

Nome : _____

Número USP : _____

Assinatura : _____

Professor : Severino Toscano do Rêgo Melo

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| 1 | |
| 2 | |
| Total | |

Questão 1: (a) Calcule $\frac{d}{dx} \left(1 + \frac{1}{x}\right)^x$.

(b) Seja $f(x) = \begin{cases} \frac{\arctan x}{x} & \text{se } x \neq 0 \\ 1 & \text{se } x = 0 \end{cases}$. Verifique que $f'(0) = 0$.

$$a) \frac{d}{dx} \left(1 + \frac{1}{x}\right)^x = \frac{d}{dx} e^{x \ln\left(1 + \frac{1}{x}\right)} = e^{x \ln\left(1 + \frac{1}{x}\right)} \cdot \frac{d}{dx} \left[x \ln\left(1 + \frac{1}{x}\right) \right]$$

$$= \left(1 + \frac{1}{x}\right)^x \left[\ln\left(1 + \frac{1}{x}\right) + x \frac{1}{1 + \frac{1}{x}} \cdot \left(-\frac{1}{x^2}\right) \right] =$$

$$= \left(1 + \frac{1}{x}\right)^x \left[\ln\left(1 + \frac{1}{x}\right) - \frac{1}{x+1} \right]$$

$$b) f'(0) = \lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0} \frac{\frac{\arctg x}{x} - 1}{x} =$$

$$= \lim_{x \rightarrow 0} \frac{\arctg x - x}{x^2} \underset{\uparrow}{=} \lim_{x \rightarrow 0} \frac{\frac{1}{1+x^2} - 1}{2x} =$$

L'Hopital

$$= \lim_{x \rightarrow 0} \frac{x^2}{2x(1+x^2)} = \lim_{x \rightarrow 0} \frac{x}{2(1+x^2)} = 0.$$

Questão 2: Seja $f(x) = x^4 - 4x^3 + 20$, $x \in \mathbb{R}$.

(a) Analise o sinal de f' e o sinal de f'' .

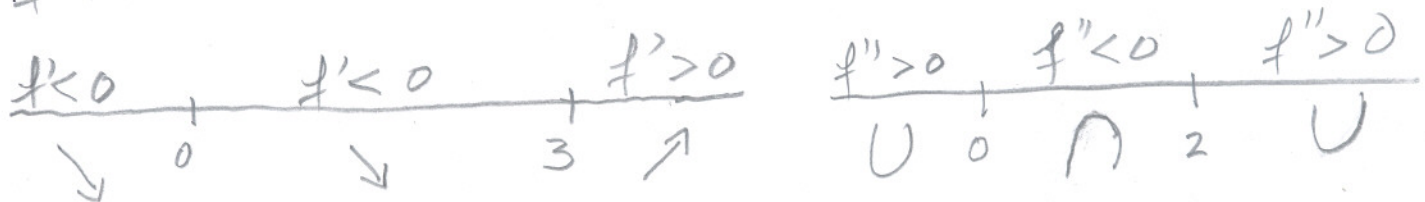
(b) Esboce o gráfico de f , indicando claramente os pontos críticos e os pontos de inflexão.

$$(a) f'(x) = 4x^3 - 12x^2 = 4x^2(x - 3)$$

$$f''(x) = 12x^2 - 24x = 12x(x - 2)$$

$$f'(x) = 0 \Leftrightarrow x = 0 \text{ ou } x = 3$$

$$f''(x) = 0 \Leftrightarrow x = 0 \text{ ou } x = 2$$



$$(b) f(0) = 20 \quad f(2) = 16 - 32 + 20 = 4$$

$$f(3) = 81 - 108 + 20 = -7$$

