IME - Instituto de
Matemática e Estatística

2nd Workshop of the São Paulo Journal of Mathematical Sciences: J.-L. Koszul in São Paulo, His Work and Legacy

Auditorium Antônio Gilioli
Instituto de Matemática e Estatística
Universidade de São Paulo
13–14 November 2019

Program

Wednesday, November 13th

9:30-10am	Registration
10-10:50am	**C. Gorodski:** *Koszul in São Paulo, a historical view*
10:50-11:10am	Coffee-Break
11:10am-Noon	**M. N. Boyom:** *Geometry of Koszul and the Information Geometry. Correspondence from Jean-Louis Koszul*
Noon-2pm	Lunch
2-2:50pm	**Dirk Töben:** *Equivariant basic cohomology and applications*
3-3:50pm	**J. C. Baez:** *From classical to quantum and back**
3:50-4:20pm	Coffee-Break
4:20-5:10pm	**I. Struchiner:** *Structure equations for G-structures and G-structure algebroids*

Thursday, November 14th

9-9:50am	**U. Bruzzo:** *The extension problem for Lie algebroids on schemes*
9:50-10:10am	Coffee-Break
10:10-11am	**Rui L. Fernandes:** *Prequantization, differential cohomology and the genus integration*
11:10-Noon	**L. A. B. San Martin:** *Semigroups in semi-simple Lie groups and eigenvalues of second order differential operators on flag manifolds*
Noon-3pm	Lunch/Meeting of Editorial Board
3-3:50pm	**A. Wade:** *Koszul's bracket and Jacobi structures**
3:50-4:20pm	Coffee-Break

*Skype conference.

End of Activities

List of Abstracts of Talks

John Baez (University of California at Riverside, USA): *From classical to quantum and back*

Edward Nelson famously claimed that quantization is a mystery, not a functor. In other words, starting from the phase space of a classical system (a symplectic manifold) there is no functorial way of constructing the correct Hilbert space for the corresponding quantum system. In geometric quantization one gets around this problem by equipping the classical phase space with extra structure: for example, a Kähler manifold equipped with a suitable line bundle. Then quantization becomes a functor. But there is also a functor going the other way, sending any Hilbert space to its projectivization. This makes quantum systems into specially well-behaved classical systems! In this talk we explore the interplay between classical mechanics and quantum mechanics revealed by these functors going both ways.

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Michel Nguiffo Boyom (Université de Montpellier, France): *Geometry of Koszul and the Information Geometry. Correspondence with Jean-Louis Koszul*

In many mails he sent to me by 2011-2012, Jean-Louis Koszul often wondered that the Information Geometry may be linked with the Hessian Geometry. On 3 February 2012 he wrote:

> *Ce qui reste par contre un mystère absolu pour moi c'est ce que signifie au juste "Géométrie de l'Information". Et quand en plus elle est Hessienne, cela n'arrange rien. Notez que je suis habitué depuis longtemps à voir naître des terminologies bizarres et à assister à des détournements de sens audacieux, voire criminels.*

In this talk I aim to point out the foundamental role played by both the Geometry of Koszul and the Topology of Koszul in the theory of statistical models of measurable sets. The (algebraic) foundation of the Information Geometry is the KV cohomology of locally flat manifolds, (KV stands for Koszul-Vinbeg.) The current Information Geometry as in works of I-S Amari and alias as well as the Hyperbolic Geometry after Kaup, Koszul, Vey and others, are local vanishing theorems in the KV cohomology of locally flat manifolds. From the combinatorial wording, a structure of statistical model is a leaf of a tree whose root is a cohomology class in a 2nd KV cohomology space.

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Ugo Bruzzo (Scuola Internazionale Superiore di Studi Avanzati, Italy, and Federal University of Paraíba, Brazil): *The extension problem for Lie algebroids on schemes*

We study the extension problem in the general nonabelian case for Lie algebroids in the category of schemes.

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Rui Loja Fernandes (University of Illinois at Urbana-Champaign, USA): *Prequantization, differential cohomology and the genus integration*

In the work of J.-L. Koszul, Lie theory and cohomology are often blended together in unexpected ways. This talk follows the same spirit: I will survey the genus integration of a Lie algebroid A introduced recently by Ivan Contreras and myself. It is related to the canonical integration of A through abelianization, in the same way that singular homology and homotopy are related. I will discuss in some detail the special case of the prequantization Lie algebroid associated with a closed 2-form, showing how our results recover the well-known prequantization condition and that they give a slight improvement of the usual description of principal S^1-bundles with connection via differential characters. Based on joint works with Ivan Contreras and Alejandro Cabrera.

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Luiz Antônio Barrera San Martin (State University of Campinas, Brazil): *Semigroups in semi-simple Lie groups and eigenvalues of second order differential operators on flag manifolds*

An open problem in Lie group theory (and their seemigroups) is to decide whether the semigroup generated by a family of 1-parameter semigroups of a Lie group G is proper or not. This problem originated in control theory and is called the controllability problem for invariant systems on Lie groups.

This talk indicates a relationship between the controllability problem in a noncompact semi-simple Lie group G with the highest eigenvalues of certain second order differential operators on the flag manifolds of G. The operators are obtained by the infinitesimal representations induced by Banach induced representations of G.

The link between the controllability problem and the eigenvalues of differential operators is provided by probability theory on Lie groups. Consider an independent and identically distributed (i.i.d.) random sequence y_n in a semi-simple Lie group G with common law μ and form its random product $g_n = y_n \cdots y_1$.

The issue is to relate geometric properties of the semigroup S_μ generated by the support of μ to the asymptotics of the random product g_n.

The asymptotic properties of g_n are described by limits of cocycles $\rho_\lambda(g,x)$ over the flag manifolds. These cocycles are defined after the Iwasawa decomposition $G = KAN$ through the function $\rho : G \times K \to A$ given by $gu = v\rho(g,u)n$ with $v \in K$, $\rho(g,x) \in A$ and $n \in N$ and the parameter λ is a linear map on $\mathfrak{a} = \log A$.

The Lyapunov exponents of the random product g_n are defined by

$$\Lambda_\lambda(x) = \lim_{n\to\infty} \frac{1}{n} \log \rho_\lambda(g_n, x).$$

The moment Lyapunov exponent of the random product depending on $\lambda \in \mathfrak{a}^*$ and x in a flag manifold is defined by

$$\gamma_\lambda(x) = \limsup_{n\to\infty} \frac{1}{n} \log \int \rho_\lambda(g,x)\mu^n(dg).$$

where μ^n is the n-th convolution power of μ.

We assume that $\operatorname{int} S_\mu \neq \varnothing$ (that is, μ is an exposed measure). The so-called flag type of S_μ is a flag manifold associated to it that reveals several geometric and algebraic properties of the semigroup (for instance the Jordan form of its elements).

We relate the flag type of S_μ with the behavior as $p \to -\infty$ of the functions $p \mapsto \gamma_{p\lambda}(x)$.

The moment Lyapunov exponent $\gamma_\lambda(x)$ equals $\log r_\lambda$ where r_λ is the spectral radius of the operator

$$(U_\lambda(\mu)f)(x) = \int_G \rho_\lambda(g,x) f(gx)\mu(dg)$$

acting in spaces of continuous functions. Typically $U_\lambda(\mu)$ is a compact operator.

Theorem 1 $S_\mu = G$ *if and only if the map* $\lambda \to r_\lambda$ *is analytic.*

For a proper semigroup S_μ the lack of analyticity is provided by the flag type of S_μ.

The result can be applied to measures generated by an stochastic differential equation

$$dg = X(g)\,dt + \sum_{j=1}^{m} Y_j(g) \circ dW_j$$

on G where X and Y_j are right invariant vector fields. In this case the semigroup S that replaces S_μ is generated by the 1-parameter semigroup e^{tX}, $t \geq 0$, together with the 1-parameter groups e^{tY_i}, $t \in \mathbb{R}$, $i = 1, \ldots, m$. The measures given by the solution of the differential equation define on the representation spaces of G, 1-parameter semigroups of operator whose infinitesimal generators are second order differential operators. Thus controllability which means $S = G$ is equivalent to analyticity of the map giving the principal eigenvalues of the differential operators.

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Ivan Struchiner (University of São Paulo, Brazil): *Structure equations for G-structures and G-structure algebroids*

The infinitesimal data attached to a G-structure with connection is its structure equations. These equations can be interpreted as determining a Lie algebroid with extra symmetries, known as a G-structure algebroid. This correspondence is a special instance of the correspondence between degree 1 derivations of the exterior algebra of a vector bundle, and Lie algebroid structures on the vector bundle which is obtained from the Koszul formula for the exterior derivative.

In this talk I will discuss the G-structure algebroid associated to a family of G-structures with connections and how the Lie theory for G-structure algebroids can be used to understand the moduli space of G-structures in the family.

The talk is based on joint work with Prof. Rui Loja Fernandes (UIUC), available at

https://arxiv.org/abs/1907.13614

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Dirk Töben (Federal University of São Carlos, Brazil): *Equivariant basic cohomology and applications*

For a foliation the cohomology of the deRham subcomplex of basic forms is an important invariant. A natural question is how the basic cohomology is related to the dynamics of the foliation. It was discovered by Molino that a Riemannian foliation carries an intrinsic infinitesimal Lie algebra action. This allows us to introduce an equivariant version of basic cohomology. With this notion we can adapt important results from the theory of group actions like the Borel-Localization Theorem. These results help us to understand the foliation with respect to the aforementioned question. We will see applications for secondary characteristic classes and Sasakian manifolds. This talk is based on works with O. Goertsches, H. Nozawa and F.C. Caramello Jr.

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Aïssa Wade (Pennsylvania State University, USA): *Koszul's bracket and Jacobi structures*

The Koszul bracket associated to a Poisson tensor was introduced by Koszul in 1985. It induces a differential graded algebra with a bracket that has an exact generator. Since Jacobi structures are generalizations of Poisson structures, it is natural to ask whether Koszul's construction can be extended to the general context of Jacobi manifolds. In this talk, we will begin to briefly review Koszul's construction and Jacobi line bundles. Then, we will discuss applications of Koszul's construction to Jacobi bundles.

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