Second Joint Meeting of Mathematicians
Ohio State University - University of São Paulo
Talks and Abstracts

Speaker: Marcos. M. Alexandrino (IME-USP)
Title: *Closure of singular foliations: the proof of Molino's conjecture*
Abstract: A singular foliation on a Riemannian manifold M is called Riemannian if their leaves are locally equidistant. A typical example of a singular Riemannian foliaton is the decomposition of M into the orbits of an isometric group action on M. Another example is the holonomy foliation, the foliation whose leaves are orbits of parallel transports on na Euclidean vector bundle with a metric connection. In this talk we review some basic concepts and examples and give an idea of the proof of Molino's conjecture that for each singular Riemannian foliation the partition given by the closure of the leaves is again a singular Riemannian foliation. This talk is based on a joint work with Prof. Marco Radeschi (Notre-Dame).

Speaker: Fernando Antoneli (UNIFESP)
Title: *Homeostasis, Singularities and Gene Regulatory Networks*
Abstract: Homeostasis occurs in a biological or chemical system when some output variable remains approximately constant as input parameters vary over some range. The notion of homeostasis is often associated with regulating global physiological parameters like temperature in multicellular complex organisms, such as mammals. For unicellular organisms, homeostasis is related to how some internal cell state of interest (the copy number, or concentration, of an mRNA transcript or of a protein) responds to changes in the intra-cellular or extra-cellular environment. Recently, Golubitsky and Stewart ["Homeostasis, Singularities and Networks". J. Mathematical Biology, 74 (2017) 387-407] introduced the notion of "infinitesimal homeostasis" allowing the use of implicit differentiation to find regions of homeostasis in systems of differential equations. In this talk we explain how apply singularity theory to explicitly find regions of homeostasis to differential equation models associated to "motifs" (small sub-network patterns that appear with high frequency in large complex networks) in gene regulatory networks (GRN) of single-cell organisms. Joint work with Martin Golubitsky and Ian Stewart.

Speaker: Vitaly Bergelson (OSU)
Title: *Uniform distribution, generalized polynomials and the theory of multiple recurrence.*
Abstract: A classical theorem due to H. Weyl states that if P is a real polynomial such that at least one of its coefficients (other than the constant term) is irrational, then the sequence P(n), n=1,2,... is uniformly distributed mod 1. After briefly reviewing various approaches to the proof of Weyl's theorem, we will discuss some modern developments which involve "generalized polynomials", that is, functions which are obtained from the conventional polynomials by the use of the greatest integer function, addition and multiplication. As we shall see, there exists an intrinsic connection between the generalized polynomials, dynamical systems on nil-manifolds and the polynomial extensions of Szemeredi's theorem on arithmetic progressions. We will conclude with formulating and discussing some natural open problems and conjectures.

Speaker: Ugo Bruzzo (SISSA, Italy)
Title: *The Noether-Lefschetz problem — old and new*
Abstract: The Picard number of an algebraic variety X is, roughly speaking, the number of connected components of the group classifying the isomorphism classes of line bundles on X. As such, it is a deep geometric invariant of X. The classical Noether-Lefschetz theorem states that, for d \ge 4, the very general surface of degree d in projective 3-space P3 has Picard number 1, i.e., the same Picard number as the ambient variety. The aim of my talk will be to review some classical results about this problem, together with more recent developments, including some generalizations to surfaces in normal, Q-factorial 3-folds obtained in collaboration with A. Grassi and A.F. Lopez.

Speaker: Francisco C. Caramello Jr. (UFScar)
Title: *Positively curved Killing foliations via deformations*
Abstract: We will present some results on Killing foliations, a class of Riemannian foliations, obtained during the speaker's doctorate research under the supervision and with the collaboration of Prof. Dirk Töben (UFSCar). We show that a manifold admitting a Killing foliation with positive transverse curvature and maximal transverse symmetry rank fibers over finite quotients of spheres or weighted complex projective spaces. This and other similar results are obtained by deforming the foliation into a

generalized Seifert fibration while maintaining its transverse geometry, which allows us to apply results from the Riemannian geometry of orbifolds to the space of leaves. In this vein, we also obtain a transverse version of Cheng's sphere theorem for positively curved Killing foliations with maximal transverse diameter, as well as a generalization of the transverse analog of Berger's theorem for Killing foliations. Finally, we show that the basic Euler characteristic is preserved by such deformations, which provides some topological obstructions for Riemannian foliations.

Speaker: Luis Casian (OSU)
Title: *Representation Theory of Real Semi simple Lie groups and Geometry (a survey).*
Abstract: There are deep connections between the representation theory of real semi simple Lie groups and geometry of flag manifolds. I will review with simple example some of the representation theory and, using these examples, describe some of the various Important connections including a connection to the cohomology of real flag manifolds.

Speaker: Mike Davis (OSU)
Title: *Action dimensions of simple complexes of groups*
Abstract: The basic connection between group theory and topology / geometry is via the fundamental group. For any group G there is a topological space BG, with fundamental group G and with contractible universal cover. The space BG is unique up to homotopy equivalence. So, the algebraic topological invariants of BG are invariants of G. For example, the geometric dimension of a discrete, torsion-free group G is the minimum dimension of a model for BG by a CW complex. The action dimension of G is the minimum dimension of a model for BG by a manifold. I will discuss some recent work with Kevin Schreve and Giang Le (and earlier work with other collaborators) in which we compute the action dimension for Artin group and for graph products of groups. The method is based on a classical obstruction of van Kampen for embedding a simplicial complex in a Euclidean space of some given dimension.

Speaker: Andrzej Derdzinski (OSU)
Title: *Riemannian Yang-Mills connections (joint work with Paolo Piccione)*
Abstract: In a real or complex vector bundle E endowed with a fibre metric h, over a compact Riemannian manifold (M,g), the Yang-Mills functional YM associates with every metric connection the L^2 norm squared of its curvature tensor. The critical points of YM are called the Yang-Mills connections, and are of particular interest when dim M = 4. The case of "Riemannian Yang-Mills connections" (where E = TM, h = g, and the Levi-Civita connection of g is a Yang-Mills connection) is also of interest, and has been studied since the late 1970s. This talk presents a summary of equivalent characterizations and known results about the latter case, followed by some details of an ongoing project aimed at understanding the metrics g on compact four-manifolds which have the above property and lie outside of the five familiar classes of examples.

Speaker: Peter Edward Hazard (Post-doc - USP)
Title: *Quadratic irrationals, Lévy constants, and dynamical zeta functions.*
Abstract: The Lévy constant of an irrational real number, when it exists, is a positive real number which measures the rate of growth of the denominators of the sequence of best rational approximations. In 1936, Paul Lévy showed that this real number exists (and is constant) for almost every irrational. The set of exceptional irrationals, such as the algebraic real numbers, was investigated much later. For instance, Jäger and Liardet showed that the Lévy constant of a quadratic irrational always exists. In this lecture we will give a new proof of this result and show a link between Lévy constants and the topological entropy and dynamical zeta functions of certain hyperbolic toral automorphisms. Part of this work is joint with A Belova (Uppsala University, Sweden).

Speaker: Edson de Faria (USP)
Title: *On slow growth and entropy-type invariants*
Abstract: We discuss a generalization of topological entropy in which the usual exponential growth-rate function is replaced by an arbitrary gauge function. This generalized topological entropy had previously been described by Galatolo in 2003 – up to a choice of notation in the defining formulas – which in turn is essentially the same as that described by Zhao and Pesin in 2015 (that involves a re-parameterization of time). One of the

main motivations for studying this new set of invariants comes from the need to distinguish maps with zero (standard) topological entropy. In such cases, if the dynamics is not equicontinuous, then there exists at least one gauge for which the corresponding generalized entropy is positive. After illustrating this simple qualitative criterion, we perform a more quantitative study of the growth of orbits in some low-dimensional examples of zero-entropy maps. Our examples include period-doubling maps in dimension one, and maps of the annulus built from circle homeomorphisms having an exceptional minimal set. This talk is based on joint work with P. Hazard and C. Tresser.

Speaker: Albert Fisher (USP)
Title: *Infinite measures in dynamics with fractal-like return times*
Abstract: In the most often studied types of dynamics, one looks for an appropriate finite ergodic invariant measure, where we can apply ideas from ergodic theory. Normalizing this to total mass one (a probability measure), then the Birkhoff ergodic theorem tells us that "time average equal space average", so the asymptotic frequency of time that a point spends in a subset is equal to the measure of that set, i.e. to the probability of that "event". There are, however, other dynamical systems where the natural recurrent ergodic measures are infinite. In that case, the time average for returns to any finite measure subset must be zero. Equivalently, the set of return times of a point has density zero. At first such sets may seem uninteresting as they are so sparse. However, there can be a lot of interesting stucture, for example they may have a polynomial growth rate with exponent $0<d<1$ and may imitate the geometry of a fractal set of dimension d. And indeed, ergodic maps can be found with this type of fractal-like return behavior. In the nicest cases, this leads to a new statement of the type "time average equals space average": an order-two ergodic theorem. We will survey some examples coming from probability theory (renewal processes), geometry (the horocycle flow of an infinite area Riemann surface), and dynamics (adic transformations). Interesting examples of this last last come from ``nested circle rotations": one irrational rotation embedded as a measure-zero Cantor subset of another. Joint work with Marina Talet.

Speaker: Marty Golubitsky (OSU)
Title: *Two Properties of Network Solutions*
Abstract: In this talk I will discuss two robust properties of solutions to networks of systems of ordinary differential equations: phase-shift synchrony in periodic solutions and singularities in projections of families of equilibria. Both topics developed in response to applications: the first motivated by locomotor central pattern generators for animal gaits (not discussed in this talk) and the second by homeostasis (which occurs when a property of a stable equilibrium remains approximately constant as an external parameter is varied). We show that rigid phase-shift synchrony can occur in network dynamics only when the network has symmetry and that homeostasis can be understood mathematically by use of the normal form and unfolding theories of elementary catastrophe theory.

Speaker: Claudio Gorodski (USP)
Title: *Boundary in the orbit space*
Abstract: Consider a proper and isometric action of a Lie group G on a complete Riemannian manifold M. The orbit space X=M/G carries a canonical stratification by orbit types. The boundary of X is defined to be the closure of the union of all strata of codimension one. The presence or absence or boundary in the orbit space is related to other geometric/algebraic properties of the action. We can linearize the situation as follows. It is easy to see that the boundary is non-empty if and only if there exists a point p in M such that the orbit space of the slice representation at p has non-empty boundary. In this talk, we explain how some insights about the geometry of compact Riemannian symmetric spaces can be used to classify orthogonal representations of compact Lie groups whose orbit space has a non-empty boundary. Joint work with A. Kollross (Stuttgart) and B. Wilking (Münster).

Speaker: Yuji Kodama (OSU)
Title: *Combinatorics and geometry of two-dimensional wave patterns*
Abstract: Let Gr(N,M) be the real Grassmann variety defined by the set of all N-dimensional subspaces of R^M. Each point on Gr(N,M) can be represented by an NxM matrix A of rank N. If all the NxN minors of A are nonnegative, the set of all points associated with those matrices forms the totally nonnegative part of the Grassmannian, denoted by Gr(N,M)^+. In this talk, I start to give a realization of Gr(N,M)^+ in terms of the (regular)

soliton solutions of the KP (Kadomtsev-Petviashvili) equation which is a two-dimensional extension of the KdV equation. The KP equation describes small amplitude and long waves on a surface of shallow water. I then construct a cellular decomposition of Gr(N,M)^+ with the asymptotic form of the soliton solutions. This leads to a classification theorem of all solitons solutions of the KP equation, showing that each soliton solution is uniquely parametrized by a derrangement of the symmetric group S_M. Each derangement defines a combinatorial object which enable us to give a classification of the "entire" spatial patterns of the soliton solutions coming from the Gr(N,M)^+ for asymptotic values of the time. The talk is elementary, and shows interesting connections among combinatorics, geometry and integrable systems.

Speaker: Facundo Memoli (OSU)
Title: *Algebraic methods for characterizing dynamic networks*
Abstract: When studying flocking/swarming behaviors in animals one is interested in quantifying and comparing the dynamics on the clustering structures induced by the coalescence and disbanding of animals in different groups. Motivated by this we study the problem of obtaining homology based summaries of time dependent network data. These algebraic summaries admit a classification in terms of finite configurations of points in R^2. We study their stability under a suitable variant of the Gromov-Hausdorff distance.

Speaker: Eric Ossami (student - USP)
Title: *The Ising model and the phase transition*
Abstract: Ising model, invented by Lenz and studied by Ising, is one of the first model that was studied in order to attempt to derive a phase transition by thermodynamical formalism. In 1922, Ising showed that the model on the one-dimensional lattice has absence of a phase transition at any temperature. However, after 14 years, Peierls showed the presence of a phase transition at low temperature for d-dimensional lattice with $d>1$. We are going to introduce the Ising model and present some classical results, as well as recent results when the model has presence of inhomogeneous external field.

Speaker: Grzegorz A. Rempala (OSU)
Title of preliminary talk: *Stochastic Models of Infectious Diseases*
Abstract: In this introductory talk aimed at graduate and upper level undergraduate students, I will describe the basic concepts of stochastic reaction networks and their relevance to modeling infect$ show examples of applications to the classical susceptible-infectious-removed (SIR) model.
Title of main talk: *Correlation equations, US elections, and modeling SIR-epidemics on dynamic random graphs*
Abstract: Stochastic SIR-type epidemic processes on random graphs are a special class of interaction networks that have become of interest lately for modeling contact-type epidemics (Ebola, HIV, election choices etc). I will discuss a particular case of the SIR epidemic evolving on a configuration model random graph with given degree distribution. In particular, I will describe the relevant large graph limit result which yields the law of large numbers (LLN) for the edge-based SIR process and is useful in building a "network-free" SIR Markov hybrid model for epidemic parameters inference.

Speaker: Nimish A. Shah (OSU)
Title: *Equidistribution of expanding translates of shrinking curves and Dirichlet's approximation theorem*
Abstract: The dynamics of subgroup actions on finite volume quotient spaces of Lie groups, also called homogeneous dynamics, is intimately connected to various questions on Diophantine approximation. Via proving a result on equidistribution of limits of expanding translates of analytic curves on the space of unimodular lattices in R^(n+1), we showed that for almost all points on any analytic curve on R^n which is not contained in a proper affine subspace, the Dirichlet's theorem on simultaneous approximation cannot be improved. Very recently, in a joint work with Pengyu Yang, by proving a novel result on linear dynamics and equidistribution of expanding translates of shrinking curves, we have managed to generalize the non-improvablility result for points on regular smooth curves. At the heart of such results lies Ratner's theorem for unipotent flows. In this talk we will try to explain some of the concepts involved this circle of ideas.

Speaker: Gaetano Siciliano (USP)
Title: *On a quasilinear Schrödinger equation with an almost critical nonlinearity*
Abstract: We consider, on a bounded and smooth domain in R^3, a quasilinear Schrödinger operator under a p-power nonlinearity. On the boundary of the domain we assume homogeneous Dirichlet boundary condition. By using the Ljusternick-Schnirelmann Theory we show that, whenever p approaches the critical Sobolev exponent related to the problem, the number of positive solutions is estimated below by the LS category of the domain. This result is obtained in collaboration with G.M. Figueiredo (UnB) and B.U. Severo (UFPB).