Compact routing schemes in complex networks

Compact routing schemes for small world graphs proposals

Santiago Viertel & André Luís Vignatti

Department of Informatics — Federal University of Paraná

{sviertel,vignatti}@inf.ufpr.br — +55 (41) 3361-3569

UNIVERSIDADE FEDERAL DO PARANÁ

Abstract

Compact routing schemes are new solutions on routing for large networks. Thorup and Zwick presented a universal compact routing scheme [5] that requires small amount of memory for routing tables. It is possible to get improvements over that routing scheme if the network topology is known. Complex networks appear naturally in many systems. There is a specialized compact routing scheme for power law complex networks [2] that requires less amount of memory to routing tables than the universal compact routing scheme. Besides that, there are other types of complex networks, such as small world networks. The scientific community have not studied well these specific complex networks in the scope of compact routing yet.

Introduction

Determining routes and computing distances in networks are recurring problems. Dijkstra presented a polynomial time algorithm that solves that kind of problems. However, in some applications, the polynomial time required to find minimal paths is too much. There are evidences that node quantity of the Internet is growing so that some current solutions will become inefficient soon. Some examples are the creation of IPv6, the Big Data existence and the natural formation of large complex networks. The scientific community presented some new approaches, which the compact routing schemes stand out. These solutions involve:

- A network preprocessing;
- The generation of a routing table with sublinear size for each node;
- A same constant time routing algorithm that is executed in each node;
- Routings done by not necessarily minimal paths with size limited by a slight bound.

Compact Routing Schemes

A *routing scheme* is a distributed algorithm capable to deliver messages from any node to any other node of a network. A routing scheme is *compact* if the routing tables of all the network nodes have sublinear size. A good use of compact routing schemes is delivering messages mainly in complex networks, which quantity of nodes is large, or in networks which the nodes have little memory and low processing power.

Thorup and Zwick presented a universal compact routing scheme [5] that works well for all graph topology. Its operation is based on subsets of vertex set V, which the authors named as *centers* and *clusters*. Essentially, a delivery from a node u to a node v is done, firstly, by a minimum path from u to the center w closer to v and, after, by a minimum path from w to v, as showed in Figure 1.

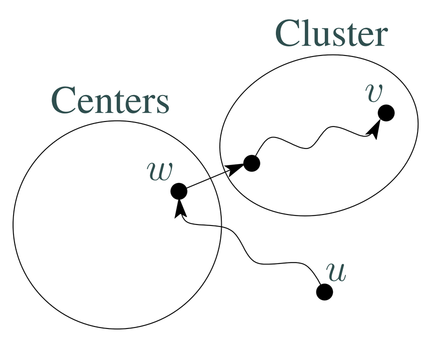

Figure 1: Thorup and Zwick scheme routing

Despite that a compact routing scheme demands small amount of memory for all nodes and has a slight path stretch bound, it is possible to get better results with specialized routing schemes. There are compact routing schemes that improved stretch-space trade-off when the instances are trees or planar graphs. Besides that, it is possible to get improvements through specialized routing schemes for graphs made by random graph generators such as Aiello et al. power law generative model [1].

Complex Networks

Complex networks are networks naturally modeled by real world systems. World Wide Web with web pages (nodes) and hyperlinks (edges) and food webs with animal species (nodes) and the relations predator-prey (edges) are some examples. Complex networks can be classified according to the existence of some specific topological features.

Power Law Graphs

The vertex degree distribution of *power law graphs* follows a power law curve, as illustrated in Figure 2. It means that a power law network has a large number of nodes with low degree and a few nodes with extremely high degree. Aiello et al. [1] proposed a well known random power law graph generative model. Theoretical researchers can use that model to compute analytical results from it.

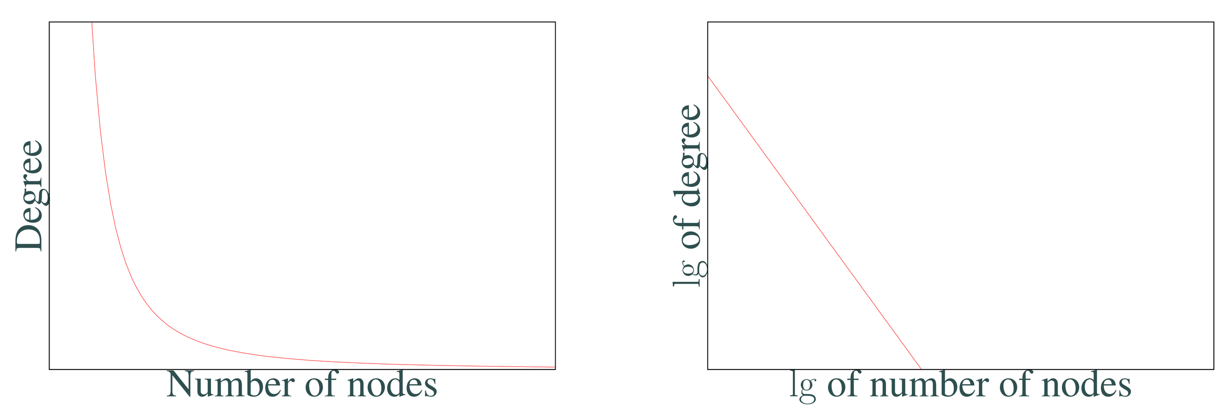

Figure 2: Vertex degree distribution of power law graphs

Chen et al. [2] developed a specialized compact routing scheme for power law graphs based on the Thorup and Zwick scheme. The authors changed the random sample procedure of the center nodes choice to a procedure that selects the highest degree nodes. They reached better results in routing table space than the Thorup and Zwick scheme.

Small World Graphs

A *small world graph* is a network which most of nodes can reach any other through small paths. This structural feature is called as *small world phenomenon*. Milgram awoke the scientific community interest about the small world phenomenon after his research publication [4]. He made an experiment that people had to forward a letter to one of their friends aiming that the letter would have to reach its specific destiny. Milgram discovered that the path done by the letter is generally short.

Besides the small world phenomenon, these networks have another topological feature: the *clustering*. It means that there are groups of strongly linked nodes and these groups are interconnected by some long range links. Researchers proposed some random small world graph generative models:

- **Watts-Strogatz model** [6]: It builds an undirected ring lattice which all the n nodes are connected to k closest nodes and each edge of each node is rewired with probability p to any other node chosen at random. This model generates from regular ($p = 0$) to totally random ($p = 1$) graphs. The Figure 3 shows a graph built with parameters $n = 12$ and $k = 4$.
- **Kleinberg's model** [3]: It builds a directed $n \times n$ grid which each node is connected to all nodes within lattice distance p and to q other nodes with probability proportional to their lattice distance d raised to the r^{th} inverse power (d^{-r}). The nodes within p lattice distance of a node u are the *local contacts* of u and the q nodes chosen at random are the *long-range contacts* of u. The Figure 3 shows a node with its $q = 4$ long-range contacts.

Both small world generative models build clustered graphs with the small world phenomenon feature.

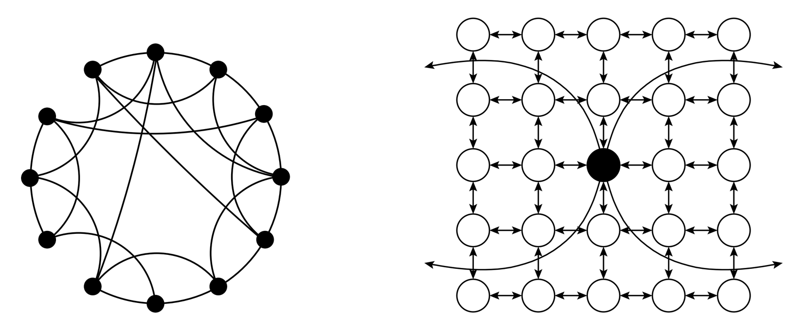

Figure 3: Watts-Strogatz (left) and Kleinberg (right) models

Objective and Future Directions

This work objective is to create a new compact routing scheme based on the Thorup and Zwick universal compact routing scheme specialized on graphs built by Kleinberg's model. We hope to get some improvements over the Thorup and Zwick universal compact routing scheme table size analytical bound. Some future doctoral work:

- Propose a specialized compact routing scheme for graphs generated by Kleinberg's model with routing based on myopic search.
- Compute the expected routing table size bound to the case which a graph generated by Kleinberg's model is submitted to Thorup and Zwick universal compact routing scheme.
- Make a small modification on the Thorup and Zwick universal compact routing scheme aiming to improve the analytical table size bound for the cases in which the graph was generated by Kleinberg's model.
- Make experiments in order to compare the table size of the new compact routing scheme and the Thorup and Zwick universal compact routing scheme.

References

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