Pricing interest rate derivatives under monetary policy changes

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(Based on joint work with M. Avellaneda)

Motivation

1. IT-regime is characterized by scheduled meetings;
2. Standard interest rate models do not incorporate schedule events;
3. Stochastic timed jumps (jump-diffusion models) are not consistent with scheduled events;
4. Traded instruments have embedded market expectation about policy changes;
5. IR futures and options must be priced consistently.

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It is a well know result that under no-arbitrage zero coupon price is given by:

$$B_t = \mathbb{E}(e^{-\int_t^T r_s ds} | \mathcal{G}_t) \tag{1}$$

if we assume that $(r_t)_{t \geq 0}$ is an affine process, zero coupon bond prices can be obtained using the Laplace transform, as Duffie et al. (2003).

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A possible form to incorporate Central Bank's decisions regarding the target rate is by assuming that the resulting overnight rate is a semimartigale where the discontinuous component captures monetary decisions. A standard construction when one adopt semimartigales to model asset prices is to assume that the jump component is resulting from a sequence of inaccessible stopping times, in this case one have a randomly timed jump.

Our model does not include randomly timed jumps in prices, but we adopted a deterministic time events instead. We assume there is a deterministic counting process $N(t)$, counting the number of predictable events that occur up to time t:

$$N(t) := \sum_{j} \mathbb{1}_{\{\tau_j \leq t\}}$$

where $(\tau_j)_{j\geq 1}$ are increasing predictable stopping times.

Without any assumption on the jump size distribution we can write the interest rate dynamics like:

$$\mathrm{d}r_t = \mu(r_t)\mathrm{d}t + \sigma(r_t)\mathrm{d}W_t + \eta(r_{t^-})\mathrm{d}J(t), \quad t > 0. \qquad (2)$$

Where $\mu(\cdot)$ is the drift, $\sigma(\cdot)$ the diffusion coefficient and is the $\eta(\cdot)$ jump impact parameter. Here J is a compound jump process:

$$J(t) = \sum_{j=1}^{N(t)} \theta_j$$

and $r_{t^-} = \lim_{s \uparrow t} r_s$.

Motivation

Plugging equation (2) into (1) we have that bond prices are given by:

$$B_t = \mathcal{L}(r_t) \times \mathcal{L}(\theta_t) \tag{3}$$

Where $\mathcal{L}$ is the Laplace transform.

The process $(r_t)_{t \geq 0}$ describe the continuous part of overnight interest rate and $(\theta_t)_{t \geq 0}$ incorporate the scheduled events.

We can rewrite equation (1) as:

$$B_t = \mathbb{E}(e^{-\int_t^T r_s ds - \theta(T-u)} | \mathcal{G}_t) \tag{4}$$

Intuitively the expression above means that changes in target rate only affects the level of $(r_t)_{t \geq 0}$ but not its volatility. This assumption is not too strong, because:

1. overnight interest rate are determined by Interbank transactions, and
2. without any deterioration in Banks' credit quality, the new target rate will not increase the volatility of the borrow/lending rate among banks with same creditworthiness.

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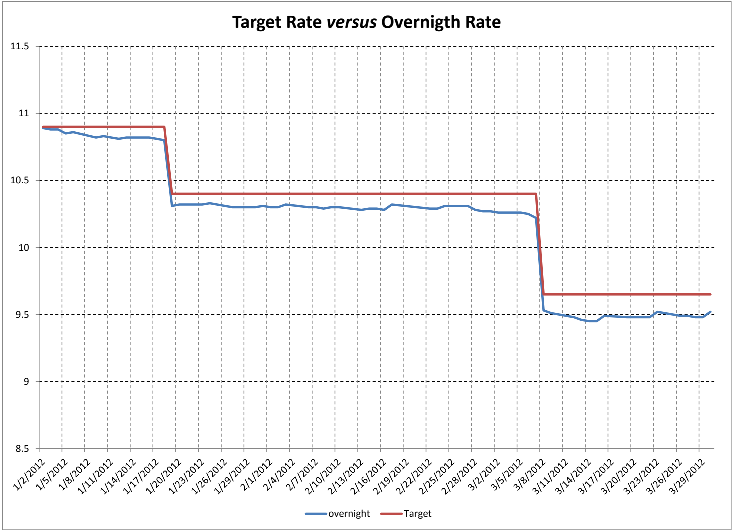

Target Rate *versus* Overnigth Rate
11.5
11
10.5
10
9.5
9
8.5
1/2/2012
1/5/2012
1/8/2012
1/11/2012
1/14/2012
1/17/2012
1/20/2012
1/23/2012
1/26/2012
1/29/2012
2/1/2012
2/4/2012
2/7/2012
2/10/2012
2/13/2012
2/16/2012
2/19/2012
2/22/2012
2/25/2012
2/28/2012
3/2/2012
3/5/2012
3/8/2012
3/11/2012
3/14/2012
3/17/2012
3/20/2012
3/23/2012
3/26/2012
3/29/2012
overnight
Target

The evolution of $(\theta_t)_{t\geq 0}$

1. Central bank decision about θ are not time independent;
2. The values of θ tend to reflect the current monetary policy pursued by the Central Bank (tight and loosing cycles);
3. To incorporate simultaneously uncertainty and dependence on $(\theta_t)_{t\geq 0}$ (Central bank decisions) it is treated as a Discrete Time Markov Chain (DTMC) of order k

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For sake of simplicity we assume that $(\theta_t)_{t\geq 0}$ is an ergodic Markov Chain of order one. Ergodicity is not a strong assumption, because:

1. one can always write a k order DTMC as a first order DTMC;
2. periodicity is not a rational behavior under a IT-regime, and
3. set $\mathcal{A}$ given by all potential values of Central Bank's decision about $(\theta_t)_{t\geq 0}$ is finite.

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The evolution of $(\theta_t)_{t\geq 0}$

For a discrete value random variable, its Laplace transform is given by:

$$\mathcal{L}(\theta_u) := \sum_i e^{-\theta_{u,i}} \times \mathbb{Q}(\theta_u = i) \tag{5}$$

Here $i \in \mathcal{A}$. Typical elements of $\mathcal{A}$ are $i = k \times 0.0025$ such that $k \in \mathbb{Z}$. Additionally, once θ is DTMC its marginal distribution $\mathbb{Q}(\theta_u = i)$ over $\mathcal{A}$ at time u is described by:

$$\mathbb{Q}(\theta_u = i) = \sum_j \mathbb{Q}(\theta_u = i | \theta_s = j)\mathbb{Q}(\theta_s = j) \tag{6}$$

Where transition probabilities $\mathbb{Q}(\theta_u = i | \theta_t = j)$ satisfy the Chapman Kolmogorov equation for two consecutive Central Bank meetings $s < t < u$.

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The evolution of $(\theta_t)_{t\geq 0}$

A convenient simplification arise in equation (6) when there exist just one scheduled meeting before the bond maturity. In this case, $\theta_s \in \mathcal{G}_t$ and equation (6) simplifies to:

$$\mathbb{Q}(\theta_u = i) = \mathbb{Q}(\theta_u = i|\theta_s = j) \tag{7}$$

Because $\mathbb{Q}(\theta_s = j)$ assume just two outcomes $\{0, 1\}$. We have $\mathbb{P}(\theta_s = j) = 1$ if $\theta_s = j$ was the decision taken by Central bank at meeting s and zero otherwise. Such simplification is important to calibrate the transition probabilities from market prices.

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1. Interest rate options products provide market participants the right payoff to bet on Central bank futures decisions about the target rate.
2. If a binary options is available, investors can make bets on futures values of $(\theta_t)_{t\geq 0}$ at time t by buying/selling binary options on overnight interest rate expiring in the next business day after a scheduled meeting u.
3. Binary options are generally considered "exotic" instruments and there is no liquid market for trading these instruments between their issuance and expiration.
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Regardless the option type, i.e, binary, vanilla or Asian, the non-arbitrage price including expectation about changes in monetary policy is given by:

$$Call(T, K, r_t) = \mathbb{E}^{\mathbb{Q}}[(A_T(\theta_T) - K)^+ | \mathcal{G}_t] \tag{8}$$

where A_T can be either the overnight interest rate at time T or its average value. As seen before r_T depends on all previous values of θ.

The expectation in (8) is calculated over the joint density of (r_T, θ_T) which might be quite complicate because θ_T is a DTMC and therefore the joint density will be a mixture of continuous and discrete variables.

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We can rewrite equation (8) as:

$$Call(T,K,r_t) = \int_{\Omega\times\mathcal{A}} [(A_T(\theta_T)-K)^+] f(A_T,\theta)\mathrm{d}(A_T,\theta) \tag{9}$$

$$\int_{\mathcal{A}} \left[\int_{\Omega} [(A_T(\theta_T)-K)^+] f(A_T|\theta)\mathrm{d}A_T \right] g(\theta)\mathrm{d}\theta \tag{10}$$

So conditioning $f(\bullet,\bullet)$ on θ we can solve the expectation above as a classic Black & Scholes problem for every value $\theta \in \mathcal{A}$;

This strategy of conditioning on all possible values of θ_T is conceptually equivalent to Merton (1976) to price option where jumps are present. Assuming that A_T has a lognormal distribution, we have:

$$Call(T, K, r_t) = \sum_i BS(A_T(\theta_T = i), K, T, \sigma)\mathbb{Q}(\theta_T = i) \qquad (11)$$

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Model Calibration: simulated monetary decision data

1. We performed a Monte Carlo simulation to assess its quality to extract market beliefs about Central Bank decision;
2. We assume different values for the elements of $\mathcal{A}$ for 2 consecutive meetings;
3. We assume that overnight interest rates are Guassian;
4. For every set $\mathcal{A}$ we combine all elements to describe futures decision of Central Bank. For instance, if $\mathcal{A} = \{-25bps, 0, +25bps\}$ we have a vector of dimension 8×2 corresponding to all 2-combinations from elements of set $\mathcal{A}$.

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1. For every possible combination of monetary decision we use equation (4) and (3) to simulate bond prices at time t and latter we solve the optimization problem:

$$\text{argmin } (\hat{B}_t - \check{B}_t)^2 \; s.t: \begin{cases} \sum_j \mathbb{P}(\theta_u = i | \theta_s = j) = 1 \\ \mathbb{P}(\theta_u = i | \theta_s = j) \geq 0, \forall j \end{cases} \quad (12)$$

2. where $\hat{B}_t$ is obtained by plugging the values of $\mathcal{A}$ into (4) with different values for initial overnight rate r_t. $\check{B}_t$ is the predicted bond price using (3).
3. The first constraint assures that the sum of each line in the transition matrix is equal to 1; and .
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1. The output from the optimization problem is a vector of dimension 8×2 corresponding to all 2-combinations from elements of set $\mathcal{A}$.
2. Results from the simulation exercise are in tables 1 and 2:

	1^{st} *Meeting*	2^{nd} *Meeting*
$\mathcal{A} = \{-25bps, 0, +25bps\}$	100%	100%
$\mathcal{A} = \{-25bps, 0, +50bps\}$	100%	100%
$\mathcal{A} = \{-50bps, 0, +25bps\}$	100%	99%

Table: Calibration exercise for simulated monetary decision. Initial overnight interest rate, $r_t = 10\%$

1. We assume that the bond maturity is 4 months and Central Bank Meetings are scheduled every month.

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Model Calibration: Real market data

We calibrate the model to Brazilian data for some reasons:

1. There is a very liquid market for overnight interest rate in Brazil, both for futures and options;
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1. The overnight interest rate futures(Ticker: DI1) traded at BM&FBOVESPA is one of the most liquid short-term interest rate contracts in emerging markets worldwide (ADTV 1,3 M);
2. DI futures are quoted in terms of rates and are traded in basis-point, but positions are recorded and tracked by the present value of contract, called PU;
3. For a given day t the present value is obtained by discounting the notional value of the contract by the expected overnight interest rate from t up to the day prior to expiration, T.
4. Therefore, at time t we can calculate the present value (PU) of a DI-futures with expiration date of T as:

$$PU_t = \mathbb{E}(e^{-\int_t^T r_s ds} | \mathcal{G}_t) \times 100,000 \tag{13}$$

From equation (13) we verify that the DI futures is very similar to a zero-coupon bond, except that it pays margin adjustments every day.

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From equation (13) we verify that the DI futures is very similar to a zero-coupon bond, except that it pays margin adjustments every day.

Model Calibration: Real market data

1. The overnight interest rate futures(Ticker: DI1) traded at BM&FBOVESPA is one of the most liquid short-term interest rate contracts in emerging markets worldwide (ADTV 1,3 M);
2. DI futures are quoted in terms of rates and are traded in basis-point, but positions are recorded and tracked by the present value of contract, called PU;
3. For a given day t the present value is obtained by discounting the notional value of the contract by the expected overnight interest rate from t up to the day prior to expiration, T.
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1. We will calibrate our models as we were in January/2012;
2. We assume that $\mathcal{A} = \{-50bps, 0, +25bps\}$;
3. We calibrate the model for every day in January to extract the market probabilities of the two next COPOM decisions.
4. The first two COPOM meeting in 2012 are scheduled for January 18 and March 7;
5. Tables below exhibit the transitions matrix implied into DI futures. We do not report all transition matrix due to lack of space.

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Model Calibration: Real market data

	$\theta = U$	$\theta = D$	$\theta = N$
$\theta = U$	0.73	0.13	0.14
$\theta = D$	0.00	0.87	0.13
$\theta = N$	0.33	0.33	0.34

Table: Implied transition matrix - 1/2/2012

	$\theta = U$	$\theta = D$	$\theta = N$
$\theta = U$	0.74	0.14	0.12
$\theta = D$	0.00	0.87	0.13
$\theta = N$	0.33	0.33	0.34

Table: Implied transition matrix - 1/10/2012

$$\mathcal{A} = \{D = -50bps, N = 0, N = +25bps\}$$

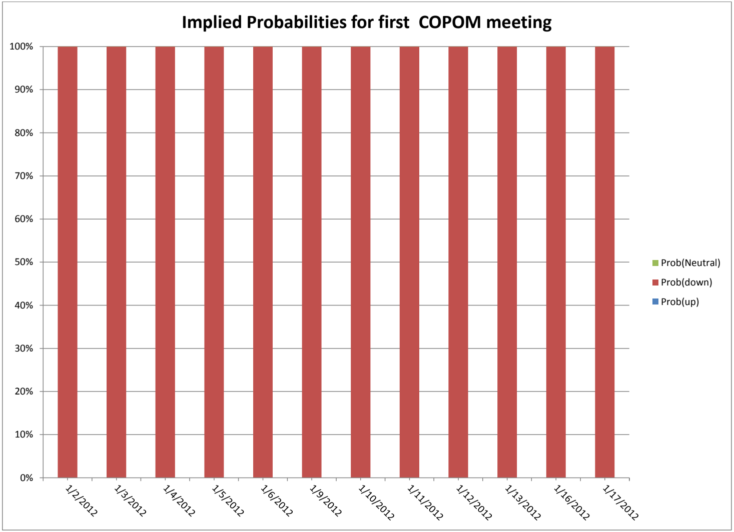

Figure: Implied Probabilities for COPOM's decision - Scheduled meeting for 1/18/2012

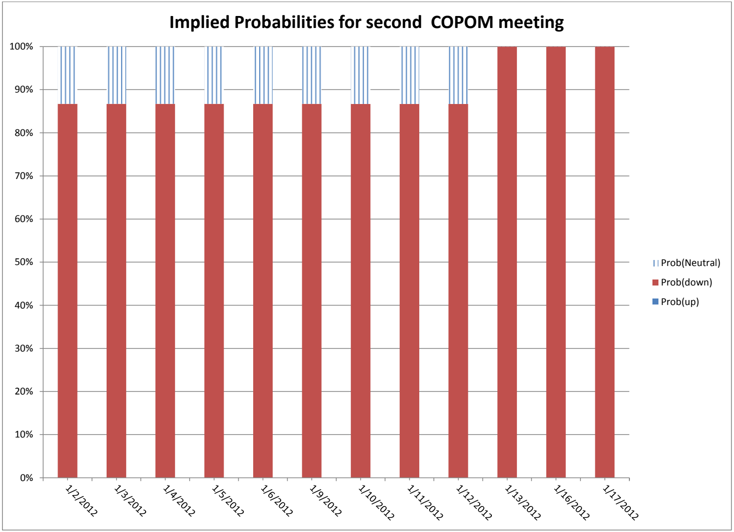

Figure: Implied Probabilities for COPOM's decision - Scheduled meeting for 3/7/2012

Final Remarks

1. In this paper, we develop a model to incorporate monetary announcements for pricing interest rate options;
2. The model formulates future monetary decision and options pricing in a consistent way;
3. We calibrate the model to Brazilian Data;
4. The model can be applied to other central banks, such as the European Central Bank and the Bank of England which announce their policy decisions at regularly scheduled meetings.

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