Combinatorial Optimization Models for Global Value Chains

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Introduction

Over the last thirty years international trade and the production of goods and services were dramatically changed by what became known as Global Value Chains. Driven by a sharp reduction in transportation and telecommunication costs and aimed at reducing their overall costs, companies started to include third countries in what were essentially local production processes. Accordingly, instead of simply relying upon eventual advantages offered by local manufacturing, they started to split production among a network of international partners. From the production of the simplest components to the assembling of an entire product, such a fragmentation of production relied mostly on the availability of cheaper labour elsewhere, at least initially. Furthermore, as it progressed, it changed the notion of competitiveness, moving from a local sphere to the regional and global ones.

There are several valuable works in the field of global value chain. Most of them are uses intensively and cannot be dissociated from the World Input Output Tables (WIOTs) [2]. These tables do allow us a deeper look at trade connectivity, despite their inherent European bias - 27 out of 40 countries involved are European. Our research relies heavily on these tables, used under some minor adjustments.

We propose a new analytical method to obtain information on regional and global value chains. It is based on the combined use of two Combinatorial Optimization models that are directly applied to WIOTs. We apply our models to the 1995, 2000, 2005 and 2010 WIOTs, that span a period of time that is crucial to trade connectivity.

Purpose

- Propose and use a new analytical tool that allows us a more detailed picture of supply-chain trade to be taken, particularly from the year 1995 onwards.
- Evaluate the degree of international connectivity, either regionally or globally and determine its trend over the years. In particular, we are able to compare regional and global supply chains and visualize their relative individual importance.
- Identify *Key Sectors* over both regional and global value chains.

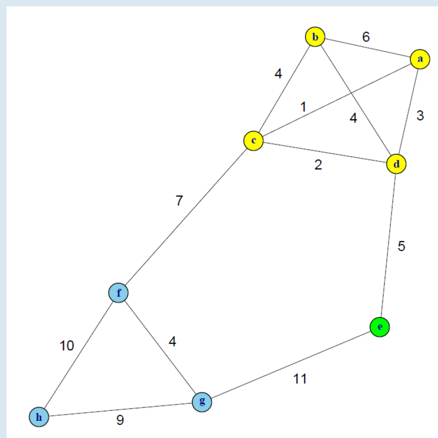
Models

Let $G = (V, E)$ be a graph and $M = [m_{ij}]_{nxn}$ an input-output matrix associated with it. The vertices V are in one-to-one correspondence with the sectors $N = 1, \ldots, n$ of M. Having supply-chain trades in mind, we propose two models:

1. **The Maximum Weight Clique Problem (MWCP)**
2. ***t*-Key Economic Sectors Problem (*t*-KESP)**

The first model intends to identify the most relevant value-chain of an input-output table. We take into account the existence or not of a trade between every pair of distinct economic sectors and evaluate the intensity of such connectivity through the amount of exchangeable values. The second model, in turn, identifies *Key Sectors* for the supply-trade chain returned from the previous model.

Example



Graph G representing 8 sectors from $\mathbf{M}$
Clique $\{f, g, h\}$ is the most relevant

Maximum Weight Clique Problem [4]

The formulation associates distinct sets of variables $\{x_i \in \mathbb{R}^+ : i \in V\}$ and $\{y_e \in \mathbb{R}^+ : e \in E\}$ respectively with the vertices and the edges of G.

$$x_i = \begin{cases} 1, & \text{if vertex } i \in V \text{ belongs to the clique} \\ 0, & \text{otherwise} \end{cases}$$

Similarly,

$$y_e = \begin{cases} 1, & \text{if edge } e \in E \text{ belongs to the clique} \\ 0, & \text{otherwise} \end{cases}$$

The feasibility set for the formulation is defined over a polyhedral region $\mathcal{R}_1$ given by

$$\begin{gathered} y_e \leq x_k,\, e = \{i, j\} \in E,\, k \in \{i, j\}, \\ y_e \geq x_i + x_j - 1,\, e = \{i, j\} \in E, \\ x_i + x_j \leq 1,\, \{i, j\} \notin E, \\ 0 \leq x_i \leq 1,\, i \in V, \\ 0 \leq y_e \leq 1,\, e \in E, \end{gathered}$$

while the formulation is

$$\text{maximize } \{\sum_{e \in E} c_e y_e : (\mathrm{x}, \mathrm{y}) \in \mathcal{R}_1 \cap (\mathbb{Z}^{|V|}, \mathbb{R}^{|E|})\}.$$

t-Key Economic Sectors Problem

Our t-KESP model is not restricted to the square submatrices of M associated with clique induced subgraphs of G. We simplify the presentation applying our model directly over graph G. A formulation for our model involves two distinct sets of variables. Namely, $\{z_i \in \mathbb{R} : i \in V\}$ for the vertices of G and $\{h_e \in \mathbb{R} : e \in E\}$ for its edges.

$$z_i = \begin{cases} 1, & \text{if vertex } i \in V \text{ is key} \\ 0, & \text{otherwise} \end{cases}$$

Additionally,

$$h_{e=\{i,j\}} = \begin{cases} 1, & \text{if } i \text{ or } j \text{ is key} \\ 0, & \text{otherwise} \end{cases}$$

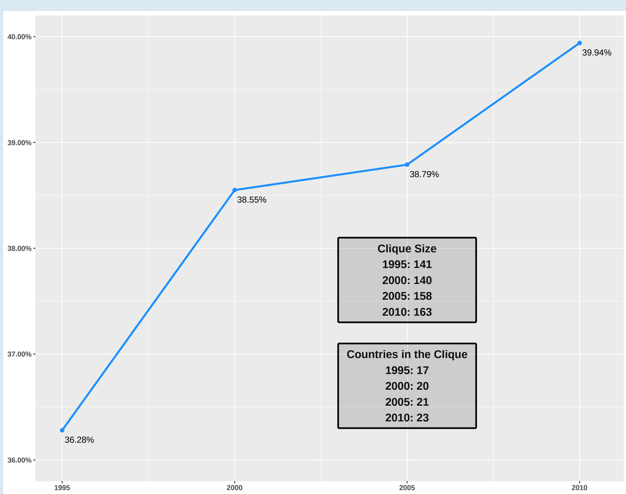
For a given pre-defined integral valued parameter $t \in [1, (n-1)]$, consider a polyhedral region $\mathcal{R}_2$ defined as follows:

$$\begin{gathered} \sum_{i \in V} z_i = t, \\ h_e \leq z_i + z_j,\, e = \{i, j\} \in E, \\ 0 \leq h_e \leq 1,\, e \in E, \\ 0 \leq z_i \leq 1,\, i \in V. \end{gathered}$$

A formulation for t-KESP is then given by

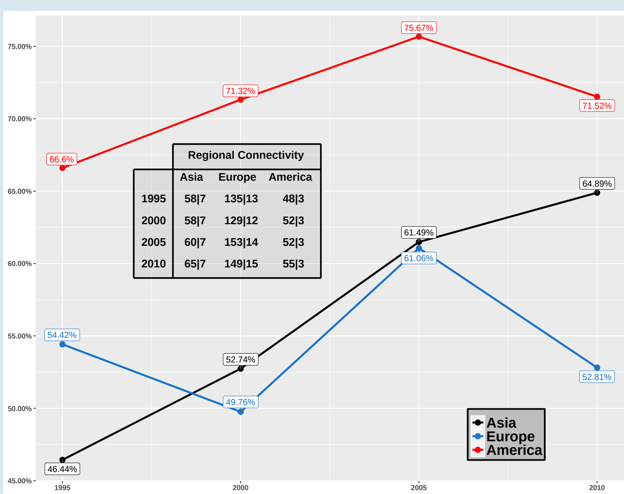
$$\text{maximize } \{\sum_{e \in E} c_e h_e : (\mathrm{h}, \mathrm{z}) \in \mathcal{R}_4 \cap \mathbb{Z}^{(\frac{n(n-1)}{2} + n)}\}.$$

Global Connectivity

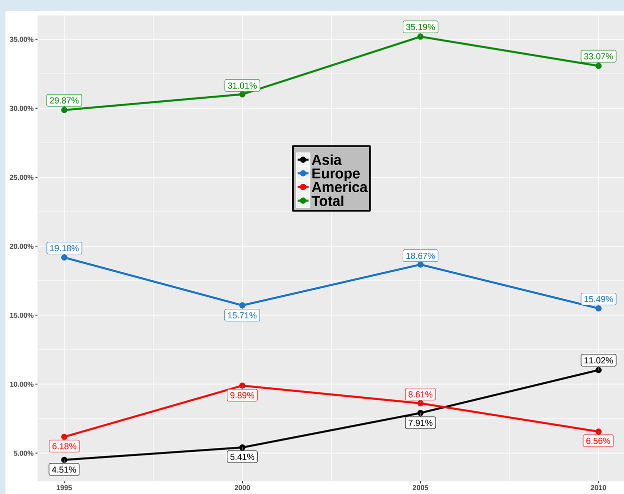


Clique % of the Overall Volume Traded

Regional Connectivity

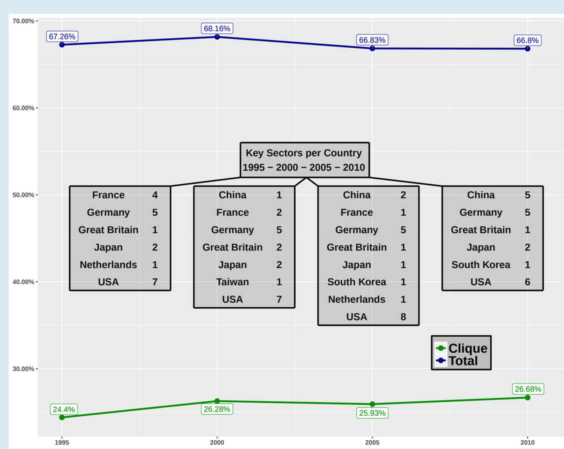


Clique % of Overall Volume Traded in each Region



Regional Clique Shares of Total World Trade

Key Sectors ($t = 20$)



Top - % Key Sectors on World Trade
Bottom - % Key Sectors on World Clique

References

[1] R.E. Baldwin and J. Lopez-Gonzalez. *Supply-chain trade: A portrait of global patterns and several testable hyphoteses.* World Economy, 2015.

[2] Dietzenbacher. E., B. Los, R. Stehrer, M. Timmer, and G. de Vries. *The construction of world input-output tables in the wiod project.* Economic Systems Research, 25:71-98, 2013.

[3] B. Los, M.P. Timmer, and G.J. de Vries. *How global are global value chains?* J. Econ. Perspect.,55:66-92, 2015.

[4] K. Park, K. Lee, and S. Park. *An extended formulation approach to the edge-weighted maximal clique problem.* European Journal of Operational Research, 95:671-682, 1996.