

**POLYHEDRA ASSOCIATED WITH IDENTIFYING CODES IN GRAPHS**Argiroffo, Gabriela¹; Bianchi, Silvia¹; Lucarini Yanina^{1,2}¹ Departamento de Matemática - ECEN -FCEIA - UNR² CONICET - Rosario

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Identifying codes in graphs**Introduction**

$G = (V, E)$
 $C \subset V \rightarrow \chi^C \in \{0, 1\}^V$ incidence vector of C
 $N[i] = \{j \in V : d(i, j) \leq 1\}$
If C is an identifying code then

- $N[i] \cap C \neq \emptyset \quad \forall i \in V$
- $(N[i] \Delta N[j]) \cap C \neq \emptyset \quad \forall i \neq j, i, j \in V$

The minimum identifying code problemThe minimum identifying code problem in G can be formulated as

$$\min \mathbf{1}^T x : x \in P_{ID}(G)$$

and the identifying number of G is

$$\gamma^{ID}(G) = \min \mathbf{1}^T x : x \in P_{ID}(G)$$

This is a hard problem for several graph classes.

Polyhedra associated with identifying codes $M_{ID}(G)$: matrix whose rows are the incidence vectors of:

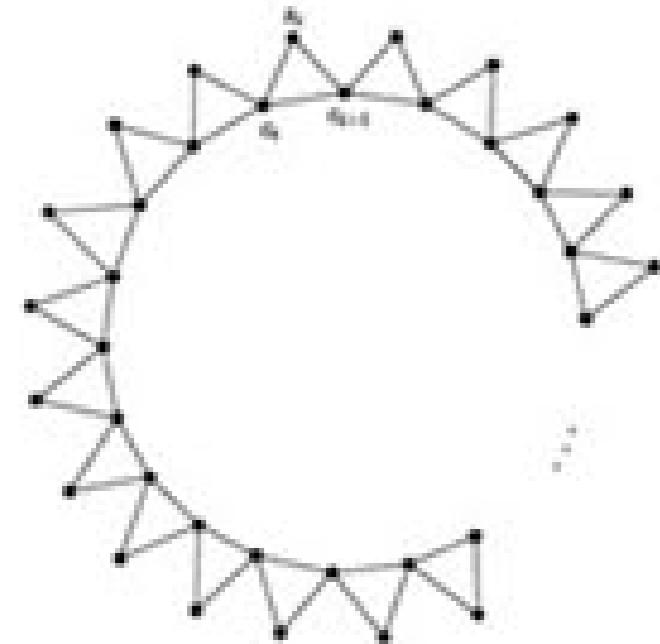
- $N[i] \quad \forall i \in V$
- $N[i] \Delta N[j] \quad \forall i, j \in V$

If $C \subset V$ is an identifying set then

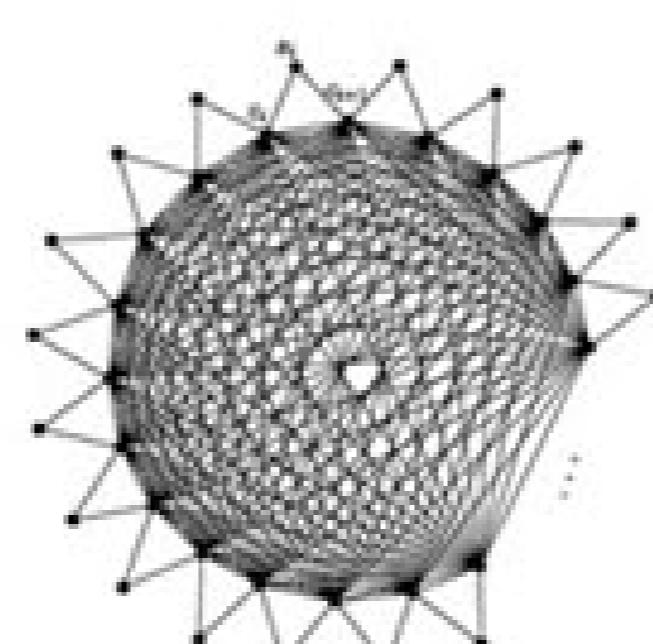
$$M_{ID}(G)\chi^C \geq \mathbf{1}$$

 $C_{ID}(G)$: rows submatrix of $M_{ID}(G)$ without dominating rows. The identifying code polyhedron of G

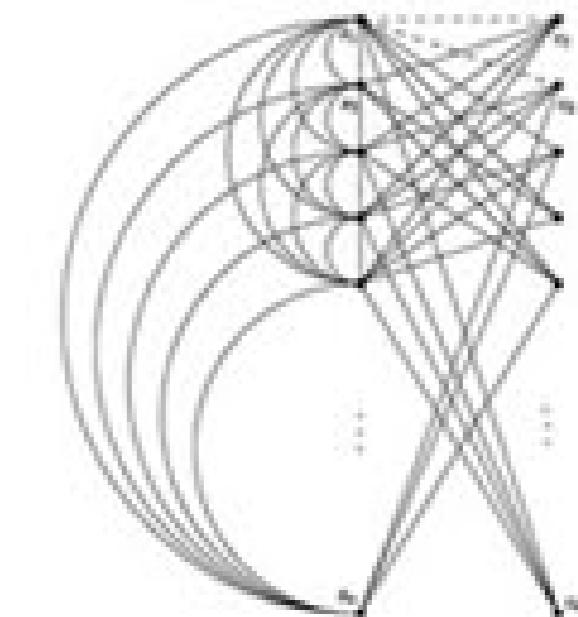
$$P_{ID}(G) = \text{conv}\{x \in \mathbb{Z}^V : C_{ID}(G)x \geq \mathbf{1}\}$$

Hypergraph $\mathcal{H} = (V, \mathcal{E})$: hypergraph with $V = \{1, \dots, n\}$ and $\mathcal{E} \subseteq 2^V$ $M(\mathcal{H}) \rightarrow$ incidence matrix $H_{ID}(G) \rightarrow$ identifying code hypergraph, hypergraph such that $M(H_{ID}(G)) = C_{ID}(G)$ $\mathcal{C} = (V', \mathcal{E}') \rightarrow$ hypercycle of length m in \mathcal{H} , is a sequence $i_1 E_1 i_2 \dots i_m E_m i_1$ of m distinct vertices and m edges with $\{i_j, i_{j+1}\} \in E_j$ and $i_{m+1} = i_1$ **Different families of graphs** $M_n = (C \cup S, E)$: n -suns

$C \rightarrow$ hole, $S \rightarrow$ stable
 $s_i \in S$ is adjacent to exactly two vertices $c_i, c_{i+1} \in C$

 $S_n = (C \cup S, E)$: complete suns

$C \rightarrow$ clique, $S \rightarrow$ stable
 $s_i \in S$ adjacent to exactly two vertices $c_i, c_{i+1} \in C$

 \bar{S}_n : co - suns \bar{S}_n complement of S_n **Results**Theorem: Let $M_n = (C \cup S, E)$ n -suns with $n \geq 4$, then

$$C_{ID}(M_n) = \begin{pmatrix} I & I \\ C_w & I \end{pmatrix}$$

 C_w is a circulant matrix whose first row is $(0, 1, 0, \dots, 0)$. Moreover, $H_{ID}(M_n) = C_{2n}$ and $C_{ID}(M_n) = C_{2n}^2$.Corollary: For $M_n = (C \cup S, E)$ with $n \geq 4$, $P_{ID}(M_n) = Q(C_{ID}(M_n))$ and $\gamma^{ID}(M_n) = n$ whose $Q(C_{ID}(M_n))$ is the linear relaxation.Lemma: Let $S_n = (C \cup S, E)$ a complete sun with $n \geq 4$. The hyperedges $N[s_i]$, $N[s_{i+1}]$ and $N[s_i] \Delta N[s_{i+1}]$ are a hypercycle in $H_{ID}(S_n)$ that induces the facet $x(\{c_i, c_{i+1}, c_{i+2}, s_i, s_{i+1}\}) \geq 2$ of $P_{ID}(S_n)$.Theorem: For a complete suns $S_n = (C \cup S, E)$ with $n \geq 4$ the stable set S is a minimum identifying code and $\gamma^{ID}(S_n) = n$.By definition of \bar{S}_n , we obtain the followings hyperedges of $H_{ID}(\bar{S}_n)$

- $N[s_i] = (C \setminus \{c_i, c_{i-1}\}) \cup \{s_i\}$
- $N[s_i] \Delta N[s_j] = \{c_{i-1}, c_i, c_{j-1}, c_j, s_i, s_j\}$ in particular $N[s_i] \Delta N[s_{i+1}] = \{c_{i-1}, c_{i+1}, s_i, s_{i+1}\}$
- $N[c_i] \Delta N[c_j] = \{s_i, s_{i+1}, s_j, s_{j+1}\}$ in particular $N[c_i] \Delta N[c_{i+1}] = \{s_i, s_{i+2}\}$

Theorem: The identifying code number of \bar{S}_n with $n \geq 7$ is $n - 1$.**Bibliography**

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- [4] E. Balas, S. M. Ng: On the set covering polytope: I. All the facet with coefficients in 0, 1, 2, Mathematical Programming 43, (1989), pp. 57-69.