On some assumptions of Null Hypothesis Statistical Testing (NHST)

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Outline

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- to discuss the **classical statistical model** and **statistical hypotheses**,
- to present some **limitations of the classical p-value** with numerical examples,
- to introduce **an alternative measure of evidence**, called s-value, that overcomes some limitations of the p-value.

The classical statistical model

The classical statistical model is:

$$(\Omega, \mathcal{F}, \mathcal{P}),$$

where:

- Ω is the space of possible experiment outcomes,
- $\mathcal{F}$ is a σ-field of Ω,
- $\mathcal{P}$ is a family of non-random probability measures that **possibly** explain the experiment outcomes.

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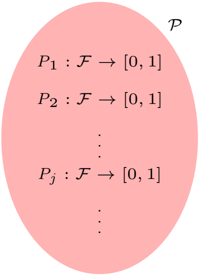

The quantity of interest is $g(P)$.
For instance:

$$g(P) = E_P(Z),$$

$$g(P) = P(Z_1 \in B | Z_2 \in A),$$

etc.

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Conditional, **marginal** and **joint** distributions can be used to make inferences about γ.

Take $\mathcal{P} = \{P_0\}$ and build your joint probability P_0 from:

- $\gamma \sim f_0(\cdot)$ (with no unknown constants),
- $X|\gamma \sim f_1(\cdot|\gamma)$

Now, you are ready to be a **hard core Bayesian**!

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(or simply $\boxed{H_0 : \text{"}P \in \mathcal{P}_0\text{"}}$*)*

Under a parametric model, there exists a finite dimensional set Θ such that:

- $\boxed{\boldsymbol{\mathcal{P} \equiv \{P_\theta : \ \theta \in \Theta\}}}$, where $\Theta \subseteq \mathbb{R}^p$, $p < \infty$,
- $\boxed{\boldsymbol{H_0 : \theta \in \Theta_0}}$, where $\Theta_0 \subset \Theta$ and $\mathcal{P}_0 \equiv \{P_\theta : \ \theta \in \Theta_0\}$.

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In the last context, we can **choose**[1] between H_0 and H_1 — Neyman and Pearson approach.

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A classical statistician may also test Bayesian hypotheses. Rather than p-values, they would use estimated conditional probabilities.

P-value definition

The p-value for testing the classical null hypothesis H_0 is defined as follows

$$p(\mathcal{P}_0, x) = \sup_{P \in \mathcal{P}_0} P\Big(T_{H_0}(X) > T_{H_0}(x)\Big)$$

where T_{H_0} is a statistic such that the more discrepant is H_0 from x, the larger is its observed value.[2]

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$p(\mathcal{P}_0, x) \approx 0$ indicates that the best case in H_0 provides a small probability to more "extreme events" than the observed one.

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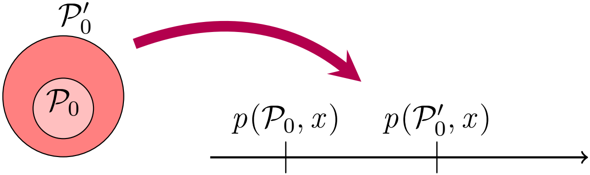
P-value limitations

Consider two null hypotheses H_0 : "$P \in \mathcal{P}_0$" and H_0' : "$P \in \mathcal{P}_0'$" such that $H_0 \implies H_0'$. Then, we would expect that:



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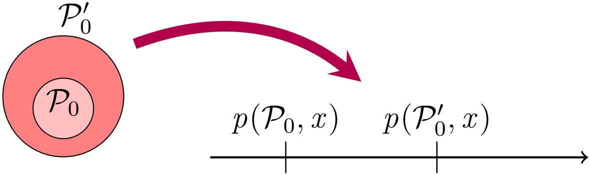

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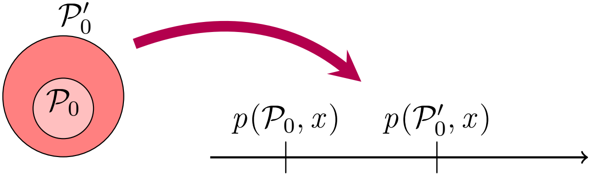

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The previous p-value **is not monotone** over the set of null hypotheses/Sets.

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- under $H_0' : \mu_1 = \mu_2$ is

$$T_{H_0'}(X) = \frac{n}{2}(\bar{X}_1 - \bar{X}_2)^2 \sim \chi^2_1,$$

where $\bar{X} = (\bar{X}_1, \bar{X}_2)^\top$ is the maximum likelihood estimator for $\boldsymbol{\mu}$.

P-values do not respect monotonicity

Observed sample $(\bar{x}_1, \bar{x}_2)$	$\bar{x}_1 - \bar{x}_2$	$H_0 : \boldsymbol{\mu} = \mathbf{0}$ p-value	$H_0' : \mu_1 = \mu_2$ p-value
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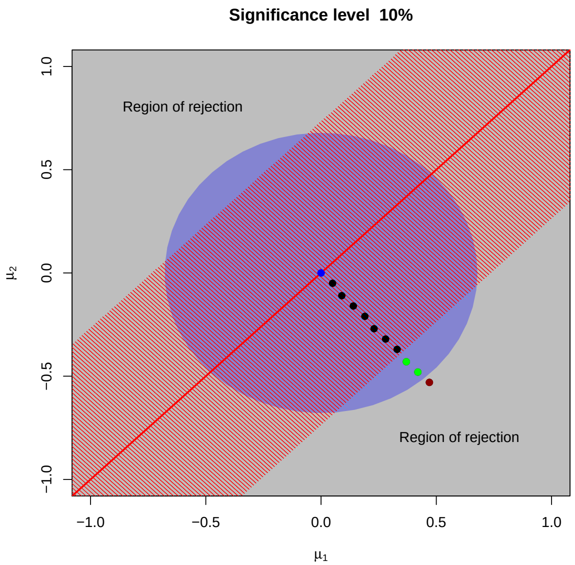
Level curves (contour curves)



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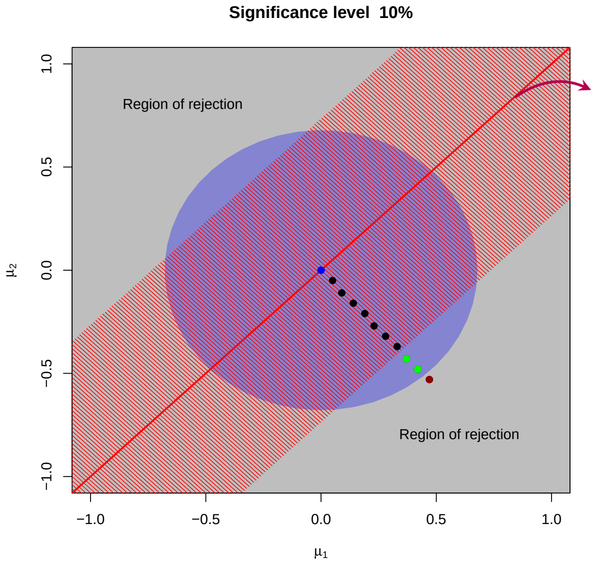

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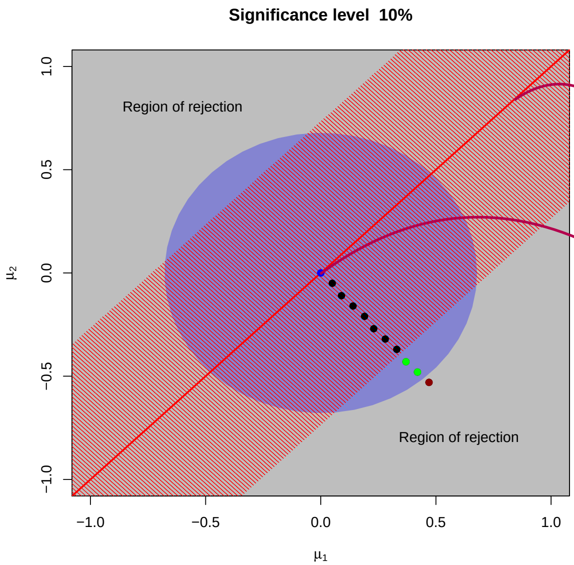

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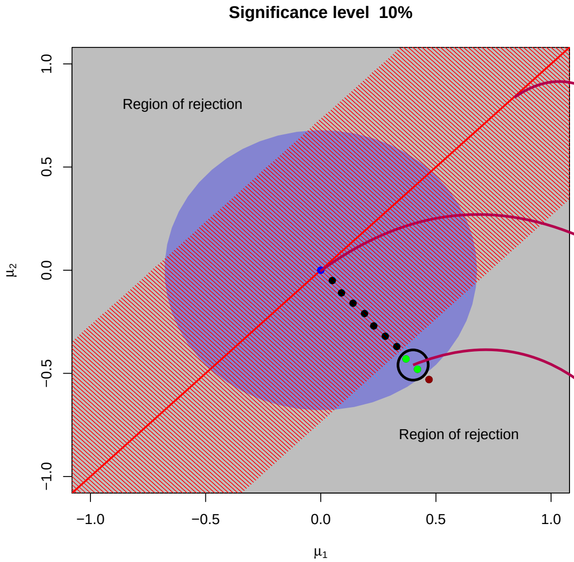

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Rejection of H_0' but no rejection of H_0

$\Downarrow$

non-coherent conclusion

An alternative measure of evidence (parametric case)

In what follows, we present an alternative measure called **s-value** to overcome the previous issue (Patriota, 2013, FSS, 233).

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The **s-value** is a function $s : 2^{\Theta} \times \mathcal{X} \to [0, 1]$ such that

$$s(\Theta_0, x) = \begin{cases} \sup\{\alpha \in (0,1) : \ \Lambda_\alpha(x) \cap \Theta_0 \neq \varnothing\}, & \text{if } \Theta_0 \neq \varnothing, \\ 0, & \text{if } \Theta_0 = \varnothing. \end{cases}$$

where Λ_α is a confidence set for θ with confidence level $1 - \alpha$ with some “nice” properties.

Interpretation

Interpretation: $s = s(\Theta_0, x)$ is the largest significance level α (or $1 - s$ is the smallest confidence level $1 - \alpha$) for which the confidence set and the set $\overline{\Theta_0}$ have at least one element in common.

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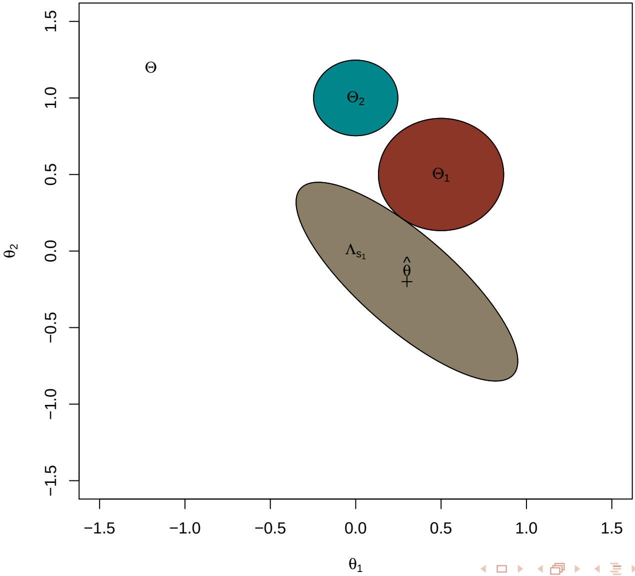
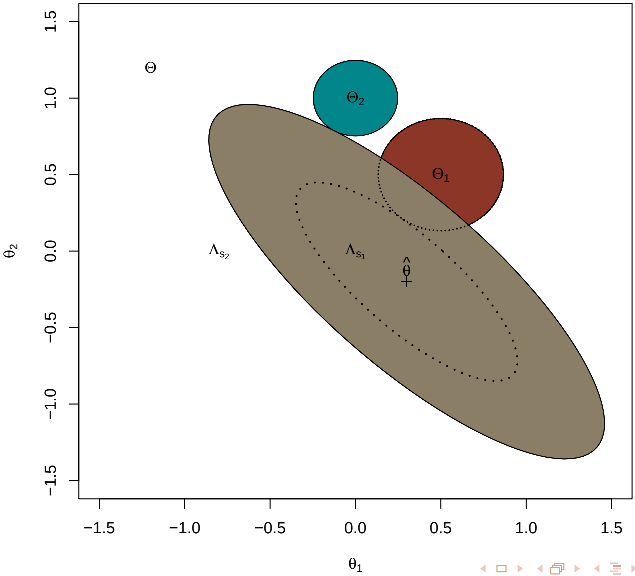
Large values of s indicate that **there exists at least one** element in Θ_0 close to the center of Λ_α (e.g., close to the ML estimate).

Small values of s indicate that **ALL** elements of Θ_0 are far away from the center of Λ_α.

Graphical illustration: $s_1 = s(\Theta_1, x)$



Graphical illustration: $s_2 = s(\Theta_2, x)$



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4. If $\Theta_1 \subseteq \Theta$, then $s(\Theta_1, x) = \sup_{\theta \in \Theta_1} s(\{\theta\}, x)$,
5. $s(\Theta_1, x) = 1$ or $s(\Theta_1^c, x) = 1$:
    - if $\widehat{\theta} \in \overline{\Theta_1}$ (closure of Θ_1), then $s(\Theta_1, x) = 1$,
    - if $\widehat{\theta} \in \overline{\Theta_1^c}$, then $s(\Theta_1^c, x) = 1$.

where $\hat{\theta}$ is an element of the center of Λ_α, i.e., $\hat{\theta} \in \bigcap_\alpha \Lambda_\alpha(x)$.

Decisions about H_0

Let Φ be a function such that:

$$\Phi(\Theta_0) = \langle s(\Theta_0), s(\Theta_0^c)\rangle.$$

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$\Phi(\Theta_0) = \langle 1, 1 \rangle \implies$ **total ignorance** about H_0.

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Property: If the statistical model is regular and the confidence region is built from a statistics $T_\theta(X)$ that converges in distribution to χ^2_k, then:

$$s_a = 1 - F_k(F_{H_0}^{-1}(1 - p_a)),$$

where $p_a = 1 - F_{H_0}(t)$ is the asymptotic p-value to test H_0.

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The confidence set Λ_α is given by

$$\Lambda_\alpha(x) = \{\mu \in \mathbb{R}^2 : \ T_{\boldsymbol{\mu}}(x) \leq F_2^{-1}(1-\alpha)\},$$

where F_2 is the cumulative chi-squared distribution with two degrees of freedom.

Numerical illustration

Observed sample		$H_0 : \boldsymbol{\mu} = \mathbf{0}$	$H_0' : \mu_1 = \mu_2$	
$(\bar{x}_1, \bar{x}_2)$	$\bar{x}_1 - \bar{x}_2$	p/s-value	p-value	s-value
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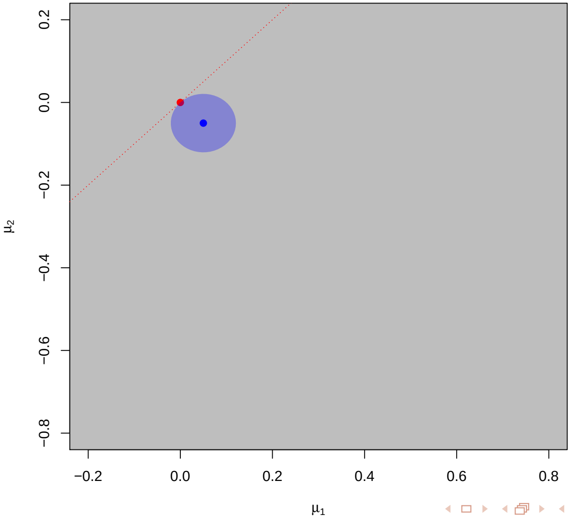
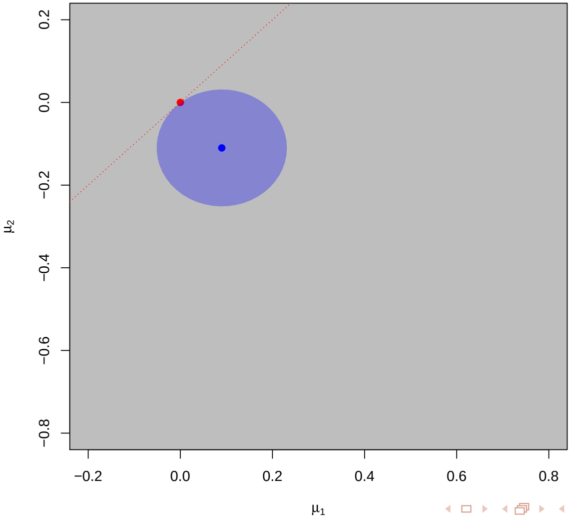
(0.19,-0.21)	0.4	0.6697	0.3711	0.6703
(0.23,-0.27)	0.5	0.5331	0.2636	0.5353
(0.28,-0.32)	0.6	0.4049	0.1797	0.4066
(0.33,-0.37)	0.7	0.2926	0.1175	0.2938
(0.37,-0.43)	0.8	0.2001	0.0736	0.2019
(0.42,-0.48)	0.9	0.1308	0.0442	0.1320
(0.47,-0.53)	1.0	0.0813	0.0253	0.0821

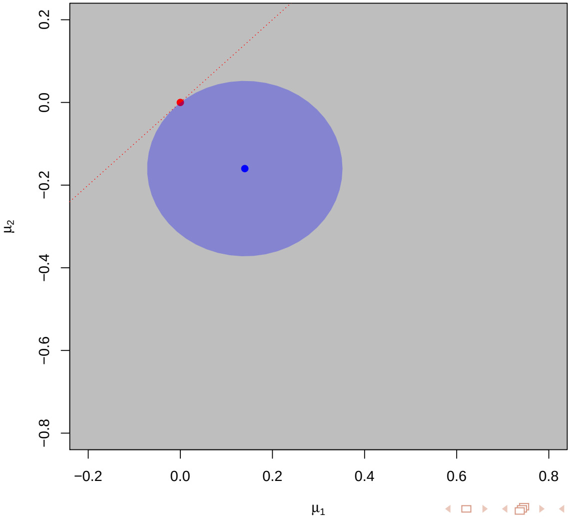
Graphical illustration: $s(\{\mu_1 = \mu_2\}, x_1) = 0.9753$



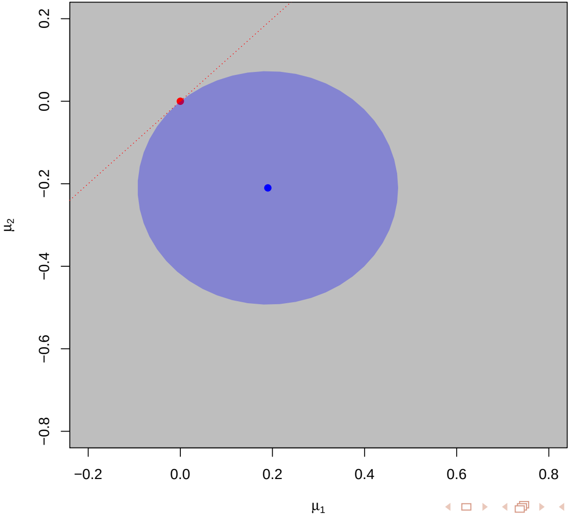
Graphical illustration: $s(\{\mu_1 = \mu_2\}, x_2) = 0.9048$



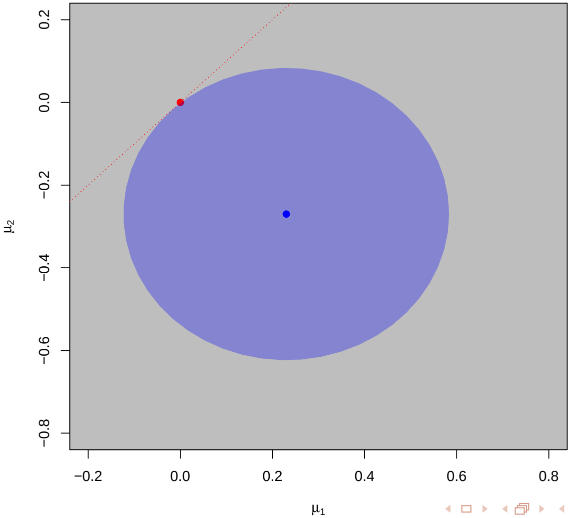
Graphical illustration: $s(\{\mu_1 = \mu_2\}, x_3) = 0.7985$



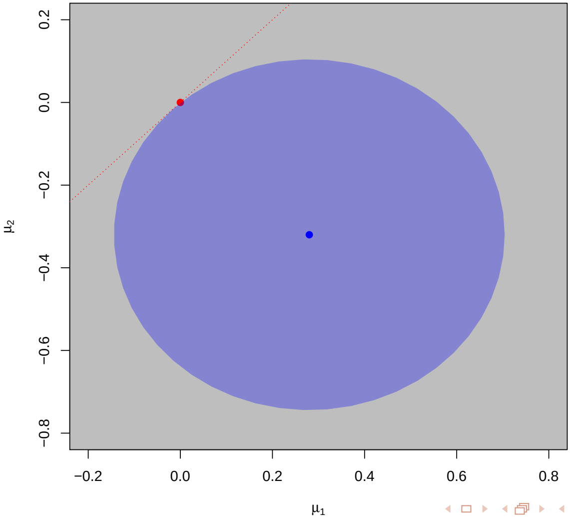
Graphical illustration: $s(\{\mu_1 = \mu_2\}, x_4) = 0.6703$



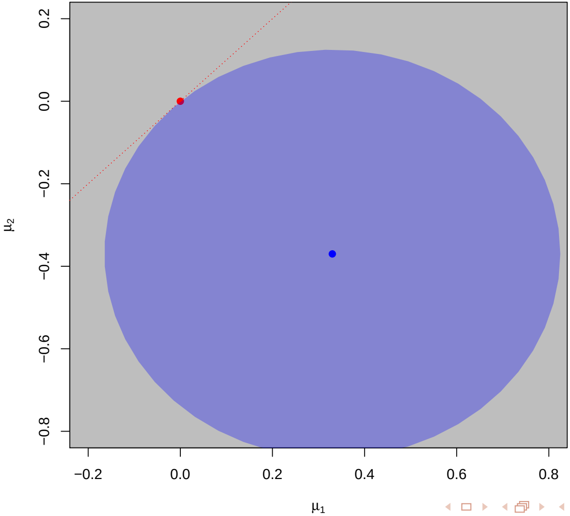
Graphical illustration: $s(\{\mu_1 = \mu_2\}, x_5) = 0.5353$



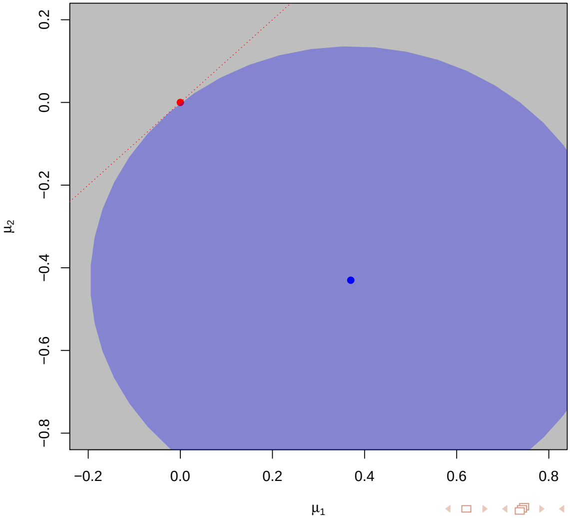
Graphical illustration: $s(\{\mu_1 = \mu_2\}, x_6) = 0.4066$



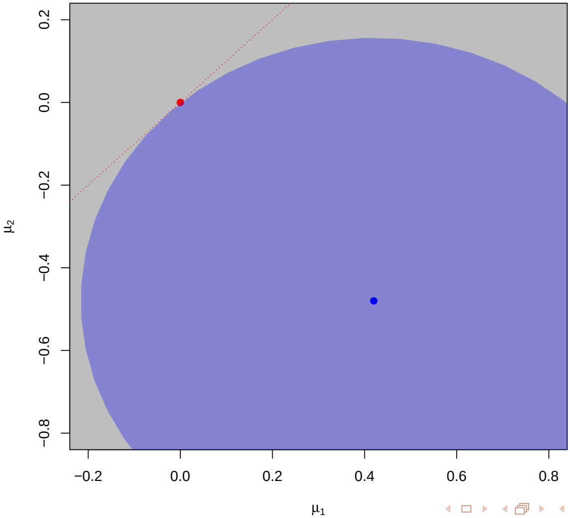
Graphical illustration: $s(\{\mu_1 = \mu_2\}, x_7) = 0.2938$



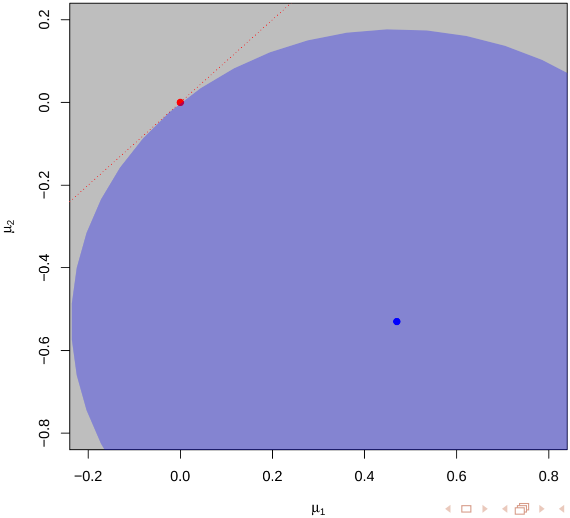
Graphical illustration: $s(\{\mu_1 = \mu_2\}, x_8) = 0.2019$



Graphical illustration: $s(\{\mu_1 = \mu_2\}, x_9) = 0.1320$



Graphical illustration: $s(\{\mu_1 = \mu_2\}, x_{10}) = 0.0821$



Final remarks

The s-value:

- can be applied directly whenever the log-likelihood function is concave by the formula $s = 1 - F(F_{H_0}(1 - p))$
- is a possibilistic measure and can be studied by means of the Abstract belief Calculus ABC (Darwiche, Ginsberg, 1992).
- can be justified by *desiderata* (more basic axioms).
- avoids the p-value problem of non-monotonicity.
- is a classic alternative to the FBST (Pereira, Stern, 1998).

References:

Darwiche, A.Y., Ginsberg, M.L. (1992). A symbolic generalization of probability theory, AAAI-92, Tenth National Conference on Artificial Intelligence.

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Pereira, C.A.B., Stern, J.M. (1999). Evidence and credibility: Full Bayesian significance test for precise hypotheses, Entropy, 1, 99–110.