Variable Length Error Correcting Codes

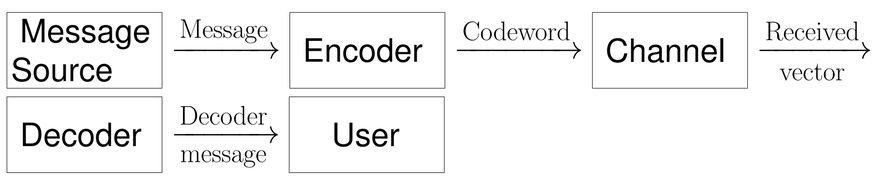
Natália Pedroza[1]
Jayme L. Szwarcfiter[1] **Paulo E. D. Pinto**[2]
[1] UFRJ [2] UERJ
Rio de Janeiro, Brasil
nataliaps@cos.ufrj.br

São Paulo School of Advanced Science on Algorithms, Combinatorics and Optimization

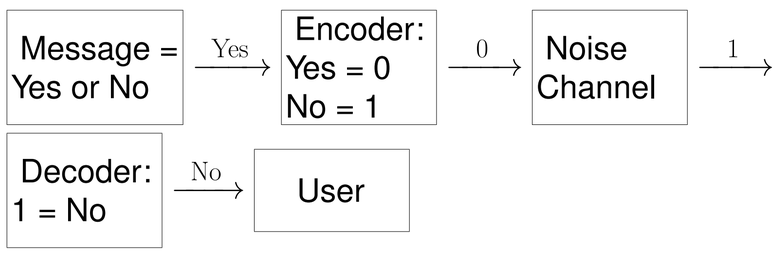
Introduction

Code is a system of rules to convert information — such as a letter, word, sound, image — into another form or representation for communication through a channel — such as telephone line, high frequency radio link, satellite communication link.

- $A = \{a_1, \cdots, a_s\} \rightarrow$ symbols to be codificated;
- $F_q = \{0, \cdots, 1\} \rightarrow$ alphabet with q elements.
- $c = f_1 f_2 \cdots f_l$, with $f_i \in F_q \rightarrow$ *codeword*;
- q-*ary Code* $C = \{c_1, \cdots, c_M\} \rightarrow$ set of codewords.

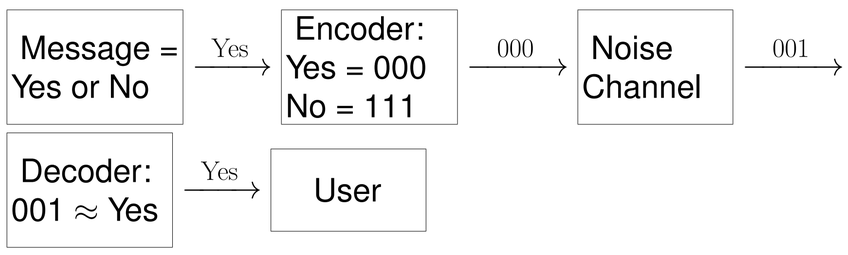


When messages are transmitted through a noisy communication channel — noise may be human error, lightning, thermal noise, imperfections in equipment — errors may ocurr so that the data received is different from that sent. For example:



Error Correction

If the data is encoded by adding a certain amount of redundancy to the message, so that the original message can be recovered even if a certain amount of errors has occurred.



Definition: The Hamming distance between two vectors x and y is the number of places in which they differ. For example, $d(00111, 10100) = 3$.

Theorem: A code with Hamming distance d is capable to correct up $\left\lfloor \frac{d-1}{2} \right\rfloor$ errors.

An important problem in coding theory is to find the largest code of given length n and given minimum distance d. The table shows the results for some n and d and the upper and lower bounds for the open cases.

n \ d	3	5	7	9	11	13	15
5	4	2	1	1	1	1	1
6	8	2	1	1	1	1	1
7	16	2	2	1	1	1	1
8	20	4	2	1	1	1	1
9	40	6	2	2	1	1	1
10	72	12	2	2	1	1	1
11	144	24	4	2	2	1	1
12	256	32	4	2	2	1	1
13	512	64	8	2	2	2	1
14	1024	128	16	4	2	2	1
15	2048	256	32	4	2	2	2
16	2816-3276	256-340	36	6	2	2	2
17	5632-6552	512-673	64-72	10	4	2	2
18	10496-13104	1024-1237	128-135	20	4	2	2
19	20480-26168	2048-2279	256	40	6	2	2
20	40960-43688	2560-4096	512	42-47	8	4	2
21	81920-87333	4096-6941	1024	64-84	12	4	2
22	147456-172361	8192-13674	2048	80-150	24	4	2
23	294912-344308	16384-24106	4096	128-268	48	6	4
24	2^{19}-599184	16384-47538	4096-5421	192-466	52-55	8	4
25	2^{20}-1198368	32768-84260	4104-9275	384-836	64-96	14	4
26	2^{21}-2396736	65536-157285	8192-17099	512-1585	128-169	28	6
27	2^{22}-4792950	131072-291269	16384-32151	1024-2817	178-288	56	8

Variable Length Error Correcting Codes

Traditionally, codes with fixed length have been used. From the last few years, a new trend of error correcting codes with variable length — VLEC — started to be considered, such as in [1, 2].

The construction of optimum error correcting codes, both with fixed and variable word length, using a minimum amount of bits is still an unsolved problem.

In our attempt to solve this issue, we developed an optimum way to concatenate two fixed length binary codes to build a VLEC code. Based on this technique we were able to discover a family of VLEC codes which has a lower cost than the corresponding fixed length codes and those proposed in the literature. We also found some VLEC codes through exhaustive search with the property, shown below.

C_3 =	C_5 =	C_6 =	C_{10} =
0000	0000	00000	000000
01110	011100	11011	011110
10111	101101	011011	101011
	110110	011100	110101
	111011	101010	0100111
		101101	0110010
			1001101
			1011000
			1100100
			1110001

The average codeword length from the codes found are shown on the next table.

M	3	5	6	9	10	11	12	17	18	19	21	22	23
Fixed	5	6	6	7	7	7	7	8	8	8	9	9	9
Variable	4,67	5,6	5,66	6,44	6,6	6,63	6,66	7,35	7,67	7,95	8,29	8,55	8,78

References

[1] V. Buttigieg. *Variable-Length Error-Correcting Codes*. PhD thesis, Department of Electrical Engineering, University of Manchester, England, 1995.

[2] Richa Gupta. *Variable Length Error Correcting Codes and Reversible Variable Length Codes: Analysis and Applications*. PhD thesis, Jaypee Institue of Information Technology, Noida, Índia, 2013.