**20,245**	35.36	**20,245**	0.02	20,646	0.00	**20,245**	0.47
miniDAFJS28	**20,635**	0.02	**20,635**	0.04	**20,635**	0.00	**20,635**	0.00	**20,635**	0.00	21,693	0.00	**20,635**	0.38
miniDAFJS29	**19,201**	0.01	**19,201**	0.00	**19,201**	1.39	**19,201**	0.00	**19,201**	0.00	19,448	0.00	**19,201**	0.31
miniDAFJS30	**21,552**	0.06	**21,552**	0.21	21,698	46.08	**21,552**	63.33	**21,552**	0.15	21,645	0.00	**21,552**	0.45
$\overline{C_{\max}}$	20,600.83		20,600.83		20,636.33		20,637.30		20,836.63		21,135.53		20,629.50	
$\overline{CV}$	0.0000		0.0000		0.0008		0.0014		0.0000		0.0000		0.0028	
#best	30		30		27		28		22		6		27	
gap(%)	0.00		0.00		0.16		0.16		1.06		2.60		0.13	
$\overline{\text{gap}}$(%)	0.00		0.00		0.24		0.28		1.06		2.60		0.42	

instance	ILS–RN		ILS–CN		GRASP–RN		GRASP–CN		TS–RN		TS–CN		SA	
	$C_{\max}$	time	$C_{\max}$	time	$C_{\max}$	time	$C_{\max}$	time	$C_{\max}$	time	$C_{\max}$	time	$C_{\max}$	time
miniYFJS01	**33,132**	0.00	**33,132**	0.00	**33,132**	0.00	**33,132**	4.67	**33,132**	0.00	34,443	0.00	**33,132**	0.28
miniYFJS02	**23,100**	0.00	**23,100**	0.01	**23,100**	0.00	**23,100**	0.00	**23,100**	0.00	25,032	0.00	**23,100**	0.30
miniYFJS03	**42,896**	0.00	**42,896**	0.01	**42,896**	0.00	**42,896**	0.01	46,806	0.00	49,111	0.00	**42,896**	0.34
miniYFJS04	**24,485**	0.00	**24,485**	0.00	**24,485**	0.00	**24,485**	0.00	**24,485**	0.00	**24,485**	0.00	**24,485**	0.31
miniYFJS05	**23,597**	0.00	**23,597**	0.00	**23,597**	0.00	**23,597**	0.00	**23,597**	0.00	**23,597**	0.00	**23,597**	0.25
miniYFJS06	**28,655**	0.01	**28,655**	0.01	**28,655**	0.00	**28,655**	0.00	**28,655**	0.01	28,750	0.01	**28,655**	0.32
miniYFJS07	**42,239**	0.00	**42,239**	0.05	**42,239**	0.01	**42,239**	7.75	**42,239**	0.00	45,493	0.00	**42,239**	0.40
miniYFJS08	**31,471**	0.02	**31,471**	0.15	**31,471**	0.00	**31,471**	22.11	31,597	0.00	32,669	0.00	**31,471**	0.42
miniYFJS09	**35,250**	0.02	**35,250**	0.01	**35,250**	4.98	**35,250**	0.01	35,794	0.00	36,098	0.00	**35,250**	0.32
miniYFJS10	**26,145**	0.01	**26,145**	0.00	**26,145**	5.41	**26,145**	0.00	**26,145**	0.00	**26,145**	0.00	**26,145**	0.32
miniYFJS11	**38,545**	0.01	**38,545**	0.02	**38,545**	3.71	**38,545**	7.90	**38,545**	0.00	38,756	0.00	**38,545**	0.44
miniYFJS12	**27,895**	0.15	**27,895**	0.09	**27,895**	48.02	28,281	0.01	**27,895**	0.10	30,851	0.00	**27,895**	7.80
miniYFJS13	**28,120**	0.01	**28,120**	0.00	**28,120**	0.00	**28,120**	0.02	**28,120**	0.00	**28,120**	0.00	**28,120**	0.38
miniYFJS14	**29,682**	0.02	**29,682**	0.00	**29,682**	0.00	**29,682**	0.00	**29,682**	0.00	**29,682**	0.00	**29,682**	0.37
miniYFJS15	**41,619**	0.00	**41,619**	0.00	**41,619**	0.00	**41,619**	0.01	**41,619**	0.00	42,415	0.00	**41,619**	0.41
miniYFJS16	**31,280**	0.02	**31,280**	0.02	**31,280**	0.00	**31,280**	0.00	**31,280**	0.05	32,366	0.00	**31,280**	0.48
miniYFJS17	**40,388**	0.01	**40,388**	0.03	**40,388**	0.00	**40,388**	10.05	**40,388**	0.01	41,316	0.00	**40,388**	0.44
miniYFJS18	**26,297**	0.00	**26,297**	0.01	**26,297**	0.00	**26,297**	0.01	**26,297**	0.01	**26,297**	0.00	**26,297**	0.38
miniYFJS19	**30,717**	0.02	**30,717**	0.06	**30,717**	0.01	**30,717**	0.01	**30,717**	0.02	33,965	0.00	**30,717**	0.47
miniYFJS20	**28,832**	0.03	**28,832**	0.02	**28,832**	0.04	**28,832**	0.03	**28,832**	0.03	31,304	0.00	**28,832**	0.38
miniYFJS21	**34,811**	0.13	**34,811**	0.09	**34,811**	0.07	**34,811**	0.06	34,876	0.00	**34,811**	0.05	**34,811**	24.18
miniYFJS22	**31,702**	0.01	**31,702**	0.03	**31,702**	0.03	31,747	0.17	**31,702**	0.03	31,946	0.00	**31,702**	0.54
miniYFJS23	**38,639**	0.08	**38,639**	0.14	**38,639**	33.88	**38,639**	39.04	**38,639**	0.04	39,721	0.00	**38,639**	0.50
miniYFJS24	**28,884**	0.05	**28,884**	0.05	30,200	21.84	**28,884**	46.64	**28,884**	0.06	30,568	0.00	**28,884**	0.53
miniYFJS25	**34,231**	0.04	**34,231**	0.04	**34,231**	26.59	**34,231**	41.59	**34,231**	0.01	35,256	0.00	**34,231**	0.46
miniYFJS26	**47,519**	0.01	**47,519**	0.01	**47,519**	1.48	**47,519**	5.00	49,202	0.04	48,066	0.00	**47,519**	0.50
miniYFJS27	**34,042**	0.08	**34,042**	0.09	**34,042**	0.03	**34,042**	1.61	**34,042**	0.01	34,225	0.00	**34,042**	0.55
miniYFJS28	**32,080**	0.04	**32,080**	0.04	**32,080**	69.04	**32,080**	43.21	**32,080**	0.02	32,852	0.00	**32,080**	0.60
miniYFJS29	**36,093**	0.11	**36,093**	0.31	**36,093**	0.13	**36,093**	0.03	36,561	0.00	36,561	0.00	**36,093**	0.47
miniYFJS30	**31,888**	0.53	**31,888**	0.42	**31,888**	59.15	**31,888**	46.43	**31,888**	0.26	33,118	0.00	**31,888**	1.15
$\overline{C_{\max}}$	32,807.80		32,807.80		32,851.67		32,822.17		33,034.33		33,933.97		32,807.80	
$\overline{CV}$	0.0000		0.0000		0.0021		0.0013		0.0000		0.0000		0.0025	
#best	30		30		29		28		24		7		30	
gap(%)	0.00		0.00		0.15		0.05		0.54		3.36		0.00	
$\overline{\text{gap}}$(%)	0.00		0.00		0.43		0.17		0.54		3.36		0.24	

Table A9: Results of applying the metaheuristics to the small-sized instances with learning index $\alpha = 0.2$.

instance	ILS-RN		ILS-CN		GRASP-RN		GRASP-CN		TS-RN		TS-CN		SA	
	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time
miniDAFJS01	**19,443**	0.03	**19,443**	0.06	**19,443**	24.05	**19,443**	54.44	19,652	0.00	19,702	0.00	**19,443**	0.34
miniDAFJS02	**18,916**	0.00	**18,916**	0.02	**18,916**	0.02	**18,916**	0.05	**18,916**	0.00	21,136	0.00	**18,916**	0.25
miniDAFJS03	**17,419**	0.00	**17,419**	0.00	**17,419**	0.00	**17,419**	0.00	**17,419**	0.00	17,619	0.00	**17,419**	0.16
miniDAFJS04	**18,800**	0.00	**18,800**	0.00	**18,800**	0.00	**18,800**	0.00	**18,800**	0.00	20,124	0.00	**18,800**	0.13
miniDAFJS05	**17,596**	0.05	**17,596**	0.07	**17,596**	3.01	**17,596**	35.51	17,657	0.02	17,657	0.00	**17,596**	0.30
miniDAFJS06	**18,692**	0.03	**18,692**	0.05	**18,692**	0.00	**18,692**	0.04	**18,692**	0.00	18,952	0.00	**18,692**	0.33
miniDAFJS07	**24,256**	0.00	**24,256**	0.01	**24,256**	0.11	**24,256**	0.43	**24,256**	0.00	24,636	0.00	**24,256**	0.23
miniDAFJS08	**17,900**	0.00	**17,900**	0.00	**17,900**	0.00	**17,900**	0.00	**17,900**	0.00	**17,900**	0.00	**17,900**	0.03
miniDAFJS09	**20,797**	0.01	**20,797**	0.05	**20,797**	0.05	**20,797**	0.00	**20,797**	0.00	21,298	0.00	**20,797**	0.32
miniDAFJS10	**17,395**	0.00	**17,395**	0.00	**17,395**	0.00	**17,395**	0.10	**17,395**	0.00	19,472	0.00	**17,395**	0.22
miniDAFJS11	**25,304**	0.02	**25,304**	0.01	**25,304**	1.06	**25,304**	0.00	27,387	0.00	25,554	0.00	**25,304**	0.37
miniDAFJS12	**17,105**	0.01	**17,105**	0.01	**17,105**	1.01	**17,105**	1.98	**17,105**	0.00	17,745	0.00	**17,105**	0.25
miniDAFJS13	**14,077**	0.00	**14,077**	0.00	**14,077**	0.00	**14,077**	0.00	**14,077**	0.00	**14,077**	0.00	**14,077**	0.01
miniDAFJS14	**20,620**	0.00	**20,620**	0.00	**20,620**	0.00	**20,620**	7.65	**20,620**	0.00	20,711	0.00	**20,620**	0.28
miniDAFJS15	**18,625**	0.00	**18,625**	0.00	**18,625**	0.00	**18,625**	0.00	**18,625**	0.00	**18,625**	0.00	**18,625**	0.18
miniDAFJS16	**22,734**	0.01	**22,734**	0.01	**22,734**	0.00	**22,734**	4.24	22,939	0.00	22,939	0.00	**22,734**	0.29
miniDAFJS17	**18,253**	0.00	**18,253**	0.00	**18,253**	0.00	**18,253**	0.00	**18,253**	0.00	**18,253**	0.00	**18,253**	0.09
miniDAFJS18	**16,495**	0.00	**16,495**	0.00	**16,495**	0.00	**16,495**	0.01	**16,495**	0.00	17,270	0.00	**16,495**	0.24
miniDAFJS19	**18,474**	0.00	**18,474**	0.00	**18,474**	0.00	**18,474**	0.00	**18,474**	0.00	19,030	0.00	**18,474**	0.14
miniDAFJS20	**18,521**	0.04	**18,521**	0.03	**18,521**	7.15	**18,521**	0.64	19,587	0.00	18,602	0.00	**18,521**	0.27
miniDAFJS21^{+}	**19,430**	0.34	**19,430**	0.52	**19,430**	40.61	19,448	49.57	20,368	0.00	20,624	0.00	19,550	0.45
miniDAFJS22	**22,322**	0.04	**22,322**	0.22	**22,322**	56.51	**22,322**	84.04	22,396	0.01	22,767	0.00	**22,322**	0.42
miniDAFJS23^{+}	**20,932**	1.75	**20,932**	0.63	21,372	0.02	21,372	0.02	21,176	0.01	21,540	0.01	21,031	0.27
miniDAFJS24	**20,389**	0.03	**20,389**	0.04	**20,389**	0.01	**20,389**	0.02	20,938	0.00	**20,389**	0.01	**20,389**	0.39
miniDAFJS25	**18,400**	0.01	**18,400**	0.00	**18,400**	0.00	**18,400**	35.30	**18,400**	0.01	18,472	0.00	**18,400**	0.36
miniDAFJS26	**18,396**	0.04	**18,396**	0.02	**18,396**	17.47	**18,396**	55.17	18,870	0.01	19,427	0.00	**18,396**	0.37
miniDAFJS27	**18,501**	0.12	**18,501**	0.04	**18,501**	79.26	**18,501**	22.10	**18,501**	0.02	20,586	0.00	**18,501**	0.38
miniDAFJS28	**18,762**	0.11	**18,762**	0.06	**18,762**	0.00	**18,762**	3.62	**18,762**	0.01	20,020	0.00	18,916	0.35
miniDAFJS29	**18,253**	0.01	**18,253**	0.00	**18,253**	1.39	**18,253**	0.01	**18,253**	0.00	18,560	0.00	**18,253**	0.31
miniDAFJS30^{+}	**19,504**	0.14	**19,504**	0.46	**19,504**	31.31	20,137	30.57	**19,504**	0.27	19,659	0.00	19,618	0.38
$\overline{C_{\max}}$	19,210.37		19,210.37		19,225.03		19,246.73		19,407.13		19,778.20		19,226.60	
$\overline{CV}$	0.0000		0.0000		0.0019		0.0014		0.0000		0.0000		0.0048	
#best	30		30		29		27		20		5		26	
gap(%)	0.00		0.00		0.07		0.18		0.93		3.02		0.08	
$\overline{\text{gap}}$(%)	0.00		0.00		0.24		0.32		0.93		3.02		0.61	

instance	ILS–RN		ILS–CN		GRASP–RN		GRASP–CN		TS–RN		TS–CN		SA	
	$C_{\max}$	time	$C_{\max}$	time	$C_{\max}$	time	$C_{\max}$	time	$C_{\max}$	time	$C_{\max}$	time	$C_{\max}$	time
miniYFJS01	**31,008**	0.00	**31,008**	0.00	**31,008**	0.00	**31,008**	13.25	**31,008**	0.00	32,506	0.00	**31,008**	0.25
miniYFJS02	**22,010**	0.00	**22,010**	0.02	**22,010**	0.00	**22,010**	0.00	**22,010**	0.01	24,146	0.00	**22,010**	0.28
miniYFJS03	**38,935**	0.00	**38,935**	0.00	**38,935**	0.00	**38,935**	0.00	41,699	0.00	42,339	0.00	**38,935**	0.34
miniYFJS04	**23,774**	0.00	**23,774**	0.00	**23,774**	0.00	**23,774**	0.00	**23,774**	0.00	24,017	0.00	**23,774**	0.28
miniYFJS05	**22,843**	0.00	**22,843**	0.00	**22,843**	0.00	**22,843**	0.00	**22,843**	0.00	23,236	0.00	**22,843**	0.24
miniYFJS06	**27,366**	0.00	**27,366**	0.00	**27,366**	0.00	**27,366**	0.00	**27,366**	0.00	**27,366**	0.00	**27,366**	0.00
miniYFJS07	**38,932**	0.00	**38,932**	0.06	**38,932**	0.24	**38,932**	13.11	**38,932**	0.01	41,487	0.00	**38,932**	0.39
miniYFJS08	**29,464**	0.02	**29,464**	0.04	**29,464**	18.16	**29,464**	9.52	29,898	0.00	30,276	0.00	**29,464**	0.43
miniYFJS09	**33,763**	0.01	**33,763**	0.01	**33,763**	0.00	**33,763**	0.00	**33,763**	0.04	34,357	0.00	**33,763**	0.30
miniYFJS10	**25,072**	0.15	**25,072**	0.10	**25,072**	30.31	**25,072**	49.47	**25,072**	0.00	25,093	0.00	**25,072**	6.20
miniYFJS11	**36,307**	0.02	**36,307**	0.05	**36,307**	2.55	**36,307**	9.76	**36,307**	0.01	39,637	0.00	**36,307**	0.44
miniYFJS12	**26,219**	0.04	**26,219**	0.06	26,571	0.01	**26,219**	0.09	**26,219**	0.22	27,829	0.00	**26,219**	0.44
miniYFJS13	**25,619**	0.00	**25,619**	0.01	**25,619**	0.00	**25,619**	0.02	**25,619**	0.00	25,881	0.00	**25,619**	0.36
miniYFJS14	**27,428**	0.03	**27,428**	0.04	**27,428**	0.86	**27,428**	0.09	**27,428**	0.05	**27,428**	0.00	**27,428**	0.52
miniYFJS15	**38,256**	0.01	**38,256**	0.03	**38,256**	0.00	**38,256**	7.91	**38,256**	0.01	39,294	0.00	**38,256**	0.50
miniYFJS16	**29,442**	0.01	**29,442**	0.01	**29,442**	0.01	**29,442**	0.00	**29,442**	0.01	30,386	0.00	**29,442**	0.41
miniYFJS17	**37,465**	0.02	**37,465**	0.02	**37,465**	0.00	**37,465**	0.10	**37,465**	0.12	38,739	0.00	**37,465**	0.39
miniYFJS18	**25,067**	0.02	**25,067**	0.00	**25,067**	0.00	**25,067**	0.00	**25,067**	0.00	**25,067**	0.00	**25,067**	0.41
miniYFJS19	**29,207**	0.01	**29,207**	0.02	**29,207**	0.06	**29,207**	0.29	**29,207**	0.04	30,218	0.00	**29,207**	0.45
miniYFJS20	**27,091**	0.07	**27,091**	0.03	**27,091**	0.12	**27,091**	0.22	**27,091**	0.04	28,590	0.00	**27,091**	0.42
miniYFJS21	**32,166**	0.30	**32,166**	0.16	32,238	0.06	**32,166**	0.04	**32,166**	0.01	33,027	0.00	**32,166**	0.59
miniYFJS22	**28,985**	0.06	**28,985**	0.06	**28,985**	0.04	**28,985**	1.07	**28,985**	0.00	29,154	0.00	**28,985**	0.54
miniYFJS23^{+}	**35,441**	0.14	**35,441**	0.18	**35,441**	15.36	**35,441**	33.00	**35,441**	0.20	35,961	0.00	**35,441**	0.61
miniYFJS24	**27,023**	0.09	**27,023**	0.03	27,395	52.85	**27,023**	53.26	**27,023**	0.10	31,200	0.00	**27,023**	0.48
miniYFJS25^{+}	**32,346**	0.07	**32,346**	0.07	**32,346**	0.38	32,465	0.55	**32,346**	0.02	32,513	0.00	**32,346**	3.21
miniYFJS26	**43,452**	0.01	**43,452**	0.05	**43,452**	0.14	**43,452**	25.93	44,207	0.00	44,633	0.00	**43,452**	0.54
miniYFJS27	**31,571**	0.00	**31,571**	0.01	**31,571**	0.01	**31,571**	0.49	**31,571**	0.01	31,877	0.00	**31,571**	0.51
miniYFJS28	**30,428**	0.01	**30,428**	0.01	**30,428**	74.61	**30,428**	28.22	**30,428**	0.04	**30,428**	0.00	**30,428**	0.47
miniYFJS29	**32,826**	0.26	**32,826**	0.36	**32,826**	0.31	**32,826**	11.23	**32,826**	0.45	33,455	0.00	**32,826**	0.48
miniYFJS30	**29,848**	0.20	**29,848**	0.12	**29,848**	5.38	**29,848**	54.66	**29,848**	0.04	31,093	0.00	**29,848**	0.76
$\overline{C_{\max}}$	30,645.13		30,645.13		30,671.67		30,649.10		30,776.90		31,707.77		30,645.13	
$\overline{CV}$	0.0000		0.0000		0.0025		0.0015		0.0000		0.0000		0.0018	
#best	30		30		27		29		27		4		30	
gap(%)	0.00		0.00		0.10		0.01		0.34		3.41		0.00	
$\overline{\text{gap}}$(%)	0.00		0.00		0.28		0.16		0.34		3.41		0.12	

Table A10: Results of applying the metaheuristics to the small-sized instances with learning index $\alpha = 0.3$.

instance	ILS-RN		ILS-CN		GRASP-RN		GRASP-CN		TS-RN		TS-CN		SA		GVNSWAF	
	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time
mfjs01	**45,306**	0.01	**45,306**	0.00	**45,306**	0.50	**45,306**	0.02	46,264	0.00	46,264	0.00	**45,306**	0.25	**45,306**	0.01
mfjs02	**42,986**	0.04	**42,986**	0.01	**42,986**	4.43	**42,986**	4.89	**42,986**	0.01	**42,986**	0.00	**42,986**	0.26	**42,986**	4.13
mfjs03	**45,331**	0.03	**45,331**	0.02	**45,331**	0.90	**45,331**	0.68	**45,331**	0.01	**45,331**	0.00	**45,331**	0.74	**45,331**	0.08
mfjs04	**52,012**	1.23	**52,012**	1.20	52,480	116.49	**52,012**	197.36	**52,012**	0.30	54,075	0.01	52,630	75.60	**52,012**	65.89
mfjs05	**47,630**	0.23	**47,630**	0.12	**47,630**	133.82	**47,630**	0.22	**47,630**	0.02	**47,630**	0.02	49,988	0.47	**47,630**	0.34
mfjs06	**59,523**	18.24	**59,523**	3.35	**59,523**	124.16	**59,523**	226.29	**59,523**	1.41	60,854	0.01	60,402	0.43	**59,523**	0.40
mfjs07	**80,877**	76.04	**80,877**	36.62	82,438	24.41	81,453	28.70	**80,877**	14.62	82,686	0.06	81,371	3.15	81,364	40.51
mfjs08	80,687	205.78	**80,305**	70.59	82,481	140.18	83,273	76.08	**80,305**	216.74	83,095	0.06	82,031	1.98	82,842	7.91
mfjs09	96,922	28.84	**96,236**	110.23	100,332	90.60	99,159	160.79	98,028	93.66	99,576	0.58	97,358	4.42	98,417	76.61
mfjs10	109,183	149.45	**107,489**	227.23	115,762	271.38	114,670	38.10	110,314	165.43	110,721	0.28	109,326	25.97	114,495	11.20
$\overline{C}_{\max}$	66,045.70		65,769.50		67,426.90		67,134.30		66,327.00		67,321.80		66,672.90		66,990.60	
$\overline{CV}$	0.0013		0.0017		0.0056		0.0027		0.0000		0.0000		0.0094		0.0083	
#best	7		10		5		6		7		3		3		6	
gap(%)	0.28		0.00		1.75		1.41		0.66		2.05		1.33		1.25	
$\overline{\text{gap}}$(%)	0.48		0.19		2.32		1.86		0.66		2.05		2.26		2.22	

instance	ILS-RN		ILS-CN		GRASP-RN		GRASP-CN		TS-RN		TS-CN		SA		GVNSWAF	
	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time
MK01	**3,507**	173.24	**3,507**	81.29	3,529	19.87	3,530	62.29	3,529	0.68	3,714	0.00	3,529	3.50	3,714	0.28
MK02	2,347	38.47	2,327	176.39	2,348	147.70	2,404	203.40	2,332	17.24	2,372	0.09	**2,305**	84.67	2,348	31.99
MK03	**17,316**	5.97	**17,316**	1.31	**17,316**	1.61	**17,316**	0.07	**17,316**	1.78	17,665	0.05	**17,316**	2.83	**17,316**	8.31
MK04	**5,173**	112.01	**5,173**	49.49	5,255	189.35	5,187	197.41	5,665	0.79	5,792	0.05	**5,173**	11.75	5,725	3.24
MK05	13,674	260.60	13,666	187.22	13,747	291.55	13,648	72.05	13,657	38.48	13,877	0.27	**13,627**	94.51	13,710	119.96
MK06	5,045	183.79	5,040	114.41	5,360	129.50	5,429	59.20	5,055	287.61	**4,982**	23.00	**4,982**	184.62	5,380	61.45
MK07	11,589	173.35	11,551	259.27	11,950	291.84	12,055	162.89	11,488	266.54	11,612	14.97	**11,220**	256.56	11,551	42.73
MK08	39,002	263.25	**38,999**	245.98	39,213	198.56	39,181	135.52	39,660	1.29	39,307	0.51	39,177	49.61	39,026	265.08
MK09	25,263	237.96	**24,919**	43.07	27,103	135.39	26,211	180.98	28,421	278.25	30,456	0.06	24,970	200.21	25,545	253.28
MK10	16,913	153.92	**16,802**	218.45	19,141	204.30	19,186	202.84	16,855	255.05	18,605	3.03	17,060	171.62	18,359	131.33
MK11	46,820	208.18	46,708	144.04	47,161	54.49	47,180	78.61	**46,599**	44.05	46,800	5.82	46,602	133.49	47,091	203.88
MK12	39,877	128.22	**39,869**	32.84	39,955	190.20	39,911	120.55	42,093	0.56	42,232	0.05	39,949	63.10	39,907	104.38
MK13	32,585	216.79	32,410	234.61	35,149	185.28	34,757	284.68	32,349	221.11	**31,960**	109.03	32,954	218.41	32,967	299.84
MK14	52,376	209.56	**52,349**	165.80	53,269	1.04	52,514	212.09	56,617	6.21	56,143	0.90	52,531	271.72	52,406	109.93
MK15	28,343	223.43	**28,069**	98.35	30,525	275.34	30,278	186.99	28,313	279.37	31,336	1.18	28,916	204.11	29,577	299.38
$\overline{C}_{\max}$	22,655.33		22,580.33		23,401.40		23,252.47		23,329.93		23,790.20		22,687.40		22,974.80	
$\overline{CV}$	0.0066		0.0025		0.0069		0.0048		0.0000		0.0000		0.0049		0.0182	
#best	3		9		1		1		2		2		6		1	
gap(%)	0.82		0.47		4.28		3.99		3.15		5.81		0.63		3.44	
$\overline{\text{gap}}$(%)	1.52		0.78		5.25		4.69		3.15		5.81		1.19		5.44	

instance	ILS-RN		ILS-CN		GRASP-RN		GRASP-CN		TS-RN		TS-CN		SA		GVNSWAF	
	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time
sfjs01	**6,459**	0.00	**6,459**	0.00	**6,459**	0.00	**6,459**	0.00	**6,459**	0.00	**6,459**	0.00	**6,459**	0.00	**6,459**	0.00
sfjs02	**10,271**	0.00	**10,271**	0.00	**10,271**	0.00	**10,271**	0.00	**10,271**	0.00	**10,271**	0.00	**10,271**	0.00	**10,271**	0.00
sfjs03	**20,623**	0.00	**20,623**	0.00	**20,623**	0.00	**20,623**	0.00	**20,623**	0.00	21,716	0.00	**20,623**	0.00	**20,623**	0.00
sfjs04	**33,429**	0.00	**33,429**	0.00	**33,429**	0.00	**33,429**	0.00	**33,429**	0.00	34,483	0.00	**33,429**	0.00	**33,429**	0.00
sfjs05	**11,006**	0.00	**11,006**	0.00	**11,006**	0.00	**11,006**	0.00	**11,006**	0.00	12,107	0.00	**11,006**	0.00	**11,006**	0.00
sfjs06	**29,926**	0.00	**29,926**	0.00	**29,926**	0.00	**29,926**	0.00	31,835	0.00	32,057	0.00	**29,926**	0.00	**29,926**	0.00
sfjs07	**37,824**	0.00	**37,824**	0.00	**37,824**	0.00	**37,824**	0.00	**37,824**	0.00	**37,824**	0.00	**37,824**	0.00	**37,824**	0.00
sfjs08	**23,842**	0.00	**23,842**	0.00	**23,842**	0.00	**23,842**	0.00	**23,842**	0.00	**23,842**	0.00	**23,842**	0.01	**23,842**	0.00
sfjs09	**19,406**	0.00	**19,406**	0.00	**19,406**	0.00	**19,406**	0.00	**19,406**	0.00	**19,406**	0.00	**19,406**	0.01	**19,406**	0.00
sfjs10	**49,368**	0.00	**49,368**	0.00	**49,368**	0.00	**49,368**	0.00	**49,368**	0.00	**49,368**	0.00	**49,368**	0.06	**49,368**	0.00
$\overline{C}_{\max}$	24,215.40		24,215.40		24,215.40		24,215.40		24,406.30		24,753.30		24,215.40		24,215.40	
$\overline{CV}$	0.0000		0.0000		0.0000		0.0000		0.0000		0.0000		0.0000		0.0000	
#best	10		10		10		10		9		6		10		10	
gap(%)	0.00		0.00		0.00		0.00		0.64		2.56		0.00		0.00	
$\overline{\text{gap}}$(%)	0.00		0.00		0.00		0.00		0.64		2.56		0.00		0.00	

Table A11: Results of applying the metaheuristics and the method introduced in [45] to classical instances of the FJS with learning effect and with linear routes only, with learning index $\alpha = 0.1$.

instance	ILS-RN		ILS-CN		GRASP-RN		GRASP-CN		TS-RN		TS-CN		SA		GVNSWAF	
	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time
mfjs01	**43,208**	0.01	**43,208**	0.01	**43,208**	2.14	**43,208**	1.04	**43,208**	0.01	44,880	0.00	**43,208**	0.33	**43,208**	0.01
mfjs02	**41,273**	0.07	**41,273**	0.08	**41,273**	5.09	**41,273**	0.00	**41,273**	0.01	45,208	0.00	**41,273**	0.30	**41,273**	47.06
mfjs03	**43,412**	0.11	**43,412**	0.12	**43,412**	0.31	**43,412**	0.16	**43,412**	0.05	**43,412**	0.00	**43,412**	0.39	**43,412**	4.23
mfjs04	**47,717**	2.80	**47,717**	0.90	49,024	78.72	49,024	0.19	**47,717**	0.74	**47,717**	0.09	49,921	0.50	**47,717**	1.16
mfjs05	**45,670**	0.13	**45,670**	0.06	**45,670**	0.09	**45,670**	0.04	**45,670**	0.18	**45,670**	0.00	**45,670**	0.44	**45,670**	1.88
mfjs06	**56,743**	3.25	**56,743**	6.66	**56,743**	135.74	57,005	281.92	57,093	4.14	57,889	0.00	57,055	178.01	**56,743**	3.24
mfjs07	**73,865**	11.61	**73,865**	6.71	75,640	262.56	74,821	50.66	**73,865**	0.60	77,533	0.01	74,646	70.58	**73,865**	0.73
mfjs08	74,561	165.80	**74,359**	246.39	76,148	275.69	76,815	3.04	75,880	7.06	78,362	0.17	75,032	1.54	75,032	269.43
mfjs09	88,457	180.68	**87,676**	9.06	92,100	219.33	90,970	63.40	89,991	137.38	90,853	0.54	**87,676**	57.99	91,006	7.48
mfjs10	97,980	129.30	97,929	206.55	100,069	164.40	102,337	214.69	**97,780**	90.39	98,459	0.31	98,058	158.98	99,630	38.54
$\bar{C}_{\max}$	61,288.60		61,185.20		62,328.70		62,453.50		61,588.90		62,998.30		61,595.10		61,755.60	
$\overline{CV}$	0.0008		0.0006		0.0043		0.0010		0.0000		0.0000		0.0120		0.0152	
#best	7		9		5		4		7		3		5		7	
gap(%)	0.14		0.02		1.49		1.62		0.53		3.01		0.74		0.66	
$\overline{\text{gap}}$(%)	0.21		0.10		2.14		1.72		0.53		3.01		2.15		2.38	

instance	ILS-RN		ILS-CN		GRASP-RN		GRASP-CN		TS-RN		TS-CN		SA		GVNSWAF	
	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time
MK01	**3,041**	65.31	**3,041**	85.94	3,047	64.27	3,048	141.02	3,052	8.72	3,179	0.01	**3,041**	26.52	3,201	0.72
MK02	**2,118**	28.65	**2,118**	37.89	2,120	92.83	2,129	97.06	**2,118**	6.64	2,134	0.13	**2,118**	8.97	2,129	9.10
MK03	**14,781**	2.17	**14,781**	0.33	**14,781**	1.71	**14,781**	0.16	**14,781**	2.68	14,908	0.09	**14,781**	3.08	**14,781**	2.68
MK04	**4,483**	53.69	**4,483**	94.48	4,484	184.38	4,484	192.52	4,738	9.04	4,983	0.03	**4,483**	5.74	4,853	1.24
MK05	10,885	160.49	10,892	266.12	10,945	189.24	10,894	220.20	10,875	61.39	10,902	3.61	**10,818**	272.66	10,940	210.49
MK06	4,377	149.74	4,243	83.62	4,499	266.52	4,592	58.84	4,958	30.18	4,255	20.77	**4,134**	289.16	4,591	32.71
MK07	9,469	222.13	**9,393**	23.84	9,642	153.59	9,728	87.87	9,427	21.45	9,857	0.14	**9,393**	22.45	9,531	20.83
MK08	29,784	217.05	**29,758**	98.74	30,184	181.23	29,841	159.05	30,962	4.87	31,816	0.20	29,868	242.44	29,895	290.79
MK09	20,641	295.19	**20,359**	206.55	21,454	148.53	21,176	13.41	23,186	55.94	23,806	0.52	20,435	127.97	20,939	284.25
MK10	**13,648**	275.95	13,694	185.13	15,879	145.53	15,179	81.71	14,079	231.51	13,770	16.78	14,246	217.07	14,974	145.47
MK11	35,747	185.40	**35,680**	212.31	36,110	49.28	36,238	181.46	36,093	1.58	35,813	16.15	35,756	25.72	35,893	284.24
MK12	31,539	218.99	**31,512**	181.46	31,654	169.42	31,592	258.89	32,605	2.07	33,358	0.23	31,625	11.56	31,586	144.52
MK13	26,428	290.21	26,238	292.55	28,472	129.83	27,598	58.96	**25,764**	197.92	26,690	8.26	26,471	45.26	27,285	122.26
MK14	39,864	299.22	**39,829**	148.40	40,464	277.76	40,047	263.88	41,968	7.49	43,123	0.23	40,030	70.85	39,891	222.06
MK15	23,815	300.52	**23,442**	141.00	24,740	285.86	24,859	46.53	23,472	146.44	25,612	0.71	23,862	148.48	24,466	245.22
$\bar{C}_{\max}$	18,041.33		17,964.20		18,565.00		18,412.40		18,538.53		18,947.07		18,070.73		18,330.33	
$\overline{CV}$	0.0059		0.0021		0.0068		0.0047		0.0000		0.0000		0.0042		0.0166	
#best	5		11		1		1		3		0		7		1	
gap(%)	0.89		0.37		3.69		3.14		3.87		5.20		0.72		3.46	
$\overline{\text{gap}}$(%)	1.58		0.63		4.53		3.88		3.87		5.20		1.18		5.03	

instance	ILS-RN		ILS-CN		GRASP-RN		GRASP-CN		TS-RN		TS-CN		SA		GVNSWAF	
	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time
sfjs01	**6,328**	0.00	**6,328**	0.00	**6,328**	0.00	**6,328**	0.00	**6,328**	0.00	**6,328**	0.00	**6,328**	0.00	**6,328**	0.00
sfjs02	**9,872**	0.00	**9,872**	0.00	**9,872**	0.00	**9,872**	0.00	**9,872**	0.00	**9,872**	0.00	**9,872**	0.00	**9,872**	0.00
sfjs03	**19,281**	0.00	**19,281**	0.00	**19,281**	0.00	**19,281**	0.00	**19,281**	0.00	20,027	0.00	**19,281**	0.00	**19,281**	0.00
sfjs04	**31,553**	0.00	**31,553**	0.00	**31,553**	0.00	**31,553**	0.00	32,472	0.00	32,472	0.00	**31,553**	0.00	**31,553**	0.00
sfjs05	**10,198**	0.00	**10,198**	0.00	**10,198**	0.00	**10,198**	0.00	**10,198**	0.00	11,209	0.00	**10,198**	0.00	**10,198**	0.00
sfjs06	**28,024**	0.00	**28,024**	0.00	**28,024**	0.00	**28,024**	0.00	**28,024**	0.00	**28,024**	0.00	**28,024**	0.00	**28,024**	0.00
sfjs07	**36,075**	0.00	**36,075**	0.00	**36,075**	0.00	**36,075**	0.00	**36,075**	0.00	**36,075**	0.00	**36,075**	0.00	**36,075**	0.00
sfjs08	**22,515**	0.00	**22,515**	0.00	**22,515**	0.00	**22,515**	0.00	**22,515**	0.00	**22,515**	0.00	**22,515**	0.03	**22,515**	0.01
sfjs09	**17,552**	0.00	**17,552**	0.00	**17,552**	0.00	**17,552**	0.00	**17,552**	0.00	**17,552**	0.00	**17,552**	0.01	**17,552**	0.00
sfjs10	**47,323**	0.00	**47,323**	0.00	**47,323**	0.00	**47,323**	0.00	**47,323**	0.00	**47,323**	0.00	**47,323**	0.09	**47,323**	0.00
$\bar{C}_{\max}$	22,872.10		22,872.10		22,872.10		22,872.10		22,964.00		23,139.70		22,872.10		22,872.10	
$\overline{CV}$	0.0000		0.0000		0.0000		0.0000		0.0000		0.0000		0.0000		0.0000	
#best	10		10		10		10		9		7		10		10	
gap(%)	0.00		0.00		0.00		0.00		0.29		1.67		0.00		0.00	
$\overline{\text{gap}}$(%)	0.00		0.00		0.00		0.00		0.29		1.67		0.00		0.00	

Table A12: Results of applying the metaheuristics and the method introduced in [45] to classical instances of the FJS with learning effect and with linear routes only, with learning index $\alpha = 0.2$.

instance	ILS-RN		ILS-CN		GRASP-RN		GRASP-CN		TS-RN		TS-CN		SA		GVNSWAF	
	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time
mfjs01	**40,508**	0.02	**40,508**	0.03	**40,508**	3.74	**40,508**	1.20	42,562	0.00	41,785	0.00	**40,508**	0.37	**40,508**	0.18
mfjs02	**38,996**	0.05	**38,996**	0.05	**38,996**	2.80	**38,996**	1.60	**38,996**	0.00	39,834	0.00	**38,996**	0.29	**38,996**	4.26
mfjs03	**41,318**	0.06	**41,318**	0.04	**41,318**	72.50	**41,318**	14.85	**41,318**	0.00	44,254	0.00	**41,318**	0.26	**41,318**	7.82
mfjs04	**44,869**	2.16	**44,869**	0.60	**44,869**	79.94	46,048	61.15	46,048	0.03	46,558	0.01	46,048	25.11	**44,869**	101.21
mfjs05	**44,376**	0.21	**44,376**	0.06	**44,376**	90.04	44,738	20.87	**44,376**	0.33	44,738	0.00	**44,376**	28.30	**44,376**	0.29
mfjs06	**53,618**	2.31	**53,618**	10.19	53,760	84.79	53,760	37.32	53,760	0.39	57,329	0.00	**53,618**	184.69	**53,618**	184.69
mfjs07	**69,086**	9.26	**69,086**	4.61	70,577	192.72	70,523	80.50	**69,086**	12.88	74,941	0.01	**69,086**	0.84	70,583	0.02
mfjs08	**68,053**	152.28	**68,053**	188.36	71,413	4.05	70,333	154.93	69,845	12.97	70,333	0.26	68,362	11.09	**68,053**	74.29
mfjs09	**79,947**	3.22	80,277	249.67	83,975	92.83	82,272	203.95	80,983	27.37	81,939	0.83	80,897	1.69	81,411	40.30
mfjs10	89,188	217.74	**88,515**	195.67	91,930	114.34	91,477	158.34	89,644	41.99	90,178	1.03	90,419	7.67	90,144	117.97
$\bar{C}_{\max}$	56,995.90		56,961.60		58,172.20		57,997.30		57,661.80		59,188.90		57,362.80		57,387.60	
$\overline{CV}$	0.0010		0.0008		0.0035		0.0038		0.0000		0.0000		0.0105		0.0158	
#best	9		9		5		3		4		0		6		7	
gap(%)	0.08		0.04		1.63		1.54		1.32		4.01		0.64		0.58	
$\overline{\text{gap}}$(%)	0.21		0.16		2.09		1.97		1.32		4.01		1.89		2.64	

instance	ILS-RN		ILS-CN		GRASP-RN		GRASP-CN		TS-RN		TS-CN		SA		GVNSWAF	
	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time
MK01	**2,618**	111.30	**2,618**	79.93	2,669	26.88	2,668	40.52	2,669	38.63	2,793	0.00	**2,618**	10.59	2,714	1.38
MK02	**1,920**	7.79	**1,920**	3.27	**1,920**	80.62	1,921	89.79	**1,920**	1.45	1,932	0.03	**1,920**	1.90	1,921	1.74
MK03	**12,439**	3.89	**12,439**	9.44	**12,439**	3.27	**12,439**	2.08	**12,439**	11.02	12,669	0.03	**12,439**	3.99	12,481	2.88
MK04	3,877	296.61	3,885	298.13	3,904	117.29	3,917	237.10	3,917	6.97	4,282	0.08	**3,872**	253.43	4,246	4.82
MK05	8,693	256.96	8,692	246.56	8,736	276.98	8,716	163.93	8,708	56.12	8,688	3.02	8,678	63.11	**8,675**	266.29
MK06	3,609	168.58	3,611	232.13	3,830	266.93	3,784	120.00	3,866	188.73	**3,556**	40.62	**3,556**	148.27	4,183	267.07
MK07	7,913	46.15	7,914	224.48	8,055	103.05	8,015	216.96	7,938	19.54	8,036	0.17	**7,879**	54.92	7,938	7.96
MK08	23,265	117.12	**23,240**	198.23	23,413	30.22	23,442	246.91	23,813	12.18	24,437	0.21	23,333	44.37	23,385	172.23
MK09	16,983	219.10	**16,761**	121.54	17,571	206.91	17,332	260.19	18,955	47.76	19,508	0.22	16,833	61.28	17,214	48.30
MK10	11,576	263.74	**11,172**	130.85	12,995	278.38	12,732	234.83	11,828	297.77	12,504	2.51	11,625	155.35	12,039	220.25
MK11	27,604	208.94	**27,542**	125.25	27,786	235.45	27,990	57.88	27,692	81.59	27,644	10.97	27,694	74.08	27,757	275.70
MK12	25,138	187.88	**25,097**	127.56	25,301	30.19	25,197	72.47	25,882	2.39	26,101	0.28	25,207	44.11	25,188	268.75
MK13	21,633	143.11	21,150	227.99	22,728	141.57	22,682	15.97	**21,020**	133.81	21,773	3.22	21,466	47.61	22,182	214.19
MK14	30,597	134.76	**30,550**	94.94	31,067	283.03	30,815	124.47	31,674	131.73	31,223	1.77	30,778	88.62	30,672	280.78
MK15	19,882	260.19	**19,775**	196.63	20,792	10.26	20,570	79.55	20,477	266.85	24,081	0.19	19,900	269.25	21,345	56.19
$\bar{C}_{\max}$	14,516.47		14,424.40		14,880.40		14,814.67		14,853.20		15,281.80		14,519.87		14,796.00	
$\overline{CV}$	0.0062		0.0022		0.0049		0.0040		0.0000		0.0000		0.0035		0.0180	
#best	3		10		2		1		3		1		6		1	
gap(%)	0.75		0.21		3.46		2.99		3.02		5.82		0.63		3.88	
$\overline{\text{gap}}$(%)	1.50		0.49		4.23		3.54		3.02		5.82		0.89		5.50	

instance	ILS-RN		ILS-CN		GRASP-RN		GRASP-CN		TS-RN		TS-CN		SA		GVNSWAF	
	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time	$C_{\max}$	Time
sfjs01	**6,206**	0.00	**6,206**	0.00	**6,206**	0.00	**6,206**	0.00	**6,206**	0.00	**6,206**	0.00	**6,206**	0.00	**6,206**	0.00
sfjs02	**9,498**	0.00	**9,498**	0.00	**9,498**	0.00	**9,498**	0.00	**9,498**	0.00	**9,498**	0.00	**9,498**	0.00	**9,498**	0.00
sfjs03	**18,062**	0.00	**18,062**	0.00	**18,062**	0.00	**18,062**	0.00	**18,062**	0.00	**18,062**	0.00	**18,062**	0.00	**18,062**	0.00
sfjs04	**29,852**	0.00	**29,852**	0.00	**29,852**	0.00	**29,852**	0.00	30,648	0.00	30,648	0.00	**29,852**	0.00	**29,852**	0.00
sfjs05	**9,465**	0.00	**9,465**	0.00	**9,465**	0.00	**9,465**	0.00	**9,465**	0.00	10,288	0.00	**9,465**	0.00	**9,465**	0.00
sfjs06	**26,281**	0.00	**26,281**	0.00	**26,281**	0.00	**26,281**	0.00	**26,281**	0.00	**26,281**	0.00	**26,281**	0.00	**26,281**	0.00
sfjs07	**34,443**	0.00	**34,443**	0.00	**34,443**	0.00	**34,443**	0.00	**34,443**	0.00	**34,443**	0.00	**34,443**	0.00	**34,443**	0.00
sfjs08	**21,309**	0.00	**21,309**	0.00	**21,309**	0.00	**21,309**	0.00	21,715	0.00	**21,309**	0.00	**21,309**	0.02	**21,309**	0.00
sfjs09	**15,973**	0.00	**15,973**	0.00	**15,973**	0.00	**15,973**	0.00	**15,973**	0.00	**15,973**	0.00	**15,973**	0.01	**15,973**	0.00
sfjs10	**45,450**	0.00	**45,450**	0.00	**45,450**	0.00	**45,450**	0.00	**45,450**	0.00	**45,450**	0.00	**45,450**	0.01	**45,450**	0.00
$\bar{C}_{\max}$	21,653.90		21,653.90		21,653.90		21,653.90		21,774.10		21,815.80		21,653.90		21,653.90	
$\overline{CV}$	0.0000		0.0000		0.0000		0.0000		0.0000		0.0000		0.0000		0.0000	
#best	10		10		10		10		8		8		10		10	
gap(%)	0.00		0.00		0.00		0.00		0.46		1.14		0.00		0.00	
$\overline{\text{gap}}$(%)	0.00		0.00		0.00		0.00		0.46		1.14		0.00		0.00	

Table A13: Results of applying the metaheuristics and the method introduced in [45] to classical instances of the FJS with learning effect and with linear routes only, with learning index $\alpha = 0.3$.