Online Network Leasing Problems

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Abstract

In the leasing optimization model, resources are leased for K different time periods, instead of being acquired for unlimited duration. The goal is to use these temporary resources to maintain a dynamic infrastructure that serves n requests while minimizing the total cost.

We propose and study leasing variants of some network design problems. **Currently we have an algorithm for the Online Connected Facility Leasing problem which is $O(K \lg n)$-competitive if the scaling factor $M = 1$, and a $O(\lg K \lg |V|)$-competitive algorithm for the Online Steiner Network Leasing problem.**

Introduction

The **leasing optimization model** [8, 2, 9, 1] is a recently proposed subcategory of combinatorial optimization. This framework is useful for modeling **real-world problems** in which we face a **varying demand** and **resources have a limited lifetime or are outsourced**, such as in cloud services. Instead of being acquired for unlimited duration, as in traditional optimization problems, **a resource may be *leased* for different time periods** (e.g., a day, a week, a month). For example, a startup providing a social media service may face alternated periods of high demand and inactivity, therefore it is usually cheaper to lease a cloud service (such as Amazon AWS, Google App Engine or Microsoft Azure) to provide network availability than to build its own private infrastructure. This model may be applied to both offline and online settings; **we are particularly interested in online algorithms**.

Previous Work

The seminal work on leasing optimization is a paper by Meyerson on the **Parking Permit** (PP) problem [8]. In this problem, we are interested in the **leasing of a single resource**. It is natural to suppose an **economics of scale**, i.e., that it is cheaper to lease a resource for a longer period than for smaller periods totalizing an equivalent time. Supposing we have K lease types, Meyerson showed a deterministic $\Theta(K)$-competitive algorithm and a randomized $\Theta(\lg K)$-competitive algorithm. In the same paper, Meyerson showed how to solve the leasing version of the **Online Steiner Forest** (OSF) problem, obtaining a $O(\lg K \lg |V|)$-competitive algorithm by approximating the underlying metric by an HST [4] and by solving problem PP independently in each edge of the HST.

Anthony and Gupta [2] presented approximation algorithms for offline leasing versions of several NP-hard network design problems. For the **Facility Leasing** (FLE) problem, they obtained a $O(K)$-approximation, where K is the number of lease types.

Nagarajan and Williamson [9] improved the result for the offline version of problem FLE, obtaining a 3-approximation. For the online version (OFLE), the authors showed an algorithm with competitive factor $O(K \lg n)$, where n is the number of client requests.

Koutris [7] reviewed previous results on network leasing problems, and gave a randomized $O(K \cdot \lg n / \lg \lg n)$-competitive algorithm for problem OFLE.

Abshoff, Kling, Markarian, auf der Heide and Pietrzyk [1] also improved the result on problem OFLE, presenting an online $O(\delta_K \lg \delta_K)$-competitive algorithm (δ_K is the longest lease duration).

Hu, Ludwig, Richa and Schmid [5] presented a bidimensional generalization of problem PP, and gave an offline approximation algorithm and a deterministic $\Theta(K)$-competitive online algorithm. However, their algorithms are pseudopolynomial on the maximum resource demand.

Current Results

Online Connected Facility Leasing

We propose a leasing variant of the **Online Connected Facility Location** (OCFL) problem.

The input of this problem consists of:

- a **graph** $G = (V, E)$ with a **distance function** $d : V \times V \mapsto \mathbb{R}_+$;
- K **leasing types**, with **lengths** $\delta_1, \ldots, \delta_K \in \mathbb{N}$;
- a set $F \subseteq V$ of **potential facilities** and a special **root facility** $r \in F$, and a **cost** γ_f^k for leasing facility f for δ_k units of time ($\gamma_r^K = 0$);
- a **scaling factor** $M \in \mathbb{R}_+$;
- an **online** sequence $D_0, \ldots, D_{T-1} \subseteq V$ of **client sets**.

Every time a set D_t of clients arrives we must:

- optionally **lease some facilities**, obtaining a set $X \subseteq F \times [K] \times \{0, \ldots, T-1\}$;
- **connect** every leased facility to the root r, obtaining an **edge set** $\mathcal{T} \subseteq E$;
- choose an **active** facility lease $a(j, t) \in X$ to serve each client $j \in D_t$;

We are interested in online algorithms, and we wish to minimize

$$\sum_{(f,k,\hat{t}) \in X} \gamma_f^k + \sum_{t=0}^{T-1} \sum_{j \in D_t} d(a(j,t), j) + M \cdot \sum_{e \in \mathcal{T}} d(e).$$

We propose the following algorithm, which is based on the algorithm for problem OCFL by San Felice *et al.* [10], and uses the algorithm for problem OFLE by Nagarajan and Williamson [9] as a subroutine (NWFLE).

Algorithm OCFLE-ALG$(G, d, r, F, K, \gamma, \delta, M)$

set $\delta_r^K \leftarrow 0$, $X \leftarrow \emptyset$, $\mathcal{D} \leftarrow \emptyset$, $\mathcal{D}^{\mathcal{T}} \leftarrow \emptyset$, $\mathcal{T} \leftarrow (\{r\}, \emptyset)$
initialize NWFLE with $(G, d, F, K, \gamma, \delta)$; let (X', a') be the virtual solution
upon receipt of D_t do
    if $t \mod \delta_K = 0$ then $X \leftarrow X \cup \{(r, K, t)\}$; send (r, t) to NWFLE
    for each $j \in D_t$ do
        send (j, t) to NWFLE and update the virtual solution (X', a')
        include j in $\mathcal{D}^{\mathcal{T}}$ with probability $1/M$
        if $j \in \mathcal{D}^{\mathcal{T}}$ then
            $\mathcal{T} \leftarrow \mathcal{T} \cup$ path(j, V(T)), $(f, k, \hat{t}) \leftarrow a'(j, t)$
            if $(f, k, \hat{t})$ is not opened then
                $X \leftarrow X \cup \{(f, k, \hat{t})\}$, $\mathcal{T} \leftarrow \mathcal{T} \cup \{(f, j)\}$
        choose $(f, k, \hat{t}) \in X$ active in t and closest to j
        $\mathcal{D} \leftarrow \mathcal{D} \cup \{(j, t)\}$, $a(j, t) \leftarrow (f, k, \hat{t})$
return $(X, a, \mathcal{T})$

Our algorithm is $O(K \lg n)$-competitive if $M = 1$; this result was published in [3].

Online Steiner Network Leasing

We also propose a generalization of problem PP in which there is a varying demand for multiple copies of the same resource. This problem is a special case of the **2D Parking Permit** (PP2D) problem proposed by Hu *et al.* [5], but we give an offline approximation algorithm and a randomized $\Theta(\lg K)$-competitive algorithm which are both strictly polynomial-time algorithms.

The input of this problem, which we call the **Multi Parking Permit** (MPP) problem, consists of:

- K **permit types**, with **lengths** $\delta_1, \ldots, \delta_K$ and **costs** $\gamma_1, \ldots, \gamma_K$;
- a sequence of **demands** $r_0, \ldots, r_{T-1} \in \mathbb{Z}_+$.

Every time a demand r_t arrives, we may buy some permits (maybe multiple permits of the same type and starting time), obtaining a **multiset** $S \subseteq [K] \times \{0, \ldots, T-1\}$ which must have at least r_t active permits in instant t. The objective is to minimize the total cost of the permits.

Our algorithm consists in a reduction to problem PP. This reduction is based on the **Interval Model** proposed by Meyerson: if we sort the permits by length, then δ_k must be divisible by δ_{k-1}, and permits of type k may only begin at times that are a multiple of δ_k. If we suppose the Interval Model, then our guarantee factor may increase by a ratio of at most 4 [2].

Problem MPP may be reduced to R instances of problem PP, where R is the maximum demand, by putting as much requests in the lower-indexed instances as possible. This reduction is pseudopolynomial; we may circumvent this problem by running $\lfloor \lg R \rfloor + 1$ instances and losing at most a factor of 2 on the guarantee ratio. Thus, we obtain a constant-factor approximation algorithm for the offline setting, and, for the online setting, a deterministic $\Theta(K)$-competitive algorithm and a randomized $\Theta(\lg K)$-competitive algorithm.

In the **Online Steiner Network Leasing** (OSNLE) problem, given a graph $G = (V, E)$ with edge costs and K lease types with uniform costs on the edges, at every instant t we receive a pair (u_t, v_t) of vertices and a demand r_t. Then, we must connect this pair with r_t edge-disjoint paths of leased edges; note that we may have multiple leases for the same edge. This problem may be solved simply by approximating the underlying metric by an HST [4]; then the problem reduces to problem MPP on each edge of the HST. We obtain a $O(\lg K \lg |V|)$-competitive algorithm.

Similarly, the leasing variant of the **Online Buy-at-Bulk Network Design** problem may be solved by using tree metric approximations and by solving problem PP2D on each edge of the HST, obtaining a $O(K \lg |V|)$ algorithm.

Conclusions

- We propose a leasing variant of the Online Connected Facility Location problem and give an algorithm which is $O(K \lg n)$-competitive for the special case in which the scaling factor $M = 1$. Our algorithm is based on [10] and [9].
- We propose a multi-demand variant of the Parking Permit problem, and we give:
  - a strictly polynomial-time algorithm with constant approximation ratio;
  - a deterministic $\Theta(K)$-competitive algorithm based on [8];
  - a randomzied $\Theta(\lg K)$-competitive algorithm based on [8].
- Our techniques may be used to turn the algorithms in [5] into stricly polynomial-time algorithms.
- We propose a leasing variant of the Online Steiner Network problem and give a $O(\lg K \lg |V|)$-competitive algorithm.
- We also point that the Online Buy-at-Bulk Network Leasing problem has a $O(K \lg |V|)$-competitive algorithm.

Forthcoming Research

We intend to solve leasing variants of other network design problems. For problem OCFLE, we wish to prove the same competitive factor for arbitrary M, maybe by modifying the algorithm so to take into account core edges when deciding whether to lease a facility.

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