Lema Se $\sum_{n=0}^{\infty} a_n z_0^n$ converge ($a_n \in \mathbb{C}$)

e se $|z| < |z_0|$, então $\sum_{n=0}^{\infty} a_n z^n$ converge absolutamente.

Dem: Pelo critério do termo geral,

$\lim_{n\to\infty} a_n z_0^n = 0$. Logo, $\exists\, n_0$

tal que

$$n \geq n_0 \implies |a_n z_0^n| < 1$$

Daí, se $n \geq n_0$,

$$|a_n z^n| = |a_n z_0^n| \cdot \left|\frac{z}{z_0}\right|^n < \left|\frac{z}{z_0}\right|^n$$

$\sum_{n=n_0}^{\infty} \left|\frac{z}{z_0}\right|^n$ é finita (série geométrica, $\left|\frac{z}{z_0}\right| < 1$)

Logo, $\sum_{n=n_0}^{\infty} |a_n z^n| \leq \sum_{n=n_0}^{\infty} \left|\frac{z}{z_0}\right|^n < \infty$

Logo, $\sum_{n=0}^{\infty} |a_n z^n| = \sum_{n=0}^{n_0-1} |a_n z^n| + \sum_{n=n_0}^{\infty} \left|\frac{z}{z_0}\right|^n < \infty$

CQD