Jacob's Ladder and Scientific Ontologies: The Logic of Scientific Discovery

or Traveling Around the Möbius Band, by Aesthetics, Ethics and Metaphysics

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http://www.ime.usp.br/~jstern/miscellanea/jmsslide/moebs.pdf
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This Presentation

I Introduction;
II Cognitive Constructivism
- an epistemological framework for the FBST,
as Popperian Falsificationism for *p*-values,
as von Neumann-Morgenstern Utility Theory or
de-Finettian Decision Theory for Bayes Factors;
III History of Chemistry; Illustrative examples;
IV Ontological Alignments (future research);
V FBST Verification of Sharp Statistical Hypotheses;
VI The Zero Probability Paradox;
VII Bibliography (previous work).

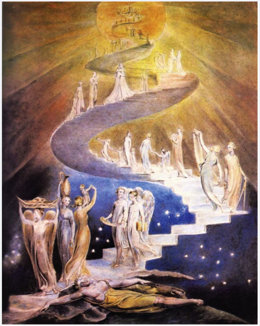
What do we want to do? Try to take over the world!
- Every grand-narrative tuns out to be flawed;
Still, some are worth the effort, providing
good intuition and powerful insights.

Cog-Con+FBST Reserach Program

Previous work of the IME-USP Bayesian research group:

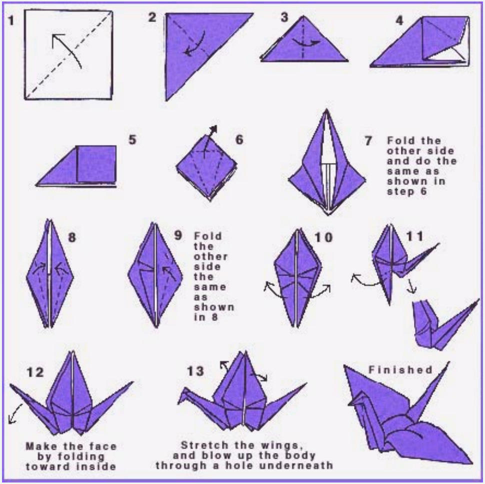
- Full Bayesian Significance Test for sharp hypotheses: Pereira and Stern (1999); Pereira et al. (2008); etc.
- Statistics, theory and methods: Diniz et al. (2011, 2012); Lauretto et al. (2003); Laureto et al. (2012); etc.
- Sparse and structured methods: Colla and Stern (2009); Inhasz and Stern (2010); etc.
- Logic and compositional aspects of *e*-values: Borges and Stern (2007); Stern (2003, 2004); Stern and Pereira (2013); etc.
- Cognitive Constructivism epistemological framework: Stern (2007a,b, 2008a,b, 2011a,b).
- Evolution of Science (and historical interpretations).
- Ontology Alignments (Diachronic, Synchronic).

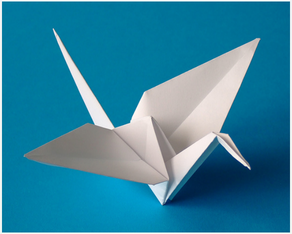
Helical Stairways, Mosaic Pavements, Heavy Anchors



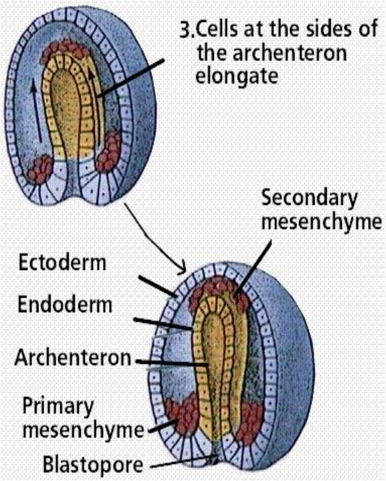
Metaphoric Imagery:
Ladder: Evolutive process of a scientific discipline;
Floors: Development stages;
Checkered pattern: Ontological organization of known objects, language semantics & syntax;
Projection: Ontolog. alignment;
Spiral case: Optimal alignment (non-dislocated landing points);
Anchor (bottom): $ev(H \mid X)$ - epistemic value of a sharp or precise statistical hypothesis, objective symbol grounding.

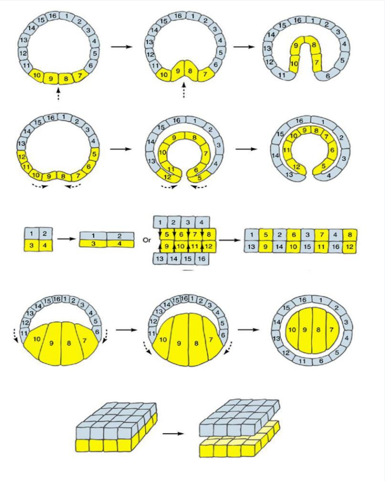
Painting: Willam Blake.
Photography: Nils Eisfeld.



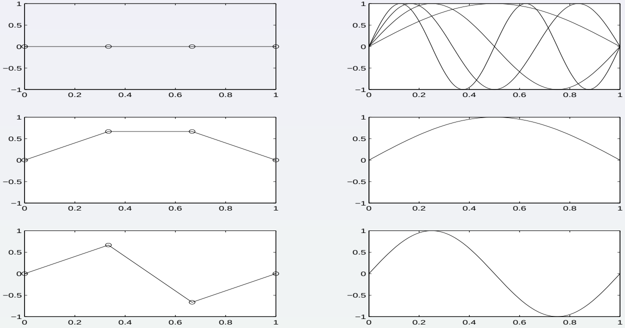


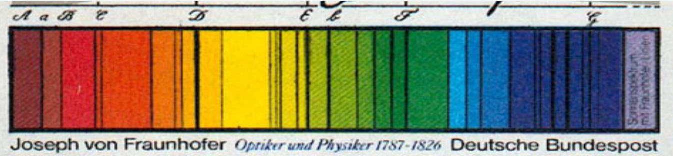
Origami folding instructions for a Crane (Tsuru).
Richard Dawkins: Play Chinese whispers game with both cranes?
Folds are: Exact, Stable, Separable and Composable!





Real cranes are also (self) assembled as an origami!
Organic morphogenesis: Gastrulation. Tissue movements:
Invagination, involution, convergent extension, epiboly, delamination.
Biology demands statistic/stochastic models, fractal geometry...





Eigen-Solutions of continuous and discrete vibrating chords.
Normal modes are: Exact, Stable, Separable and Composable!
Fraunhofer's map of sharp lines (eigen-values?) in solar spectrum.

Differential eq.for the discrete chord system's dynamics (x-displacement, h-tension, m-mass, s-spacing):

$$\ddot{x} + Kx = 0 \ , \quad w_0^2 = h/ms \ ,$$

$$K = w_0^2 \begin{bmatrix} 2 & -1 & 0 & 0 & \cdots & 0 \\ -1 & 2 & -1 & 0 & \cdots & 0 \\ 0 & -1 & 2 & -1 & \ddots & \vdots \\ 0 & 0 & -1 & \ddots & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & 2 & -1 \\ 0 & 0 & \cdots & 0 & -1 & 2 \end{bmatrix} .$$

Use the orthogonal transform. $x = Qy$ diagonalizing K, i.e., $Q'Q = I$ such that $Q'KQ = D = \text{diag}(d)$, $d = [d_1, d_2, \ldots, d_n]'$.

$$Q'(Q\ddot{y}) + Q'K(Qy) = I\ddot{y} + Dy = 0 \ \Rightarrow$$

$$\ddot{y}_k + d_k y_k = 0 \ , \ \ y_k(t) = \sin(\varphi_k + w_k t) \ .$$

The new 'normal' coordinates, $y = Q'x$, present n decoupled scalar equations for harmonic oscillators, with phase $0 \leq \varphi_k \leq 2\pi$ and angular frequency $w_k = \sqrt{d_k}$.
Eigenvectors of matrix K are columns of the decoupling operator Q, (multiples of) the un-normalized vectors z^k.
Corresponding eigenvalues are $d_k = w_k^2$, for $k = 1 \ldots n$,

$$z_j^k = \sin\left(\frac{jk\pi}{n+1}\right) \quad , \quad w_k = 2w_0 \sin\left(\frac{k\pi}{2(n+1)}\right) \quad .$$

Decoupling / composition properties: Fourier analysis.

Musical notes are precise, stable, separable and composable; the "atoms" or "building blocks" for all western musical systems. Sumerian musical notation in diatonic scale precedes functional analysis by 40 centuries!

Aesthetics: Objects are Tokens for Eigen-Solutions

Heinz von Foerster in Segal (2001, p.127, 145, 266).

- *Objects are tokens for eigen-behaviors [eigen-solutions]. This is the constructivist's insight into what takes place when we talk about our experience with objects.*
- *Eigenvalues have been found ontologically to be discrete [exact, precise, sharp], stable, separable and composable, while ontogenetically to arise as equilibria that determine themselves through circular processes. Ontologically, Eigenvalues and objects, and likewise, ontogenetically, stable behavior and the manifestation of a subject's "grasp" of an object cannot be distinguished.*
- The meaning of recursion is to run through one's own path again. Under certain conditions there exist indeed solutions which, when reentered into the formalism, produce again the same solution. These are called eigen- equilibrium- invariant- fixed-... -solution -state -behavior -point...

- Ex1: Football, passive object that interacts with a player according to FIFA's Law 2 of the Game: Spherical symmetry, 26 ± 1in size, 15 ± 1oz weight, 0.6-1.1atm inflation pressure, etc; characteristics determining the exact form of a *stable behavior*.
- Ex2: Virus (RNA), active autocatalytic objects, but not *alive*.
- Ex3: Bacterium (DNA), strange-loop that recursively renews its molecular components during its lifetime, Bertalanffy (1969).

- Autopoietic systems are organized (defined as a unity) as a network of processes of production (transform., destruction) of components that, through their interactions and transformations, continuously regenerate and realize the same network.
- This circular organization implies predictions: Interactions that took place once will take place again... Every interaction is a particular interaction, but every prediction is a prediction of a class of interactions. This makes living systems inferential systems, and their domain of interactions a cognitive domain. Maturana, Varela (1980, p.10, 78-79, 84).

Scientific Life or Auto-Poietic Diagram

Experiment				Theory
Operation-alization	$\Leftarrow$	Experiment design	$\Leftarrow$	Hypotheses formulation
$\Downarrow$				$\Uparrow$
Effects observation		**True/False** eigen-solution		**Innovative** interpretation
$\Downarrow$				$\Uparrow$
Data acquisition	$\Rightarrow$	**Metaphysical** explanation	$\Rightarrow$	Statistical modeling
Sample space, X				Parameter space, Θ

Scientific (self or recursive) production diagram.

- Objects (eigen-solutions) emerging in the scientific production cycles are expressed as - *mathematical equations*.
- Good examples of such deterministic statements are Newton's, Einstein's or Maxwell's laws; Examples of probabilistic statements can be found in statistical physics, quantum mechanics, or stochastic population genetics.
- In all these examples, the equality sign ($=$) of the law, formula or hypothesis, expresses the first essential property of an eigen-solution, namely - *precision* or *sharpness*.

- Even considering that any actual experiment aiming to verify a scientific statement expressed as a mathematical equation has its design flaws and is plagued by a variety of operational imprecisions and measurement errors, there is an underlying equality relation the experiment aims to access.
- Modern technology is living proof that science has, to a great extent, been very successful at this task. (ex. Pentium CPU)

- Truth value ev($H \mid X$) for sharp hypotheses, see Part V.

Metaphysics and Latent (random) Variables

Metaphysics concerns:

- Non (directly) observable entities (beyond physical).
- Causal explanations, that is, answers for why-questions giving reasons for things being the way they are (Aristotle).
- Systematic account of possible forms of understanding, valid forms of explanation or rational principles of intelligibility (gnoseological sense).
- In Statistical models:
    - Theoretical, latent or non-observable (random) variables are Greek letters in the Parameter space;
    - versus experimental, directly observed or state variables, that are Latin letters in the Sample space.

Hamlet: My father! - methinks I see my father.
Horatio (Royal court Statistician): Where, my lord?
Hamlet: In my mind's eye!   (a metaphysical entity)
(Horatio: Fatherly variables are Greek.)

Francis Bacon (1620) title page of
Novum Organum Scientiarum
(New Instrument of Science)
showing the brave ship Verulamio
navigating the twin columns of knowledge.

Eigen-solutions are process invariants. Therefore, Scientific objects can only emerge in whole (recurrent) production cycles, including theoretical explanations and experimental methods.

Hence, v.Foerster's *Metaphysical (negative) imperative:*
Something that cannot be explained cannot be seen!

A naïve "orientation flip" to a positive aphorism:
"Explain that what you want to see" - is just wishful thinking!
Science must stand on both columns: Experiment and Theory!

Are metaphysical or abstract entities "real" or just ghosts?
Stern (2011a,b): (anti)Realism, (anti)Positivism, etc.

Reality, Language and Ontology

- Known (knowable) Object: An actual (potential) eige-solution of a system interacting with its environment.
- Objective (how, less, more): Degree of conformance of an object to essential attributes of an eigen-solution (Precision, Stability, Separability, Composability).
- Reality: A (maximal) set of objects, as recognized by a given system, when interacting with single objects or with compositions of objects in that set.
- Ontology: A carefully controlled language (vocabulary, semantics and syntax) used to re-present, communicate and think about a given reality, its forms of production, and compositional rules or relations among objects. It includes names for eigen-solutions, as well as words for experimental means and methods, and also words for theoretical and explanatory concepts.

Piaget's Problem: From Statics to Dynamics

So far, we explained how to verify a static scientific framework; However, how does science progress, making new discoveries?

That is Piaget's *central problem of knowledge construction*, namely, the *re-equilibration of cognitive structures:*

An adequate model of knowledge construction must comply with two conditions that are difficult to conciliate:

To be open to indefinite new possibilities while conserving already constructed cycles of mutual implications destined to be converted into sub-systems of an expanded system:

The issue is therefore to conciliate openness and closure.

Jean Piaget (1976, p.91), commenting on a contribution of Heinz von Foerster, and expressing the need for change, dynamics and evolution.

Life (Auto-Poietic) Imperatives

Piaget's 'central problem' was formulated answering v.Foerster. We continue the 'dialog' using five of v.Foerster's aphorisms, see Heinz von Foerster (2003, p.136, 172, 175, 227, 284, 303), as a guideline in our investigation of Piaget's problem.

(1) Metaphysical (implicit, negative) imperative:
Something that cannot be explained cannot be seen!

(2) Therapeutic Imperative:
If you want to be yourself, change!

(3) Aesthetical imperative:
If you desire to see, learn how to act!

(4) Ethical imperative:
Act so as to increase the number of choices!

(5) Organic imperative:
Maintain structural and functional integrity!

Therapy: Curing the Blind (Innovation and Change)

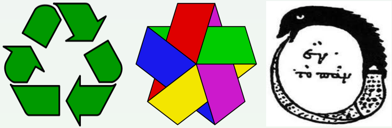
- The therapeutic imperative seeks cure for systemic blindness, revealing what is concealed, displaying what is hidden.
- However, like in a Möbius band, in order to obtain such an "orientation flip", one has to travel all the way around the strip, revisiting the aesthetical, metaphysical and ethical imperatives or, in so doing, climbing an helical stairway in Jacob's Ladder.
- The therapeutic imperative points to creative evolution, seeking sharper images of an expanded reality, looking for better scientific understanding of new phenomena, by (typically, but not necessarily in that exact order):

Speculating on alternative approaches and investigating innovative hypotheses, developing conventional means and methods and coherently incorporating unconventional ones, searching for new or sharper eigen-solutions and, if necessary, integrating all of the above in better theoretical frameworks.

Ethical Innovation and Recycling

Ethical imperative: *Act so as to increase the number of choices!*

- Necessary innovation can't be shortsighted or improvident:
  - Do not to be penny-wise and pound-foolish!
- A viable path of evolution must recycle already developed (eigen) solutions, keeping them alive even if as revised versions, replacing components of their production cycle by functional equivalents or succedanea that allow the system to retain its modus operandi (to maintain its autopoiesis).
- Conciliate Innovation and Recycling is not an easy task, with conflicting demands and inconsistent conditions: Difficult (ethical) choices need to be made.



Recycling symbol (3), Pentagram (5 foldings), Ourobouros (continuous), depicted as a Möbius band.

Autopoietic vs. Allopoietic Ethics

- Some traditions consider ethics as conformance to an externally produced (allopoietic) normative regulation, legislation, code of honor, etc.
- There is also a long standing tradition of autopoietic ethics that relates to the effort or endeavor of a living organism, individual, or abstract autopoietic system to create and retain its own (autonomous) basic set of core operations, kernel procedures or behavioral patterns, that is, to coherently maintain its modus vivendi, its form of being or way of existence (actual, potential or ideal).
- Arguably, Baruch Spinoza developed the most influential theory of autopoietic ethics in mainstream philosophy, see his Ethics (1677, Ch.III, Prop.6-11).
- DeBrander (2007) links this tradition to the Stoics.

Hierarchical Organization and Modular Structures

Herbert Simon (1996) parable of two watch makers:

Tempus sequential assembly: $r = 1000$ single elements.
Hora assembly: $m = 100 + 10 + 1 = 111$ modules of $r = 10$ parts each, organized in 3 hierarchical levels.

Deterministic environment: One minute to put a part in place.
Assembly times (minutes). Hora: 1110 ; Tempus: 1000 .

Noise: Interruption/minute with probability $p = 0.01$.
Unstable incomplete modules brake down at an interruption.
The expected (random) assembly time of a watch is:

$$\frac{m}{p}\left(\frac{1}{(1-p)^r} - 1\right) .$$

Assembly (minutes). Hora: 1,173.6 ; Tempus: 2,316,256.5 .

In a noisy world (like ours), complexity - *may* - be very toxic:
Tempus struggles while Hora prospers!

Higher Order Autopoietic System

- Systemic evolution generates complexity (toxic).

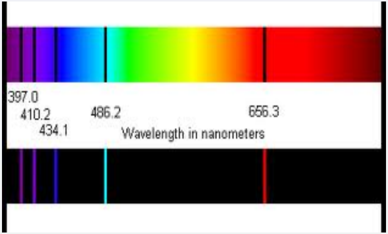
- Organic Imperative: *Maintain systemic integrity!*

Francisco Varela (1979, p.53,58), HOAS and Closure Theses:

- *There is an ever-present selective pressure for the constitution of higher-order autopoietic systems.*
- *Every autonomous system is organizationally closed.*

- Example of Self-organized, auto-assembled complex system: Beehive, (3rd order syst.), formed by the coupling of individual Bees (2nd) which, in turn, are made of individual Cells (1st).
- Niklas Luhmann applied this notion to study social systems. His basic abstraction is to look at them as autopoietic communications networks.
- The autopoietic nature of each (sub) system stipulates that one system may be aware of events at other systems, that is, be *Cognitively Open*, but is required to maintain its differentiation, that is, be *Operationally Closed*.
(a bit of a mystery, more on Part IV)

III - History of Chemistry; Illustrative examples
Example 1: Blowpipe and Spectral Analysis



An old and tricky art vs. a recent and precise technology.
(analogous to free hand-drawing vs. origami foldings)

Scientific aesthetics: Nice colors, sharp images, and beautiful tokens.

Stepping Stones for the Spectroscopy Revolution

- 1704, Isaac Newton; Spectroscopic experiments.
- 1785, David Rittenhouse; Good diffraction gratings.
- 1802, William Wollaston; Solar spectrum dark lines.
- 1814 to 1821, Joseph von Fraunhofer; Reliable spectroscopes, using prisms and diffraction gratings, first systematic spectral maps. (...wait half a century...)
- 1859, Gustav Robert Kirchhoff and Robert Wilhelm Eberhard Bunsen; Spectroscopy analysis, emission and absorption, as efficient methods for analytical chemistry.

Spectroscopy and the Chemists: A Neglected Opportunity? Sutton (1976) speculates that an earlier adoption of spectroscopic methods in chemical analysis was prevented by:

- *Unavailability of the necessary apparatus*, or (can't be!)
- *Suspicion of the consistency of the effect in the absence of any adequate theory of its cause*.

Bohr's Atom, Balmer's Formula and Kirchhoff's Law

- 1913, Niels Henrik David Bohr; First physical model of atomic structure with radiated photon energy quantization.
- 1885, Johann Jakob Balmer; First explanation for relative position of spectral lines of Hydrogen, using language and context of projective geometry and architecture design (?!) Frequencies, ν, or wavelengths, λ, of these lines are related by $R = 1.0973731568525(73)E7\ m^{-1}$, Rydberg's constant, by the integer algebraic expression:

$$\frac{\nu_{n,m}}{c} = \frac{1}{\lambda_{n,m}} = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right)\ ,\ 0 < n < m \in N\ .$$

- 1859, Gustav Robert Kirchhoff; Radiation equilibrium law: *For all bodies at the same temperature and any kind of radiation, emission power equals the absorption power.*

Minimal Seeds for Phase Transitions

- What makes this historical example so interesting is that, indeed, Kirchhoff did not provide an adequate theory for the cause of spectral emission and absorption (Bohr, 1913), not even did he provide an empirical formula for relative positions of spectral lines (Balmer, 1859).
- Instead, it seems that Kirchhoff (1859) provided the bare minimum for the breakthrough, namely, a firm handle to grasp new objects, as a general thermodynamic equality constraint.
- Kirchoff minimalist explanation triggered an abrupt, radical transformation in its cognitive field, similar to a *seeding effect* for (saturated) condensation or crystallization phenomena.
- Before the seeding, nothing happens, the world stands still...
  \- *The Double Blind* principle, or *The Blind Spot* principle:
  One does not see what one does not see. (w.the mind's eye)
- Pilpul questions: When 'could' some discoveries take place?

Example 2: Chemical Affinity and Stoichiometry

- Alchemy explains the way substances interact telling stories about their passions and desires, how they love and hate each other; wherefrom the term chemical *Affinity* can be traced.
- Affinity is the *organizing principle* of alchemy.

Guyon de Morveau's Table of Numerical Expression of Affinities

Base/ Acid	Vitriolic	Nitric	Muriatic	Acetic	Mephitic
Barytes	65	62	36	28	14
Potash	62	58	32	26	9
Soda	58	50	31	25	8
Lime	54	44	20	19	12
Ammonia	46	38	14	20	4
Magnesia	50	40	16	17	6
Alumina	40	36	10	15	2

Apologies for the following diachronic pasticcio of chemical ontologies, including late alchemy (Stahlian chemistry) and modern notations, more on Part IV.

Morveau's Algebra for Displacement Reactions

Vitriol of Potash, K_2SO_4

Sulph.of Barytes, $BaSO_4$	Vitriol.Ac.	62	Potash	Mephite of Potash, K_2CO_3
	65	+	9 (=74)	
	Barytes	14 (=76)	Meph.Ac.	

Mephite of Barytes, $BaCO_3$

$$\text{Quiescent affinities} = 65 + 9 = 74 \; <$$

$$< \; \text{Divellent affinities} = 62 + 14 = 76 \; .$$

- Morveau (1786) formalism, using experimental data in the form of occurrence (or not) of displacement reactions, can only render *mathematical inequalities*, it can only enforce finite interval bounds for the value of chemical affinities.

Stoichiometry equations

From the following ionic valencies it is easy to set up the stoichiometry balance equations for the reaction example:
- Cations: Potassium, K^{1+}; Barium, Ba^{2+};
- Anions: Carbonate, $(CO_3)^{2-}$; Sulfate, $(SO_4)^{2-}$;
- In Water: Hydrogen, H^{1+}; Hydroxide, $(OH)^{1-}$.

$$BaSO_4 + K_2CO_3 \rightarrow K_2SO_4 + BaCO_3 \ ;$$

or, in more detail, for water solution,

$$[Ba(OH)_2, H_2SO_4] + [2KOH, H_2CO_3] \rightarrow [2KOH, H_2SO_4] + [Ba(OH)_2, H_2CO_3] \ .$$

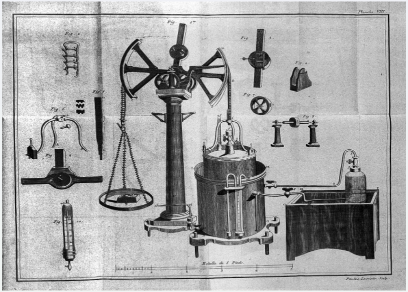
The symbol $\longrightarrow$ represents a "displacement" reaction.
The notions of reaction velocity and state equilibrium, $\rightleftharpoons$, were only introduced by Guldberg and Waage (1879), explicitly trying to rehabilitate the concept of chemical affinity within a *...general theory of chemical reactions... wherein a state of equilibrium is produced between opposing forces.*

Conservation laws and Invariant elements

- Eigen-solutions can be characterized as invariants or fixed-points for the system's operations.
- Antoine-Laurent de Lavoisier (1789): *We must admit as elements the substances into which we are capable, by any means, to reduce bodies by decomposition.*
- Jeremias Benjamin Richter (1792): Stoichiometric balance equations (algebraic form) giving the exact proportions in which elements (reagents) interact.
- Invariance Properties and Conservation Laws expressed by stoichiometric balance equations mutually support each other. Their validity can be jointly checked and confirmed by empirical gravimetric and volumetric (for gasses) measurements.
- The relation the between Invariant Elements of a theory and its Conservation Laws is a very rich area of research, see Stern (2011b) for a formal and detailed analysis.

Alchemy, Chemistry and Nicety of Experiments

- Modern chemistry "starts" with Lavoisier's definition of invariant elements and (implicit) conservation laws



in the form of stoichiometric *equality constraints*, and also with the development of new experimental methods and high precision measure. equipment like balances, gasometers, etc.

- Lavoisier (1789, Plate VIII), *Traité élémentaire de chimie.*

- Comparing their aesthetical properties, we can understand how affinity rules, formerly considered *the axioms of chemistry*, suffered a decline of prestige or decay in scientific status relative to new entities of stoichiometric equations.
- Cog-Con framework naturally shields against naïve-realist (de)reification statements that often follow a paradigm change.

From Vapor-Ware to Material States

Paraphrasing Gough (1988, p.31), we give the following simplified outline of Lavoisier's research core steps:

- These 3 steps follow the Metaphysical, Aesthetical and Ethical imperatives, around an helical stair in Jacob's Ladder.

(1) First (both historically and logically), Lavoisier formulated a theory of the gaseous state of matter that allowed him to conceive chemically distinct substances in aeriform state.

(2) Second, the development of measurement methods capable of accounting for the invisible gases leaving and entering chemical reactions, formulation of (implicit) conservation laws, and identification of invariant elements.

(3) Third, the fact that many aeriform substances existed in physical states that were nearly identical, but in chemical states that were quite divergent, prompted Lavoisier to apply the Stahlian reactive criteria of chemical identity more thoroughly and systematically than ever before.

Ethical Recycling of Stahlian Chemistry

- Lavoisier's revolution was a very ethical one!
- In 1787, an elite group of French chemists introduced a new *Method of Chemical Nomenclature*. This new language was carefully crafted to faithfully express and reliably correspond to the analytical mechanisms of the new chemistry.
- This new language has also very successful in efficiently recycling an important inventory of chemical substances and preparation methods used in Stahlian chemistry (late alchemy). allowing Lavoisier to take over, *virtually intact*, the chemistry of salts, a body of knowledge that had been developed according to *its own principles and logic* (affinity); see Holmes (1989).
- Eklund (1975) compiles a dictionary that re-presents old objects using the new nomenclature. However, translation assumes some form of functional compatibility or role-playing correspondence; It does not imply identity.

Interesting Episodes from Physics (future research)

- Fermat-Leibniz-Euler-Maupertuis principle of Least Action:
  - Formulation of efficient and objective teleological laws.
- Ludwig Boltzmann's incredible probabilistic entropy:
  - Albert Einstein's theory on Brownian motion; Conciliating thermodynamics and kinetic theory with osmotic pressure.
  - Jean Perrin's 'irrefutable' comprobatory experiments.
- Special Relativity, Space-Time and the Lorentz group:
  - Michelson-Morley experiment, and the verification of a shaply defined but unexpected null hypothesis.
  - Busting dusty synthetic-a-priori propositions.
- Werner Heisenberg's Uncertainty principle:
  - Being sure of a sharp lower bound to uncertainty, and estimating $\hbar$ to get objective epistemic probabilities.
  - Niels Bohr's new type of causation: complementarity; after Aristotelian: Material, Formal, Efficient and Final.

IV - Ontological Alignments (future research)

- Ontological alignment ≈ Translation dictionary.
Translation assumes some form of functional compatibility or role-playing correspondence; It does not imply identity.
  - Diachronic alignment: Translates between two theories from different times aiming to understand the “same things”.
  - Synchronic alignment: Translates between two co-existing theories about different things sharing some compatible objects and some “common” terms or concepts.
- Related problems:
  - (In)commensurability of scientific theories;
Feyerabend (1999) and Kuhn (2000); Trendy but unethical!
  - Objective symbol grounding in scientific ontologies.
“External” to language, but not reaching out to a *Ding an sich*.
  - Cognitive Openness / Operational Closure paradoxes.
- Can alignments be based on (algebraic) structures, internal to each ontology, for relations among objects?

Long-Range Diachronic Alignments (future research)

- Diachronic multi-alignment example: Minimum Cross Entropy (MinXEnt) formulation of thermodynamic reaction networks under stoichiometric and affinity constraints:

$$\min_p \varphi(p,q) = p^{f\prime} \log\left(p^f / q^f \right) + p^{r\prime} \log\left(p^r / q^r \right) \; ,$$

$$\text{such that } \; p^f > 0, \, p^r > 0 \; \text{ and } \; S(p^f - p^r) = b \; ;$$

where S, $m \times n$, is a generalized stoichiometric matrix;
p is a flux vector of (forward and reverse) reaction rates;
$Sp = dx/dt$ is the differential of chemical concentrations.

- At equilibrium, $\rho \log(p^f \oslash p^r) = -\Delta u, \quad \Delta u = S' u$.

Hence the system follows mass-action kinetics, implying energy conservation constraints in the same algebraic form of Morveau's additive structure (for chemical potentials)!

- MinXEnt and Minimum Information Divergence problems in Stern (2011b) used to discuss - *Ontological as Invariant*.
- Footsteps of Felix Klein, Emmy Noether, Harold Jeffreys, etc.

Synchronic Alignments (future research)

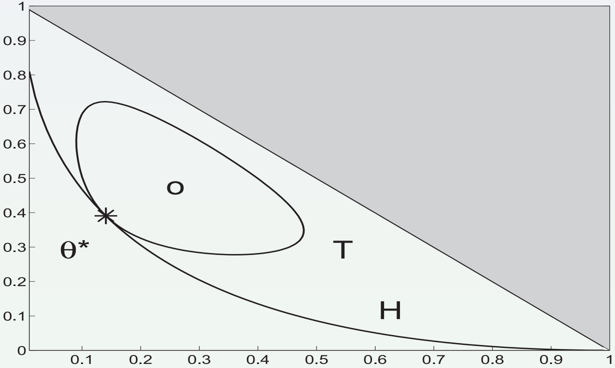
- Discuss the organization of science as a higher order autopoietic system with autonomous disciplines, for ex. Physics and Chemistry, and the related problems of:
  - Growth of complexity in evolving organisms, v.Foerster's Organic Imperative, and the need for modular structures and hierarchical organization;
  - Synchronic ontology alignments, for example: Concepts of mass, energy, entropy, etc, in Physics and Chemistry.
  - Synchronic alignments as a way to enable effective interaction or communication between sub-systems (cognitive openness), respecting nevertheless each sub-system autonomy (operational closure).

ABSURDIST Alignments (future research)

- Goldstone and coauthors (2002, 2004).
  ABSURDIST: Aligning Between Systems Using Relations Derived Inside Systems for Translation
  + "extrinsic" information.
- Only a few "good" seeds of extrinsic information concerning local correspondences, can have a ripple effect, triggering reliable global alignments.
- How to select *Key-Objects* for external seeding?
- Validation criteria for an isolated local correspondence?

Rosetta stone (196 BC):
Hieroglyphs, Demotic, Greek.
Hybrid vehicle providing synchronic and diachronic ontological alignments.

States that the truth value of the parameter, θ, of the sampling distribution, $p(x \mid \theta)$, lies in a low dimension set: The Hypothesis set, $\Theta_H = \{\theta \in \Theta \mid g(\theta) \leq 0 \wedge h(\theta) = 0\}$, has Zero volume (Lebesgue measure) in the parameter space.



Hardy-Weinberg Hypothesis

V.i - Bayesian setup

- $p(x \mid \theta)$: *Sampling distribution* of an observed (vector) random variable, $x \in \mathcal{X}$, indexed by the (vector) *parameter* $\theta \in \Theta$, regarded as a latent (unobserved) random variable.
- The model's joint distribution can be factorized either as the *likelihood function* of the parameter given the observation times the *prior* distribution on θ, or as the *posterior* density of the parameter times the observation's marginal density,

$$p(x, \theta) = p(x \mid \theta)p(\theta) = p(\theta \mid x)p(x) \ .$$

- $p_0(\theta)$: The *prior* represents our initial information.
- The *posterior* represents the available information about the parameter after 1 observation (unnormalized potential),

$$p_1(\theta) \propto p(x \mid \theta)p_0(\theta) \ ,$$

normalization constant: $c_1 = \int_\theta p(x \mid \theta)p_0(\theta)d\theta$.

- Bayesian learning is a recursive and commutative process.

- Likelihood principle: All statistical information about the observed sample is contained in the likelihood function.
- Example: Hardy-Weinberg genetic equilibrium model, see Pereira and Stern (1999).

  - n , sample size, x_1, x_3 , homozygote,
  - $x_2 = n - x_1 - x_3$, heterozygote count.
  - $\Theta = \{\theta \geq 0 \mid \theta_1 + \theta_2 + \theta_3 = 1\}$,
  - $H = \{\theta \in \Theta \mid \theta_3 = (1 - \sqrt{\theta_1}\ )^2\}$.

  $y = [0, 0, 0]$, Flat or uniform prior,
  $y = [-1/2, -1/2, -1/2]$, Invariant Jeffreys' prior,
  $y = [-1, -1, -1]$, Maximum Entropy prior.

  $$p_0(\theta) \propto \theta_1^{y_1} \theta_2^{y_2} \theta_3^{y_3} \ ;$$

  Posterior density after observations $x = [x_1, x_2, x_3]$:

  $$p_n(\theta \mid x) \propto \theta_1^{x_1+y_1} \theta_2^{x_2+y_2} \theta_3^{x_3+y_3} \ .$$

- Several priors allow nice sensitivity tests, Stern (2004).

V.ii- Full Bayesian Significance Test

- $r(\theta)$, the reference density, is a representation of no, minimal or vague information about the parameter θ. If $r \propto 1$ then $s(\theta) = p_n(\theta)$ and $\overline{T}$ is a HPDS.
- $r(\theta)$ defines the reference metric in Θ, $dl^2 = d\theta' J(\theta) d\theta$, directly from the Fisher Information Matrix,

$$J(\theta) \equiv -\mathrm{E}_{\mathcal{X}} \frac{\partial^2 \log p(x \mid \theta)}{\partial \theta^2} = \mathrm{E}_{\mathcal{X}} \left(\frac{\partial \log p(x \mid \theta)}{\partial \theta} \frac{\partial \log p(x \mid \theta)}{\partial \theta} \right) .$$

- The *surprise function*, $s(\theta) = p_n(\theta)/r(\theta)$, measures changes in the posterior relative to the reference density.
- The ‘hat’ and ‘star’ superscripts indicate unconstrained and constrained (to the hypothesis H) maximal arguments and supremal surprise values, as follows:

$$\widehat{s} = \sup_{\theta \in \Theta} s(\theta) \ , \quad \widehat{\theta} = \arg\max_{\theta \in \Theta} s(\theta) \ ,$$
$$s^* = \sup_{\theta \in H} s(\theta) \ , \quad \theta^* = \arg\max_{\theta \in H} s(\theta) \ .$$

- The surprise function's (closed, upper-bound) *v-cut*, $T(v)$, its complement, the *highest surprise function set* (HSFS) above level v, $\overline{T}(v)$, and its *rim* (aka level-v set), $M(v)$, are

$$T(v) = \{\theta \in \Theta \,|\, s(\theta) \leq v\} \ , \ \ \overline{T}(v) = \Theta - T(v) \ ,$$

$$M(v) = \{\theta \in \Theta \,|\, s(\theta) = v\} \ .$$

- If the reference density the uniform (possibly improper) density, $r(\theta) \propto 1$, then $s(\theta) \propto p_n(\theta)$ and the HSFS are standard *highest probability density sets* (HPDS)
- The statistical model's *truth function*, $W(v)$, is the cumulative probability function up to surprise level v, $0 \leq v \leq \widehat{s}$. $\overline{W}(v)$ is its complement, $\overline{W}(v) = 1 - W(v)$, and $m(v)$ is its (generalized Schwartz) derivative,

$$W(v) = \int_{T(v)} p_n(\theta) \, d\theta \ , \ \ m(v) = \frac{d}{dv} W(v) \ .$$

- Finally, the *e*-value for an hypothesis $H \subseteq \Theta$, $\mathrm{ev}(H)$, aka the *epistemic value* of hypothesis H or the statistical evidence supporting H, and its complement, $\overline{\mathrm{ev}}(H)$, are

$$\mathrm{ev}(H) = W(v^*) \ , \ \ \overline{\mathrm{ev}}(H) = 1 - \mathrm{ev}(H) \ .$$

  For the sake of simplicity, we use a relaxed notation for singleton arguments, that is, in the case of a *point hypothesis* $H = \{\theta^0\}$, writing $\mathrm{ev}(\{\theta^0\}) = \mathrm{ev}(\theta^0)$.

- The *e*-value of an hypothesis H is based on the most favorable case, $\mathrm{ev}(H) = \mathrm{ev}(\theta^*)$, a property that gives $\mathrm{ev}(H)$ a *possibilistic* character...

- The *standard* possibility measure, $\pi(H)$, introduced by Dubois and Prade (1982), coincides (in the discrete case) with $\mathrm{ev}(H)$ if $r(\theta) \propto 1$, the trivial reference density.

V.iii - Probability-Possibility Transformations

- Several important properties of $W(v)$ follow directly from the *nesting* property exhibited by the v-cuts that, in turn, give the integration range defining the truth function, see Dubois and Prade (1982),

$$u \leq v \Rightarrow T(u) \subseteq T(v) \Rightarrow W(u) \leq W(v) \ .$$

- Using this nesting property, it is easy to establish that $\text{ev}(H)$ has the desired properties of *consistency* with its underlying probability measure and *conformity* (to be similarly shaped) with its underlying surprise function, i.e.,

$$\text{Consistency:} \quad \text{ev}(H) \geq p_n(H) \ , \quad \forall \, H \subseteq \Theta \ ;$$

$$\text{Conformity:} \quad \text{ev}(\theta) \geq \text{ev}(\tau) \ \Leftrightarrow \ s(\theta) \geq s(\tau) \ , \quad \forall \, \theta, \tau \in \Theta \ .$$

A *plausibility measure*, $\text{Pl}(H)$, is defined by its *basic probability assignment*, $m : 2^\Theta \mapsto [0, 1]$, such that $\int_{S \subseteq \Theta} m(S) = 1$.
The *focal elements* of m are the subsets of the universe with non-zero basic pr.assignment, $\mathcal{F} = \{E \subseteq \Theta \mid m(E) > 0\}$.
Finally, the plausibility of $H \subseteq \Theta$, $\text{Pl}(H)$, is defined as

$$\text{Pl}(H) = \int_{E \in \mathcal{F} \mid E \cap H \neq \emptyset} m(E) \ .$$

Hence, $\text{ev}(H)$ can be characterized as a plausibility function having v-cuts of the surprise function as focal elements, $\mathcal{F} = \{T(v), 0 \leq v \leq \widehat{s}\}$, while the basic probability density assigned to $T(v)$ is obtained integrating the posterior probability density over its rim, $m(v) = \int_{M(v)} p_n(\theta) d\theta$.

A plausibility function defines its dual *belief* function as

$$\text{Bel}(\overline{H}) = \int_{E \in \mathcal{F} \mid E \subseteq \overline{H}} m(E) \ = \ 1 - \text{Pl}(H) \ .$$

- There are some traditional objections raised in decision theoretic Bayesian statistics against measures of statistical significance engendered by credibility regions, namely,

(a) Lack of invariance.

(b) Not an orthodox decision theoretic procedure(?)
-An optimal point "represents" a composite hypothesis.

(c) No need for nuisance parameter elimination procedures.

(d) Epistemological interpretation of sharp hypotheses.

(e) Traditional understandings of significance tests as coverage (or not) of a point hypothesis, H', by a credibility interval of prescribed size. H' may be obtained by "pre-processing" (under permissible rules) the original statistical model.

Many Probab.-Possibility transformations have been defined (should not have been an obstacle, but was a distraction), ex:

$$\kappa(\varphi) = \int_\Theta \min\left[p(\theta)\, p(\varphi)\right] p(\theta) d\theta \; ; \;\; \xi(\varphi) = \frac{p(\varphi)}{\widehat{p}} \; , \;\; \widehat{p} = \sup_\Theta p(\theta) \; .$$

ev(H) should not depend on the coordinate systems used to parameterize the statistical model (X, Θ or H).

- Reparameterization of H, i.e. of $h(\theta)$: Trivial.
- Consider a regular (bijective, integrable, a.s.cont.differentiable) reparameterization of Θ,

$$\omega = \phi(\theta) \ , \ \Omega_H = \phi(\Theta_H) \ .$$

The Jacobian of this coordinate transformation is

$$J(\omega) = \left[\frac{\partial\,\theta}{\partial\,\omega}\right] = \left[\frac{\partial\,\phi^{-1}(\omega)}{\partial\,\omega}\right] = \begin{bmatrix} \frac{\partial\,\theta_1}{\partial\,\omega_1} & \dots & \frac{\partial\,\theta_1}{\partial\,\omega_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial\,\theta_n}{\partial\,\omega_1} & \dots & \frac{\partial\,\theta_n}{\partial\,\omega_n} \end{bmatrix} ,$$

and the surprise funcion in the new coordinates is

$$\widetilde{s}(\omega) = \frac{\widetilde{p}_n(\omega)}{\widetilde{r}(\omega)} = \frac{p_n(\phi^{-1}(\omega))\,|J(\omega)|}{r(\phi^{-1}(\omega))\,|J(\omega)|} \ .$$

$$\text{Hence,}\quad \widetilde{s}^* = \sup_{\omega\in\Omega_H} \widetilde{s}(\omega) = \sup_{\theta\in\Theta_H} s(\theta) = s^* \ ,$$

$$T(s^*) \mapsto \phi(T(s^*)) = \widetilde{T}(\widetilde{s}^*) \ , \ \text{and}$$

$$\widetilde{\text{Ev}}(H) = \int_{\widetilde{T}(\widetilde{s}^*)} \widetilde{p}_n(\omega)d\omega = \int_{T(s^*)} p_n(\theta)d\theta = \text{ev}(H) \ , \quad \text{Q.E.D.}$$

Box and Tiao (1965, p.1470): *"It seems that we cannot hope for invariance for a genuine measure of credibility. It needs to be remembered that invariance under transformations and virtues are not synonymous. For problems which should not be invariant under transformation, a search for invariance serves only to guarantee inappropriate solutions."*

- This claim went undisputed in the statistical literature!!
- Possibilistic measures *"must be thought of as a very informal way of testing."* Harrison (1997, Sec.8.6.7, p.256,257).

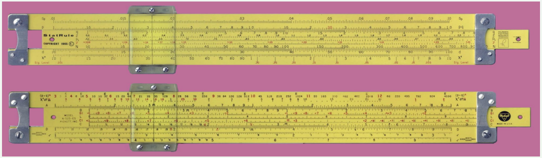
(b) Decision Theoretic Analysis

M.R.Madruga, L.G.Esteves, S.Wechsler (2001):

- Loss function, $\Lambda : \{Accept, Reject\} \times \Theta \mapsto \mathcal{R}$,
  $\Lambda(R, \theta) = a\,\mathbf{1}(\theta \in T)$, $\Lambda(A, \theta) = b + c\,\mathbf{1}(\theta \in \overline{T})$.
- Minimum loss: Accept H iff $\text{ev}(H) > \varphi = (b + c)/(a + c)$.

- ev(H) leads to an orthodox decision theoretic procedure, even if a single point, the constrained optimal estimator $\theta^* = \arg\max_H s(\theta)$, “represents” the entire hypothesis set!

- Traditionally, Bayesian procedures use only integral operators, not maximization operators. Classical p-values, like e-values, are defined using both operations, entailing pseudo-possibilistic characteristics, and leading to the *scientific tribunal* metaphor.



Pickett N525 sliderule
$(p + q)^n$, N, Z, χ^2_{df},...
and a Sig.Level scale!

(c) No need for Nuisance Parameter Elimination

- NPE is a dimensionality reduction technique.
  - Allows the "reduction" of H to dimension zero (e).
  - Difficult problems can be solved with simple devices, like the almighty Pickett N-525 Statistics Slide Rule.
- The FBST does not follow the Nuisance Parameters Elimination paradigm, working in the original parameter space, in its full dimension, breaking away from both the frequentist and the decision theoretic Bayesian tradition.
- NPE? - That's not a bug, that's a feature! ;-)
  How does a (theoretical) bug become a feature?

  Raymond Chen (Microsoft): *"One thing you quickly learn in application compatibility is that a bug once shipped gains the status of a feature, because you can be pretty sure that some program somewhere relies on it."*
- The FBST allways requires the use of numerical optimization and integration methods (MC, MCMC, etc.)

V.iv - Logic and Abstract Belief Calculi

- $\langle \Phi, \oplus, \oslash \rangle$, Support Structure; Darwiche (1992)
  Φ , Support Function, for statements on $\mathcal{U}$;
  $\mathcal{U}$, Universe of valid statements;
  0 and **1**, Null and Full support values;
- $\oplus$, Summation: Support value of the disjunction of any two logically disjoint statements from their individual supports,
  $$\neg(A \wedge B) \Rightarrow \Phi(A \vee B) = \Phi(A) \oplus \Phi(B) \ .$$
- $\otimes$, Unscaling (product): If Φ does not* reject A,
  $$\Phi(A \wedge B) = \Phi_A(B) \otimes \Phi(A) \ .$$
- $\oslash$, Scaling: Conditional support value of B given A from the unconditional support values of A and their conjunction,
  $$\Phi_A(B) = \Phi(A \wedge B) \oslash \Phi(A) \ .$$

*May want to use deFinetti-type coherent extensions (future research)

- Support structures for some Abstract Belief Calculi, Probability, Possibility, Classical Logic, Disbelief.
  $a = \Phi(A),\ \ b = \Phi(B),\ \ d = \Phi(D = B \mid A);$ (Conditional)
  $v = \Phi(V = A \vee B),\ \ w = \Phi(W = A \wedge B);$ (Join, Meet)
  $v = a \oplus b,\ \ w = d \otimes a,\ \ d = w \oslash a.$ (Sum, Prod, Scaling)

ABC	$\Phi(\mathcal{U})$	$a \oplus b$	$\mathbf{0}$	$\mathbf{1}$	$a \preceq b$	$w \oslash a$	$a \otimes b$
Pr	$[0,1]$	$a+b$	0	1	$a \leq b$	w/a	$a \times b$
Ps	$[0,1]$	$\max(a,b)$	0	1	$a \leq b$	w/a	$a \times b$
CL	$\{0,1\}$	$\max(a,b)$	0	1	$a \leq b$	$\min(w,a)$	$\min(a,b)$
DB	$\{0..\infty\}$	$\min(a,b)$	∞	0	$b \leq a$	$w-a$	$a+b$

- ABC logics (composition rules) entail scalable propagation properties, essential for building large credal networks (also for algorithm parallelization).

Logic = Truth value of Composite Statements

- H in Homogeneous Disjunctive Normal Form;
  Independent statistical Models $j = 1, 2, \ldots$
  with stated Hypotheses $H^{(i,j)}$, $i = 1, 2 \ldots$
  Structures: $M^{(i,j)} = \{\Theta^j, H^{(i,j)}, p_0^j, p_n^j, r^j\}$.

$$\mathrm{ev}(H) \;=\; \mathrm{ev}\left(\bigvee\nolimits_{i=1}^{q} \bigwedge\nolimits_{j=1}^{k} H^{(i,j)}\right) \;=$$

$$\max\nolimits_{i=1}^{q} \mathrm{ev}\left(\bigwedge\nolimits_{j=1}^{k} H^{(i,j)}\right) \;=$$

$$W\left(\max\nolimits_{i=1}^{q} \prod\nolimits_{j=1}^{k} s^{*(i,j)}\right) \;,$$

$$W(\;) = \bigotimes\nolimits_{1 \le j \le k} W^j(\;) \;.$$

- Composition operators: max and $\bigotimes$ (Mellin convolution).
- Classical logic limit: If all $s^* = 0 \vee \widehat{s}$, ev $= 0 \vee 1$.

- We analyze the relationship between the credibility, or truth value, of a complex hypothesis, H, and those of its elementary constituents, H^j, $j = 1 \ldots k$.
  This is the *Compositionality* question (ex. in analytical philosophy).
- According to Wittgenstein, (*Tractatus*, 2.0201, 5.0, 5.32):
  - Every complex statement can be analyzed from its elementary constituents.
  - Truth values of elementary statements are the results of those statements' truth-functions.
  - All truth-function are results of successive applications to elementary constituents of a finite number of truth-operations.
- Wahrheitsfunktionen, $W^j(\ )$;
  Wahrheitsoperationen, $\bigotimes$, max.

Birnbaum's Logic for Reliability Engineering

- In reliability engineering:
  One of the main purposes of a mathematical theory of reliability is to develop means by which one can evaluate the reliability of a structure (complex machine) when the reliability of its components are known.
- Composition operations:
  - Series and parallel connections;
- Belief values and functions:
  - Survival probabilities and functions.
- There are **no** *logical rules* (composition operators) for the truth values of Frequentist Statistics, *p*-values, or for those of Decision Theoretic Bayesian Statistics, Bayes Factors (in case of sharp hypotheses), since they require model specific stopping criteria or priors distributions and weights, special schemes of elimination or approximation, etc., etc.

V.vi - Bilattices and Paraconsistency

da Costa (1989, 1991), Arieli & Avron (1996), Alcantara (2002).

- Given $\langle C, \leq_c \rangle$ and $\langle D, \leq_d \rangle$, the Credibility and Doubt lattices, $B(C,D) = \langle C \times D, \leq_k, \leq_t \rangle$, has Knowledge and Truth orders:

$$\mathrm{K} : \langle c_1, d_1 \rangle \leq_k \langle c_2, d_2 \rangle \Leftrightarrow c_1 \leq_c c_2 \text{ and } d_1 \leq_d d_2 ;$$
$$\mathrm{T} : \langle c_1, d_1 \rangle \leq_t \langle c_2, d_2 \rangle \Leftrightarrow c_1 \leq_c c_2 \text{ and } d_2 \leq_d d_1 .$$

K : More information in point 2 (even if inconsistent);
T : more reason to trust in point 2 (even if with less information).

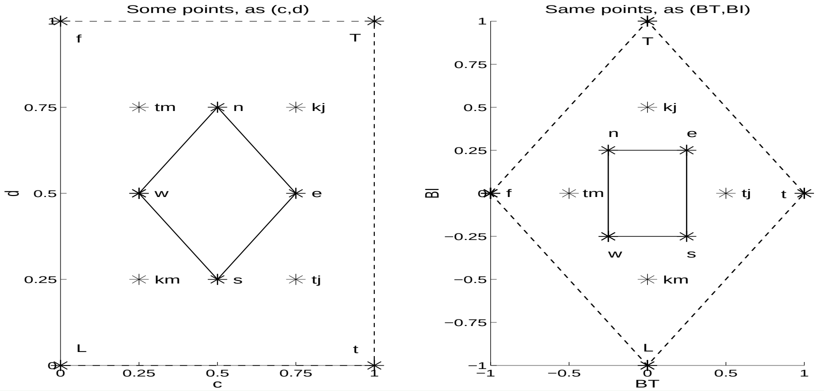
- Join, $\sqcup$, and a Meet, $\sqcap$, operators for truth and knowledge:

$$\langle c_1, d_1 \rangle \sqcup_t \langle c_2, d_2 \rangle = \langle c_1 \sqcup_c c_2, d_1 \sqcap_d d_2 \rangle ,$$
$$\langle c_1, d_1 \rangle \sqcap_t \langle c_2, d_2 \rangle = \langle c_1 \sqcap_c c_2, d_1 \sqcup_d d_2 \rangle ,$$
$$\langle c_1, d_1 \rangle \sqcup_k \langle c_2, d_2 \rangle = \langle c_1 \sqcup_c c_2, d_1 \sqcup_d d_2 \rangle ,$$
$$\langle c_1, d_1 \rangle \sqcap_k \langle c_2, d_2 \rangle = \langle c_1 \sqcap_c c_2, d_1 \sqcap_d d_2 \rangle .$$

- Unit Square, $[0,1]^2$, bilattice: Join, $\sqcup = \max$; meet, $\sqcap = \min$.

- Extreme points: t-truth, f-false, $\top$-inconsist., $\bot$-indeterm.
- Cardinal points: n-north, s-south, e-east and w-west.
- Points kj, km, tj and tm are knowledge and truth join and meet over region R, the convex hull of the cardinal points.
- Degree of Trust and Inconsistency linear reparameterization for a point $x = \langle c, d \rangle$ in the Credibility - Doubt bilattice:

$$\mathrm{BT}(\langle c, d \rangle) = c - d \ , \quad \mathrm{BI}(\langle c, d \rangle) = c + d - 1 \ .$$



Inconsistency Analysis of Bayesian Models

- Sensitivity analysis for choice of prior can engender an error-bar, see Stern (2004). However this is not a standard deviation. Rather, it is a degree of inconsistency induced by perturbations on underlying probability or possibility measures.
- Inconsistency analysis may be a good way to work with intentional or very small samples, see Lauretto et al. (2012).
- A lot of research to do...
-
-
-
-

Negation, $\neg$, and conflation, $-$, may (or not) be defined, ex.:
$\neg \langle c, d \rangle = \langle d, c \rangle$: reverses trust, preserves knowledge.
$- \langle c, d \rangle = \langle 1 - c, 1 - d \rangle$: reverses knowledge, preserves trust.

VI - ZPP: The Zero Probability Paradox.

- In decision theoretic (de Finettian) Bayesian statistics, Bayes Factors are related to “betting odds” for H. However, a sharp hypothesis has zero probability! This is the ZPP - The Zero Probability Paradox.
- The ZPP creates several technical difficulties for computing Bayes Factors, demanding many strange fixes, like
  - artificial or special purpose priors (obvious oxymoron), or
  - assignment of ad-hoc ‘handicap’ prior mass-probabilities to zero-measure sets, etc., etc.
- These fixes do not work properly, ex. Lindley’s paradox, and are case-by-case expedients that preclude consistent composition of true-values.
- Sharp hypotheses make no sense and are consistently *Not* supported (or should not be) by either the mathematical formalism, or the epistemological framework!

Physical Miracles and Mathematical Wonders

- Sharp hypotheses are fully supported in the Cognitive Constructivism epistemological framework equipped with the Full Bayesian Significance Test (FBST) methodology for computing empirical epistemic values, $\text{ev}(H \mid X)$.
- ZPP: Finding an empirical Sharp Hypotheses with $\text{ev}(H \mid X) \approx 1$ (being certain of what is most improbable) is as close as it gets to a scientific (heavenly) miracle!
- Infidels (on-believers) are hereby required to take Physics 101-104 together with Physics-Lab 101-104.
- FBST Abstract Belief Calculus has Classical Logic as its limit when all statements have full or null support value. This is a bridge that justifies Imre Lakatos (1978) view of Mathematics as quasi-empirical science, Stern (2011a).

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