COALESCENCE AND MINIMAL SPANNING TREES OF IRREGULAR GRAPHS

YEVGENIY KOVCHEGOV, PETER T. OTTO, AND ANATOLY YAMBARTSEV

Abstract. This paper concentrates on breaching the gap between the Smoluchowski coagulation equations for Marcus-Lushnikov processes and the theory of random graphs. It is known that in many cases the cluster dynamics of a random graph process can be replicated with the corresponding coalescent process. The cluster dynamics of a coalescent process (without merger history) is reflected in a auxiliary process called the Marcus-Lushnikov process. The merger dynamics of the Marcus-Lushnikov processes will correspond to a greedy algorithm for finding the minimal spanning tree in the respective random graph process. This observation allows one to express the limiting mean length of a minimal spanning tree in terms of the solutions of the Smoluchowski coagulation equations that represent the hydrodynamic limit of the Marcus-Lushnikov process corresponding to the random graph process.

We concentrate on finding the limiting mean length of a minimal spanning tree on an irregular graph. Specifically, an Erdős-Rényi random graph process on the bipartite graph $K_{\alpha[n],\beta[n]}$ is considered with $\alpha[n] = \alpha n + o(n)$ and $\beta[n] = \beta n + o(n)$. There, the following expression for the limiting mean lengths of the minimal spanning tree is derived via the Smoluchowski coagulation equations of the Marcus-Lushnikov processes with multidimensional weight vectors:

$$\lim_{n\to\infty} E[L_n] = \gamma + \frac{1}{\gamma} + \sum_{i_1\geq 1:\ i_2\geq 1} \frac{(i_1+i_2-1)!}{i_1! i_2!} \frac{\gamma^{i_1} i_1^{i_2-1} i_2^{i_1-1}}{(i_1+\gamma i_2)^{i_1+i_2}},$$

where $\gamma = \frac{\alpha}{\beta}$. This is a completely new formula for the case of an irregular bipartite graph $\gamma \neq 1$. In the case of $\gamma = 1$, the above series adds up to

$$\lim_{n\to\infty} E[L_n] = 2\zeta(3)$$

as derived in Frieze and McDiarmid [15] for a regular bipartite graph. A generalization of the approach is considered in the discussion section.