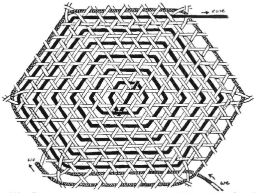
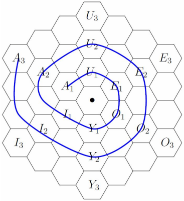
Logical Hexagons of Statistical Modalities: Probabilistic, Alethic, Hybrid & Spiral

Luís Gustavo Esteves*, Julio Michael Stern*,
Rafael Izbicki#, Rafael Bassi Stern#;
* University of São Paulo,
# Fed. Univ. of São Carlos.





http://www.ime.usp.br/∼jstern/miscellanea/jmsslide/hexa16.pdf
5th W. Congr. Square of Oppositon, Rapa Nui, Chile, Nov. 11-15, 2016;
XVIII Brazilian Logic Conference, Pirenópolis, Brazil, May 8-12, 2017;

I- Introduction;

II- Logical Hexagons of Opposing Modalities;

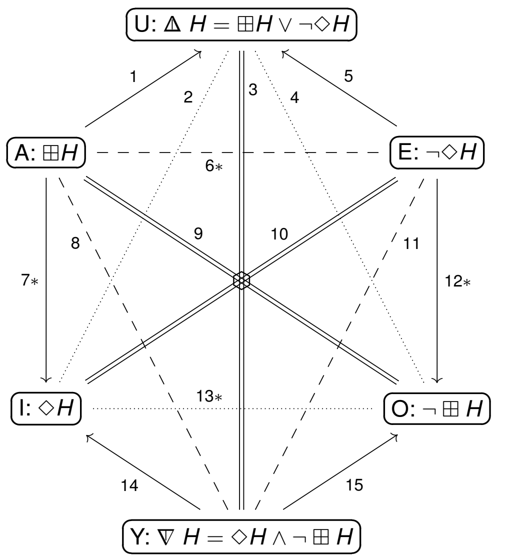
III- Testing (Accepting / Rejecting) Statistical Hypotheses,
Desirable Logical Properties of Agnostic Tests,
Failure of Probabilistic Statistical Tests;

IV- Full Coherence = (Alethic= Possib.Calculus) Region Tests,
Generalized Full Bayesian Significance Test,
GFBST Continuous Mathematics under the hood;

V- Hybrid (Alethic / Probabilistic) Relations,
Sharp Hypotheses: Importance, vs. Slackness;
Pierre Gallais' Hexagonal Spirals and Science Evolution;

VI- Final Remarks.
Problem of Induction: $\Diamond\!\!\!\!+ H$? or $\nabla\!\!\!\!* \ H = \Diamond H \wedge \neg \boxdot H$!!!
Future Research: How to measure Slackness?
References and Acknowledgments.

Logical Hexagons of Opposing Modalities



Modal Operators:
□ - Necessity,
◇ - Possibility,
△ - Contingency,
▽ - Non-Contingency;

Types:
▽ △ ◇ □ - Alethic,
⩢ ◬ ⟐ ⊞ - Probabilistic,
⧨ ◬ ⟐ ⊡ - Slackness,
⧩ ◬ ⧨ ◬ - Hybrid;

Logical Operators:
$\neg$ - Nega., $\rightarrow$ - Implic.,
$\wedge$ - Conjunction (and),
$\vee$ - Disjunction (or);

Opposition relations:
═══ Contradiction,
- - - Contrariety,
...... Sub-Contrariety.

The Problem of Induction: $\diamondplus H$ or $\Diamond H$?

$\triangle$: Accept or Reject

$\boxplus$: Accept $H \Leftrightarrow \Pr(H) \geq 1\text{-}\alpha$ $\neg\diamondplus$: Reject $H \Leftrightarrow \Pr(H) < \beta$

$\diamondplus$: Do not Reject $\neg\boxplus$: Do not Accept

∇: Agnostic $\Leftrightarrow$ Neither Accept nor Reject

Ideal world (wishful thinking), *not how it really works:*

Parameter space Θ, Posterior Probability $p_n(\theta) \propto p_0(\theta)p(X,\theta)$;

Hypotheses $H : \theta \in \Theta_H$ (relaxed notation: H for Θ_H);

Hypothesis $H \subset \Theta$ has known $\Pr(H) = \int_H p(\theta)d\theta$;

β = Pr(type II error = false negative);

$1 - \beta$ = Power = Pr(reject H if $\theta \notin H$);

α = Significance level = Pr(type I error = reject H if $\theta \in H$);

Choices for α or β:

Ronald Fisher: α = 0.05 (*), 0.02 (**), 0.01 (***);

Equal weight: Calibrate the test to minimize $\alpha + \beta$.

$\widetilde{H} = \Theta - H$, $\Pr(\widetilde{H}) = 1 - \Pr(H)$;

Slack and Sharp versions of Non-Cont. $\nabla = \Diamond \wedge \neg\Box$

$\Box$: Mandatory	Δ: Ordained	$\neg\Diamond$: Forbiden
$\Diamond$: Permitted	∇: Indifferent	$\neg\Box$: Optional

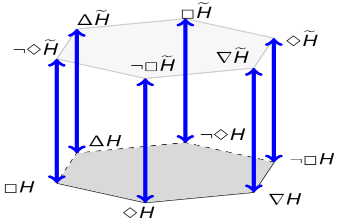
$\Box$: Inclusion	Δ: Inclu.or Exclu.	$\neg\Diamond$: Exclusion
$\Diamond$: Inclu.or Intersct.	∇: Intersection	$\neg\Box$: Exclu.or Intersct.

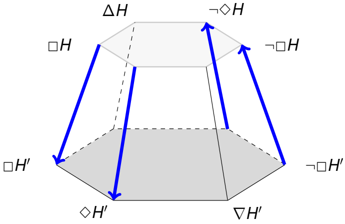
$\Box$: $x < y$	Δ: $x \neq y$	$\neg\Diamond$: $x > y$
$\Diamond$: $x \leq y$	∇: $x = y$	$\neg\Box$: $x \geq y$

- The interpretation of the ∇ modality can have a weak role (broad, vague, Slack) or a "reverse" strong role (equal, identical, Sharp)!

> Examples: Deontic relations from Gallais (1982); Order relations and set operations from Blanché (1966) & Béziau (2015).

Coherence: Logical Desiderata for Statistical Tests





Agnostic = possible case ∇H

Invertibility (for H complement):
$\Box H \iff \neg\Diamond\widetilde{H}$ and
$\nabla H \iff \nabla\widetilde{H}$

$A \leftrightarrow \widetilde{\mathrm{E}}$, $E \leftrightarrow \widetilde{\mathrm{A}}$,
$I \leftrightarrow \widetilde{\mathrm{O}}$, $O \leftrightarrow \widetilde{\mathrm{I}}$,
$U \leftrightarrow \widetilde{\mathrm{U}}$, $Y \leftrightarrow \widetilde{\mathrm{Y}}$;

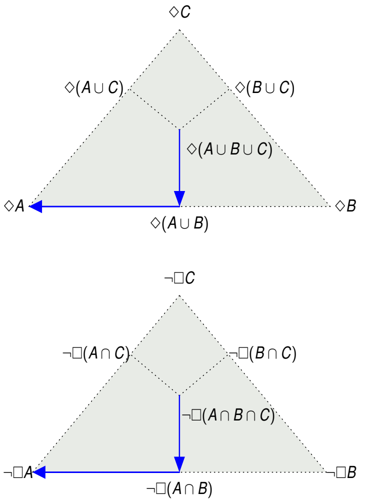
Monotonicity (for nested $H \subset H'$):

$$H \subseteq H' \Rightarrow \begin{cases} \Box H \Rightarrow \Box H' \\ \Diamond H \Rightarrow \Diamond H' \end{cases}$$

$A \leftrightarrow$ A', $I \leftrightarrow$ I',
$O' \leftrightarrow$ O, $E' \leftrightarrow$ E;

See Esteves et al. (2016).

Coherence: Logical Desiderata for Statistical Tests



Agnostic = possible case ∇H

Strong union consonance:
For every index set I,

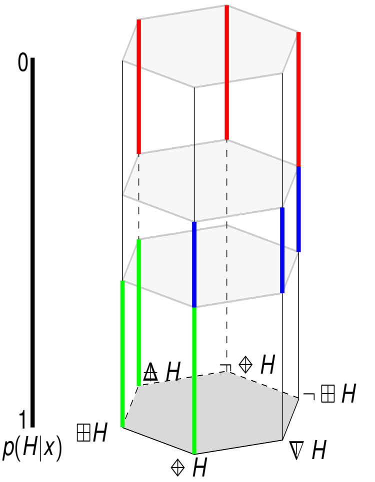
$$\Diamond(\cup_{i \in I} H_i) \Rightarrow \exists i \in I \,|\, \Diamond H_i \;;$$

Strong intersection consonance:
For every index set I

$$\neg\Box(\cap_{i \in I} H_i) \Rightarrow \exists i \in I \,|\, \neg\Box H_i \;;$$

Figures: Under strong consonance, there is at least one path from the center to a vertex of the polygon representing the indexed set of sub-hypotheses.

Failure of Decision Th. Posterior Probability Tests



Truth / Decis.	H	$\widetilde{H}$
$\boxplus H$	0	1
∇H	b	b
$\neg \oplus H$	a	0

Optimal Decision: Take
$c_1 = \max((1+a)^{-1}, b)$,
$c_2 = \min((1+a)^{-1}, b/a)$, and

Choose Probabilistic modality:

$$\begin{cases} \boxplus H & \text{, if } p_n(H|x) > c_1 \\ \neg \oplus H & \text{, if } p_n(H|x) < c_2 \\ \nabla H & \text{, otherwise.} \end{cases}$$

These tests are logically incoherent: Can calibrate constants *a* and *b* s.t. tests are invertible & monotonic, but these tests are **not** consonant!

Failures of other Standard Statistical Tests

Property\Test	ALRT	Post.Pr.	GFBST
Invertibility	X	✓	✓
Monotonicity	X	✓	✓
Consonance	X	X	✓
Invariance (Θ, H)	✓	?	✓
Consistency	✓	?	✓

> ALRT – Agnostic Likelihood Ratio Test: Slack or Sharp H;

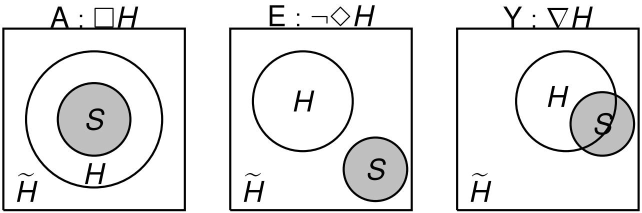
> Generalized Full Bayesian Significance Test: Slack or Sharp;

> Posterior Probability: ?=✓ for Slack H, ?=X for Sharp* H;

For details and examples: Izbicki & Esteves (2015).

* Posterior Probability tests may be extended to sharp H via Bayes Factors based on *ad hoc* prior/posterior measures defined on H. Bad idea, leading to well known paradoxes. Fully acknowledged by orthodox (decision theoretic) Bayesian statistics, that regards sharp hypotheses as *ill formulated* !

Fully Coherent (Alethic) Region Tests



$$\text{Choose Alethic modality} \begin{cases} \Box H & \text{if } S \subseteq H \\ \neg\Diamond H & \text{if } S \subseteq \widetilde{H} \\ \nabla H & \text{if } S \cap H \neq \varnothing \ \& \ S \cap \widetilde{H} \neq \varnothing \end{cases}$$

where S is a region estimator of the parameter θ, i.e., $S \subseteq \Theta$.

- Esteves (2016): Fully coherent tests *must be* region tests.
- ex: $S = \{\theta \in \Theta \mid p_n(\theta) > v\}$, Highest Probability Density Set.

> S may not be path- or simply-connected.

Generalized Full Bayesian Significance Test

- Surprise function $s(\theta) = p_n(\theta)/r(\theta)$;
- Reference density $r(\theta) \neq p_0(\theta)$, ex: Jeffreys invariant prior,

or representation of Fisher Information Metric, $dl^2 = d\theta' J(\theta) d\theta$;

- $T(v) = \{\theta \in \Theta \,|\, s(\theta) \geq v\}$, HSFS at level v.

> Highest Surprise Function Set, defining the region test.

Significance measure for hypothesis H:

- Wahrheit or truth function $W(v) = 1 - \int_{T(v)} p_n(\theta|x)\, d\theta$;
- *e*-value or Epistemic Value of H given observations X is

$\text{ev}(H \,|\, X) = W(s^*)$, where $s^* = \sup_{\theta \in H} s(\theta)$.

- GFBST: Alethic modality $\begin{cases} \Box H & \text{if ev}(\widetilde{H}) < c \\ \neg\Diamond H & \text{if ev}(H) < c \\ \nabla H & \text{otherwise.} \end{cases}$

Obs.1: $T(s^*)$ = Tangential Set, the smallest HSFS $\,|\, \Diamond H$.

Obs.2: $J(\theta) \;=\; \mathrm{E}_{\mathcal{X}} \dfrac{\partial \log r(x \,|\, \theta)}{\partial \theta} \otimes \dfrac{\partial \log r(x \,|\, \theta)}{\partial \theta}$.

- $\text{ev}(H \mid X)$ has good asymptotic properties;

> Sharp or precise hypotheses pose no special difficulties;

- $\text{ev}(H \mid X)$ is fully invariant by model reparameterization;
- $\text{ev}(H \mid X)$ can be logically computed for Coherent Structures, that is, for the series / parallel composition of statistical models and hypotheses, see Borges and Stern (2007).

Consistency and asymptotics:
Assuming a "true" (vector) parameter θ^0 for the regular (ex. H is a differentiable algebraic sub-manifold of Θ) statistical model:

- If θ^0 is an interior point of H, $\text{ev}(H \mid X) \to 1$;
- If $\theta^0 \in H$, where H is sharp, $t = \dim(\Theta)$ & $h = \dim(H)$, then as $n \to \infty$ (increasing sample size) the Standarized e-value, $\text{sev}(H \mid X)$, converges in distribution to the Uniform in $[0, 1]$:

> $\text{sev}(H \mid X) = \text{Chi2}(t, \text{Chi2}^{-1}(t - h, \text{ev}(H \mid X)\,)\,) \rightsquigarrow U_{[0,1]}$;

> $\text{Chi2}(k, x) = \Gamma(\frac{k}{2}, \frac{x}{2}) / \Gamma(\frac{k}{2}, \infty)$.

GFBST Invariance by Reparameterization of Θ

Consider a regular (bijective, integrable, a.s.cont. differentiable) reparameterization of the statistical model's parameter space, $\omega = \phi(\theta)$, $\Omega_H = \phi(\Theta_H)$, with Jacobian matrix

$$J(\omega) = \left[\frac{\partial\,\theta}{\partial\,\omega}\right] = \left[\frac{\partial\,\phi^{-1}(\omega)}{\partial\,\omega}\right] = \left[\begin{array}{ccc} \frac{\partial\,\theta_1}{\partial\,\omega_1} & \cdots & \frac{\partial\,\theta_1}{\partial\,\omega_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial\,\theta_n}{\partial\,\omega_1} & \cdots & \frac{\partial\,\theta_n}{\partial\,\omega_n} \end{array}\right] .$$

$$\check{s}(\omega) = \frac{\check{p}_n(\omega)}{\check{r}(\omega)} = \frac{p_n(\phi^{-1}(\omega))\,|J(\omega)|}{r(\phi^{-1}(\omega))\,|J(\omega)|} = s(\phi^{-1}(\omega))$$

and $\check{s}^* = \sup_{\omega\in\Omega_H} \check{s}(\omega) = \sup_{\theta\in\Theta_H} s(\theta) = s^*$. Hence,

$T(s^*) \mapsto \phi(T(s^*)) = \check{T}(\check{s}^*)$, making the significance measure

$$\check{ev}(H) = 1 - \int_{\check{T}(\check{s}^*)} \check{p}_n(\omega) d\omega = 1 - \int_{T(s^*)} p_n(\theta) d\theta = \text{ev}\,(H)$$

invariant by the reparameterization.

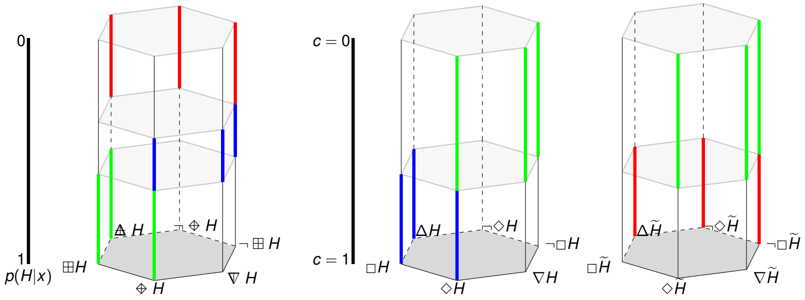
A Coherent Structure is a family, $M^{(i,j)} = \{\Theta^j, H^{(i,j)}, p_0^j, p_n^j, r^j\}$, of Independent Models, M^j, $j = 1 \ldots k$, including, for each model M^j, a set of alternative hypotheses, $H^{(i,j)}$, $i = 1 \ldots q$ (serial composition of models with parallel hypotheses).

$$\mathrm{ev}\,(H) \; = \; \mathrm{ev}\left(\bigvee_{i=1}^{q} \bigwedge_{j=1}^{k} H^{(i,j)}\right) \; = \; \max_{i=1}^{q} \; \mathrm{ev}\left(\bigwedge_{j=1}^{k} H^{(i,j)}\right)$$

$$= \; W\left(\max_{i=1}^{q} \prod_{j=1}^{k} s^{*(i,j)}\right) \; ; \qquad W = \bigotimes_{1 \leq j \leq k} W^j \; .$$

- W is the Mellin Convolution of the models' truth functions, where $[f \otimes g](y) = \int_0^\infty (1/x) f(x) g(y/x) dx$;
- If all $s^* = 0 \vee \widehat{s}$, $\mathrm{ev} = 0 \vee 1$, we get classical logic.

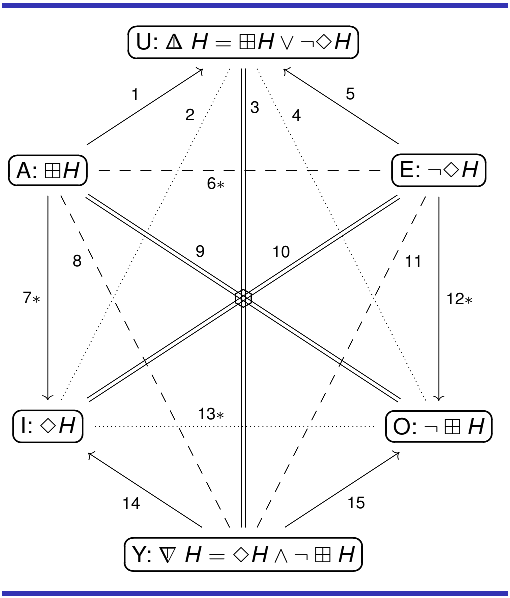
Hybrid (Alethic / Probabilistic) Relations



Setting constants $c_1 = 1 - c$ & $c_2 = c$, the modal operators defined by the GFBST and the agnostic probabilistic test obey:

- $\Box H \Rightarrow \Pr(H|X) \geq 1 - c \Rightarrow \boxplus H$;
- $\neg \Diamond H \Rightarrow \Pr(H|X) \leq c \Rightarrow \neg \diamondplus H$;

Hybrid (Alethic / Probabilistic) Relations



- Hence, setting consts. $c_1 = 1 - c$ and $c_2 = c$, $\neg\Diamond H \Rightarrow \neg \diamondplus H \Rightarrow \neg \boxplus H$, $\boxplus H \Rightarrow \diamondplus H \Rightarrow \Diamond H$, and all (stared) relations in hybrid hexagon hold! (+Consist => false hopes?)

- However, if H is sharp, $\Pr(H|X) = 0 \Rightarrow \neg \diamondplus H$ (trivial hybrid relations)

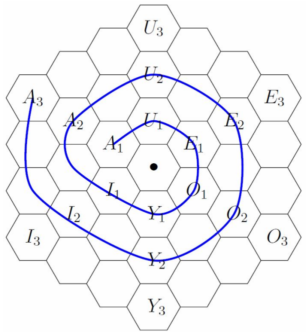
- Nevertheless, $\Diamond H$ is a consistent (s.12) outcome of the GFBST (FBST main motivation)

- Importance sharp H? Meaningful measures & versions of $\boxdot H$ $\diamondcdot H$?

From Probability to Slackness & role reversals

- Most important scientific hypotheses or Laws are Equations, and those are naturally expressed as Sharp Hypotheses;
- Motivates having new versions of $\diamond\!\!\!\!+$ & $\boxplus$ that are meaningful for precise H, with non-trivial and useful relations to $\diamond H$;
- Let $\boxdot H \Leftrightarrow \int_H r(\theta) d\theta > 0$ (Lebesgue reference volume), so that $\boxdot H$ indicates a Necessarily Slack or loose H;
- A regular (a.e. differentiable algebraic sub-manifold of Θ) hypothesis H is Sharp or precise iff $\neg \boxdot H \Leftrightarrow \Pr(H, r) = 0$.
- Role Reversal of a positive Lebesgue measure of H !
  > Zero measure means Shapness, a desirable characteristic;
  > Slackness entails **in**exactness, error, Doubt.
- $\triangledown H = \diamond H \wedge \neg \boxdot H$ reversal (s.5) from weak to strong**!!!**
  > Indeed, corroborating an H that is almost surely false is a **Miracle!!!** (Infidels required to take Physics101-104+Lab.)

Gallais' Hexagonal Spirals & Science Evolution



U: $\triangle H = \boxdot H \vee \neg \Diamond H$

A: $\boxdot H$ E: $\neg \Diamond H$

I: $\Diamond H$ O: $\neg \boxdot H$

Y: $\triangledown H = \Diamond H \wedge \neg \boxdot H$

$\Diamond$ = Credibility, ev (H), possible truth;
$\boxdot$ = Doubt, necessary slackness.

(A) A well established theory with well defined laws is put in question;

(U) Vis-à-vis an alternative class of models that, at this point, may still be somewhat vague or imprecise;

(E) The old laws are rejected as new information becomes available;

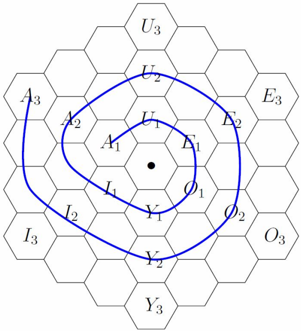
(O) Alternative class of models is taken into consideration; and a specific (precise) form is selected;

(Y) New Laws are corroborated**!!!** fundamental constants calibrated;

(I) Theory / paradigm integration,

(A') including best estimates and imprecisions (measurement errors).

Gallais’ Hexagonal Spirals & Science Evolution



U: $\triangle H = \boxdot H \vee \neg \Diamond H$

A: $\boxdot H$ E: $\neg \Diamond H$

I: $\Diamond H$ O: $\neg \boxdot H$

Y: $\nabla H = \Diamond H \wedge \neg \boxdot H$

$\Diamond$ = Credibility, ev (H), possible truth;
$\boxdot$ = Doubt, necessary slackness.

(A) Ptolemaic astronomy & system of epicycles is put in question;

(U) Circles or Oval orbits?

(E) Orbits are Not circular;

(O) Elliptical orbits (eureka);

(Y) Kepler laws**!!!**

(A’) Vortex forces in question;

(U’) Tangential or Radial?

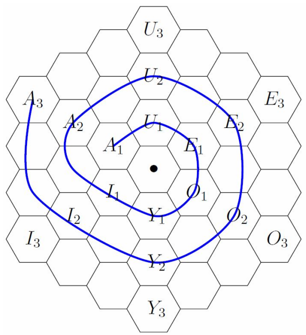
(E’) Forces are Not tangential;

(O’) Radial & inverse square;

(Y’) Newton laws**!!!**

(I’) Newtonian mechanics,

(A”) including its imprecisions.



U: $\triangle H = \boxdot H \vee \neg \Diamond H$

A: $\boxdot H$

E: $\neg \Diamond H$

I: $\Diamond H$

O: $\neg \boxdot H$

Y: $\triangledown H = \Diamond H \wedge \neg \boxdot H$

(A) Geoffroy rules and tables as axioms of chemical affinity;

(U) Ordinal or Numerical?

(E) Not ordinal;

(O) Integer affinity numbers;

(Y) Morveau rules and tables!

(I) Modern (1800) chemistry, including stoichiometry rules.

(A') Substitution reactions;

(U') Total or Parcial?

(E') Not total substitution;

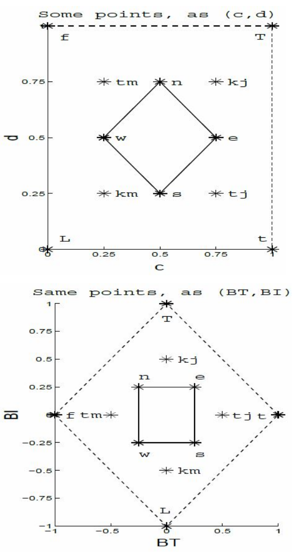
(O') Reversible equilibria;

(Y') Mass-Action kinetics**!!!**

(I') Thermodynamic networks,

(A") including its imprecisions.

Future Research: Unit Square Bilattice



$\diamond$: $c =$ ev $(H \mid X)$, Credibility, possib.truth;
$\diamond$: $d = \mu(H \mid X)$, Doubt, possibly slack.

Unit Square Bilattice in $[0,1]^2$ orders Knowledge and Trust, given coordinates Credibility and Doubt:
$B(C,D) = \langle C \times D, \leq_k, \leq_t \rangle$,
$\langle c_1, d_1 \rangle \leq_k \langle c_2, d_2 \rangle \Leftrightarrow c_1 \leq_c c_2 \ \wedge \ d_1 \leq_d d_2$,
$\langle c_1, d_1 \rangle \leq_t \langle c_2, d_2 \rangle \Leftrightarrow c_1 \leq_c c_2 \ \wedge \ d_2 \leq_d d_1$;

Alternative coordinates in $[-1,+1]^2$:
BT $(\langle c, d \rangle) = c - d$, degree of Trust;
BI $(\langle c, d \rangle) = c + d - 1$, Inconsistency.
Extreme points: Inconsistency ($\top$), truth (t), false (f), indetermination ($\perp$); Stern (2004).

- How to better use the USB to map the evolutionary path of a scientific theory?

Future Research: Measures of Slackness

- Want Possible Slackness to express a measure of doubt:
$\Diamond H \Leftrightarrow d = \mu(H \,|\, X) > \delta$

- What is the best measure $\mu(H \,|\, X)$?
> Credal sets or intervals?
> Confidence regions or intervals?
> for the theory fundamental constants or empirical calibration constants vs. instrumentation or observational error estimates?

- Could that be a good appropriate opportunity to look directly at uncertainty in the sample space?
> Some version of p-values?
> Parameter estimates and their credibility, measured in Θ, and prediction errors, measured in $\mathcal{X}$, can both have a legitimate role to play in statistical epistemology?

- Universal or case specific solutions?

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Acknowledgments

The Problem of Induction: $\diamondplus H$?
or $\;\triangledown\!\!\!\!\ast\; H = \Diamond H \wedge \neg \boxdot H$ **!!!**

- *He who wishes to solve the problem of induction must beware of trying to prove too much.*

Karl Popper, Replies to my Critics; in Schilpp (1974, Ch.32, p.1110), also quoted in Stern (2011).

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Goodby! Adiós! Adeus! Au revoir! Ko te pava kokorua!

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- The legendary Polynesian king and navigator *Hotu Matu'a* reached *Te pito te henua* (the navel of the world) some time arround 1000 CE, sailing a double hull catamaran from Mangareva, 2600 km, or the Marquesas, 3200 km away;
- Building of *Ariña ora ata tepuña*, face-living-image-idols or *moai* monoliths, lead to ecological devastation, famine, war, cultural breakdown & civilization collapse.

Goodby! Adiós! Adeus! Au revoir! Ko te pava kokorua!

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