Title: Drinfeld modules and Anderson t-motives - characteristic p analogs of abelian varieties

Abstract: Drinfeld modules and Anderson t-motives - their generalizations - form a parallel world in finite characteristic to the theory of abelian varieties over global number fields. But this analogy is far to be complete, for example, the lattice of an Anderson t-motive can be "smaller" than it should be (i.e. *exp* in (1) is not always an epimorphism, unlike the case of abelian varieties in (2)). From another side, there is a natural notion of the tensor product and Hom of Anderson t-motives, while their analogs for abelian varieties are not known yet. There exists also an analogy of the theory of Anderson t-motives and the theory of linear differential operators.

We shall give definitions of Anderson t-motives and related objects. First, we define the lattice $L(M)$ of a t-motive M, and an exact sequence

$$0 \to L(M) \to Lie(M) \stackrel{exp}{\to} E(M) \tag{1}$$

— an analog of the lattice exact sequence of a g-dimensional abelian variety A:

$$0 \to L(A) = \mathbb{Z}^{2g} \to Lie(A) = \mathbb{C}^g \to A \to 0 \tag{2}$$

Further, we define the Tate modules $T_{\mathfrak{p}}(M)$, where $\mathfrak{p}$ is prime ideal of a global functional field, Galois action on $T_{\mathfrak{p}}(M)$, and eigenvalues of Frobenius automorphisms. We consider the analogs of the upper half plane and the action of the corresponding reductive groups on them - analogs of the simplest action of $SL_2(\mathbb{Z})$ on the upper half plane, and analogs of the Eichler- Shimura theorem for this simplest case. We explain why Anderson t-motives are analogs not of generic abelian varieties, but of abelian varieties with multiplication by an imaginary quadratic field. Also, we consider definitions of some types of L-functions of t-motives (there are several types of L-functions). Finally, we mention generalizations of t-motives to sheaves over curves in characteristic p.

Some research problems will be stated.

References:

[D76] V.G. Drinfeld, Elliptic modules. Math. USSR Sb. 4 (1976) 561 – 592.

[A86] Anderson, Greg W. t-motives, Duke Math. J. 53 (2) (1986) 457 – 502.

[G96] Goss, D. Basic structures of function field arithmetic. Springer-Verlag, Berlin, 1996. xiv+422 pp.

[GL20] Grishkov A., Logachev, D. Introduction to Anderson t-motives: a survey. https://arxiv.org/pdf/2008.10657.pdf

Comments: [D76], [A86], [G96] are the basic sources of the subject, but they are difficult for a beginner.