

SEMIDEFINITE PROGRAMMING UPPER BOUNDS FOR PACKING PROBLEMS — IMPLEMENTATION EXERCISES

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This list will focus on the implementation of some of the bounds discussed in the lectures. Many softwares and programming languages can be used to implement them.

Exercise 1. Compute the theta number for a finite graph.

Exercise 2. Compute the linear programming bound for $A(n, d)$ (equation (6)) with $n = 20$, $d = 8$ and show $A(20, 8) \leq 290$ (the exact value is $A(20, 8) = 256$, as shown by Gijswijt, Mittelmann and Schrijver¹ using a stronger bound).

Exercise 3. Compute the linear programming bound for $A(n, \theta)$ (equation (10)) using sampling with $n = 12$ and $\theta = \pi/3$ and show $\tau_{12} \leq 1416$. Exactly how the “rescaling process” mentioned in page 13 of the lecture notes works? How many points are necessary to sample?

Exercise 4. Use the method of §11 to find bounds for the minimum and the maximum values of polynomial $f(x, y) = x^6 + x^2y^3 - 2xy^3 - x^2y^2 + y$ inside the unit ball.

Exercise 5. Compute the linear programming bound for $A(n, \theta)$ using the sum of squares technique from §12 (equation (18)) with $n = 12$ and $\theta = \pi/3$.

¹Gijswijt, Dion C., Hans D. Mittelmann, and Alexander Schrijver. “Semidefinite code bounds based on quadruple distances.” *IEEE Transactions on Information Theory* 58.5 (2012): 2697-2705.