Local Diagnosis

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Abstract

In (Hansson & Wassermann 1999; Wassermann 1999), we have presented operations of belief change which only affect the relevant part of a belief base. In this paper, we propose the application of the same strategy to the problem of model-based diangosis. We first isolate the subset of the system description which is relevant for a given observation and then solve the diagnosis problem for this subset.

Introduction

In (Wassermann 2000), we have shown how consistency-based diagnosis relates to belief revision and how Reiter's algorithm can be used for kernel operations of belief change. In (Hansson & Wassermann 1999), we presented the idea of local change and characterized operations of belief change that only affect the relevant part of a belief base. In (Wassermann 1999), we presented an algorithm for retrieving the relevant part of a belief base which can be used for implementing local change. In the present paper, we close the circle by showing how local change can be used for focusing the diagnosis process on the relevant part of the domain.

We will show how a diagnosis problem can be translated into an operation of kernel semi-revision. Kernel semi-revision (Hansson 1997) consists in adding new information to a database and restoring consistency if necessary. To restore consistency, the expanded database is contracted by $\perp$.

Then we will show how to use information about the structure of the device being examined in order to obtain more efficient methods of diagnosis. For this, we will use the operation of local kernel semi-revision, presented in (Hansson & Wassermann 1999), that considers only the relevant part of the database. In (Wassermann 1999), we have presented a simple method for extracting the relevant part of a structured database, which will be used in this paper.

This paper proceeds as follows: in the next two sections we give a brief introduction to kernel operations of belief change and consistency-based diagnosis. Then we show the relation between kernel semi-revision and Reiter diagnosis. Using this relation, we show how to use information about the system to focus on its relevant part during the process of diagnosis.

In the rest of this paper we consider L to be a propositional language closed under the usual truth-functional connectives and containing a constant $\perp$ denoting falsum.

Kernel Semi-Revision

Hansson introduced a construction for contraction operators, called *kernel contraction* (Hansson 1994), which is a generalization of the operation of safe contraction defined in (Alchourrón & Makinson 1985). The idea behind kernel contraction is that, if we remove from the belief base B at least one element of each α-kernel (minimal subset of B that implies α), then we obtain a belief base that does not imply α (Hansson 1994). To perform these removals of elements, we use an incision function, i.e., a function that selects at least one sentence from each kernel.

Definition 1 *(Hansson 1994) The* **kernel** *operation* $\perp\!\!\!\perp$ *is the operation such that for every set B of formulas and every formula α, $X \in B \perp\!\!\!\perp \alpha$ if and only if:*

1. $X \subseteq B$
2. $\alpha \in Cn(X)$
3. *for all Y, if $Y \subset X$ then $\alpha \notin Cn(Y)$*

The elements of $B \perp\!\!\!\perp \alpha$ are called α-kernels.

Definition 2 *(Hansson 1994) An* **incision function** *for B is any function σ such that for any formula α:*

1. $\sigma(B \perp\!\!\!\perp \alpha) \subseteq \bigcup(B \perp\!\!\!\perp \alpha)$, *and*
2. *If $\emptyset \neq X \in B \perp\!\!\!\perp \alpha$, then $X \cap \sigma(B \perp\!\!\!\perp \alpha) \neq \emptyset$.*

Semi-revision consists in adding new information to a database and restoring consistency if necessary. To restore consistency, the expanded database is contracted by $\perp$. Semi-revision consists of two steps: first the belief α is added to the base, and then the resulting base is consolidated, i.e., contracted by $\perp$. Kernel semi-revision uses kernel contraction for the second step.

Definition 3 *(Hansson 1997) The* **kernel semi-revision** *of B based on an incision function σ is the operator $?_\sigma$ such that for all sentences α:*

$$B?_\sigma\alpha = (B \cup \{\alpha\}) \setminus \sigma((B \cup \{\alpha\}) \perp\!\!\!\perp \perp)$$

Consistency-Based Diagnosis

Diagnosis is a very active area within the artificial intelligence community. The problem of diagnosis consists in, given an observation of an abnormal behavior, finding the components of the system that may have caused the abnormality (Reiter 1987).

In the area known as model-based diagnosis (Hamscher, Console, & de Kleer 1992), a model of the device to be diagnosed is given in some formal language. In this paper, we will concentrate on model-based diagnosis methods that work by trying to restore the consistency of the system description and the observations.

Although Reiter's framework is based on first-order logic, most of the problems studied in the literature do not make use of full first-order logic and can be easily represented in a propositional language. For the sake of simplicity, we will adapt the definitions given in (Reiter 1987) to only mention formulas in the propositional language L.

Basic Definitions

The systems to be diagnosed will be described by a set of propositional formulas. For each component X of the system, we use a propositional variable of the form okX to indicate whether the component is working as it should. If there is no evidence that the system is not working, we can assume that variables of the form okX are true.

Definition 4 *A* **system** *is a pair (SD,ASS), where:*

1. *SD, the system description, is a finite set of formulas of L and*
2. *ASS, the set of assumables, is a finite set of propositional variables of the form okX.*

An **observation** is a formula of L. We will sometimes represent a system by (SD,ASS,OBS), where OBS is an observation for the system (SD,ASS).

The need for a diagnosis arises when an abnormal behavior is observed, i.e., when SD$\cup$ASS$\cup${OBS} is inconsistent. A diagnosis is a minimal set of assumables that must be negated in order to restore consistency.

Definition 5 *A* **diagnosis** *for (SD,ASS,OBS) is a minimal set $\Delta \subseteq$ASS such that:*
$SD \cup \{OBS\} \cup ASS \backslash \Delta \cup \{\neg okX | okX \in \Delta\}$ *is consistent.*

A diagnosis for a system does not always exist:

Proposition 1 *(Reiter 1987) A diagnosis exists for (SD,ASS,OBS) if and only if SD$\cup${OBS} is consistent.*

Definition 5 can be simplified as follows:

Proposition 2 *(Reiter 1987) The set $\Delta \subseteq$ASS is a diagnosis for (SD,ASS,OBS) if and only if Δ is a minimal set such that $SD \cup \{OBS\} \cup (ASS \backslash \Delta)$ is consistent.*

Computing Diagnoses

In this section we will present Reiter's construction for finding diagnoses. Reiter's method for computing diagnosis makes use of the concepts of *conflict sets* and *hitting sets*. A conflict set is a set of assumables that cannot be all true given the observation:

Definition 6 *(Reiter 1987) A* **conflict set** *for (SD,ASS,OBS) is a set Conf = $\{okX_1, okX_2, ..., okX_n\} \subseteq ASS$ such that $SD \cup \{OBS\} \cup Conf$ is inconsistent.*

From Proposition 2 and Definition 6 it follows that $\Delta \subseteq$ASS is a diagnosis for (SD,ASS,OBS) if and only if Δ is a minimal set such that ASS$\backslash\Delta$ is not a conflict set for (SD,ASS,OBS).

A hitting set for a collection of sets is a set that intersects all sets of the collection:

Definition 7 *(Reiter 1987) Let $\mathcal{C}$ be a collection of sets. A* **hitting set** *for $\mathcal{C}$ is a set $H \subseteq \bigcup_{S \in \mathcal{C}} S$ such that for every $S \in \mathcal{C}$, $H \cap S$ is nonempty. A hitting set for $\mathcal{C}$ is minimal if and only if no proper subset of it is a hitting set for $\mathcal{C}$.*

The following theorem presents a constructive approach for finding diagnoses:

Theorem 1 *(Reiter 1987) $\Delta \subseteq$ASS is a diagnosis for (SD,ASS,OBS) if and only if Δ is a minimal hitting set for the collection of minimal conflict sets for (SD,ASS,OBS).*

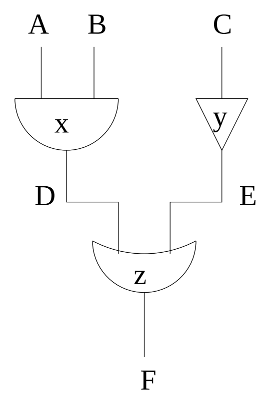

Figure 1: Circuit

Consider the circuit in Figure 1. The system description of this circuit is given by (SD,ASS), where:

$$\begin{aligned} \text{ASS} &= \{okX, okY, okZ\} \\ \text{SD} &= \{(A \wedge B) \wedge okX \rightarrow D, \\ &\quad \neg(A \wedge B) \wedge okX \rightarrow \neg D, \\ &\quad C \wedge okY \rightarrow \neg E, \\ &\quad \neg C \wedge okY \rightarrow E, \\ &\quad (D \vee E) \wedge okZ \rightarrow F, \\ &\quad \neg(D \vee E) \wedge okZ \rightarrow \neg F\} \end{aligned}$$

Suppose we have OBS=$\neg C \wedge \neg F$. This observation is inconsistent with SD $\cup$ ASS. There is only one minimal conflict set for (SD,ASS,OBS): $\{okY, okZ\}$. There are three possible hitting sets: $\{okY\}$,$\{okZ\}$, and $\{okY, okZ\}$. Reiter considers only minimal hitting sets as diagnoses, that is, either Y or Z is not working well.

Diagnosis via Kernel Semi-Revision

In (Wassermann 2000), we have shown that the standard method for finding consistency-based diagnosis, due to Reiter (Reiter 1987), is very similar to the construction of kernel semi-revision, except for the fact that Reiter only considers minimal diagnosis, which correspond to minimal values for incision functions. In this section we summarize these results.

Recall that kernel operations are based on two concepts: kernels and incision functions. The kernels are the minimal subsets of a belief base implying some sentence, while the incision functions are used to decide which elements of the kernels should be given up. Let (SD,ASS,OBS) be a system. The belief base that we are going to semi-revise corresponds to SD$\cup$ASS and the input sentence is OBS. The conflict sets are the assumables in the inconsistent kernels of SD$\cup$ASS$\cup\{$OBS$\}$. So, if B=SD$\cup$ASS, the conflict sets are given by $\{X\cap\text{ASS}|X \in (B+\text{OBS}) \perp\!\!\!\perp \bot\}$. Incision functions correspond loosely to hitting sets, the minimal hitting sets being the values of minimal incisions that return only assumables. Note that there is a difference in the status of formulas in SD and those in ASS: formulas in ASS represent expectations and are more easily retracted than those in SD (cf. Definition 8).

We can model the diagnosis problem as a kernel semi-revision by the observation. Semi-revision can be divided in two steps. First the observation is added to the system description together with the assumables. In case the observation is consistent with the system description together with the assumables, no formula has to be given up. Otherwise, we take the inconsistent kernels and use an incision function to choose which elements of the kernels should be given up.

In the case of diagnosis, we do not wish to give up sentences belonging to the system description or the observation. We prefer to give up the formulas of the form okX, where X is a component of the system. Moreover, we are interested in minimal diagnosis, so the incision should be minimal. For this, we use a special variant of incision function. We modify Definition 2 so that incisions are minimal and elements of a given set A are prefered over the others:

Definition 8 *Given a set A, an A-***minimal incision function** *is any function σ_A from sets of sets of formulas into sets of formulas such that for any set S of sets of formulas:*

1. *$\sigma_A(S) \subseteq \bigcup S$,*
2. *If $\emptyset \neq X \in S$, then $X \cap \sigma_A(S) \neq \emptyset$,*
3. *If for all $X \in S$, $X \cap A \neq \emptyset$, then $\sigma_A(S) \subseteq A$, and*
4. *$\sigma_A(S)$ is a minimal set satisfying 1,2, and 3.*

If we take A to be the set of assumables, we obtain an incision function that prefers to select formulas of the form okX over the others.

We can show that for (SD,ASS,OBS), whenever a diagnosis exists, an ASS-minimal incision function will select only elements of ASS:

Proposition 3 *Let (SD,ASS,OBS) be a system with an observation and σ_{ASS} an ASS-minimal incision function. If a diagnosis exists, then $\sigma_{ASS}((SD \cup ASS \cup OBS) \perp\!\!\!\perp \bot) \subseteq ASS$.*

Lemma 1 *The assumables that occur in an inconsistent kernel of the set SD$\cup$ASS$\cup$OBS form a conflict set for (SD,ASS,OBS) and all minimal conflict sets can be obtained in this way, i.e.:*

(i) For every $X \in (SD\cup ASS\cup OBS) \perp\!\!\!\perp \bot$, $X\cap ASS$ is a conflict set, and

(ii) For every minimal conflict set Y, there is some $X \in (SD\cup ASS\cup OBS) \perp\!\!\!\perp \bot$ such that $X\cap ASS= Y$.

Note that not every inconsistent kernel determines a minimal conflict set, since for conflict sets only the elements of ASS matter, i.e., there may be two inconsistent kernels X_1 and X_2 such that $X_1\cap$ASS is a proper subset of $X_2\cap$ASS.

Recall that given an incision function σ, the semi-revision of a set B by a formula α was given by $B?_\sigma\alpha = (B + \alpha) \setminus \sigma((B + \alpha) \perp\!\!\!\perp \bot)$. A diagnosis is given by the elements of ASS that are given up in a kernel semi-revision by the observation.

Proposition 4 *Let S=(SD,ASS,OBS) be a system and σ_{ASS} an ASS-minimal incision function.*

$(SD \cup ASS)\setminus((SD \cup ASS)?_{\sigma_{ASS}} OBS) = \sigma_{ASS}((SD \cup ASS \cup OBS) \perp\!\!\!\perp \bot)$ is a diagnosis.

Using System Structure

Suppose that instead of the circuit depicted in Figure 1, we have the circuit in Figure 2. Suppose also that we get the same observation, i.e., OBS= $\neg C \wedge \neg F$. Intuitively, only a small part of the circuit (roughly the sub-circuit at figure 1) has to be considered in order to arrive to a diagnosis.

In (Hansson & Wassermann 1999), we have extended the definition of kernel semi-revision to an operation that considers only the relevant part of a database, local kernel semi-revision. In (Wassermann 1999), we have shown how to use structure present in a database in order to find compartments and implement local kernel operations more efficiently. The key idea of the method described is to use a relation of relatedness between formulas of the belief base. In some applications, as we will see, such a relation is given with the problem. In the case of the circuit shown in Figure 2, there is a very natural dependence relation. The output of each of the components depends on the input and on whether the component is working well.

The only observation we have is $\neg C \wedge \neg F$. Since this observation is inconsistent with the system description

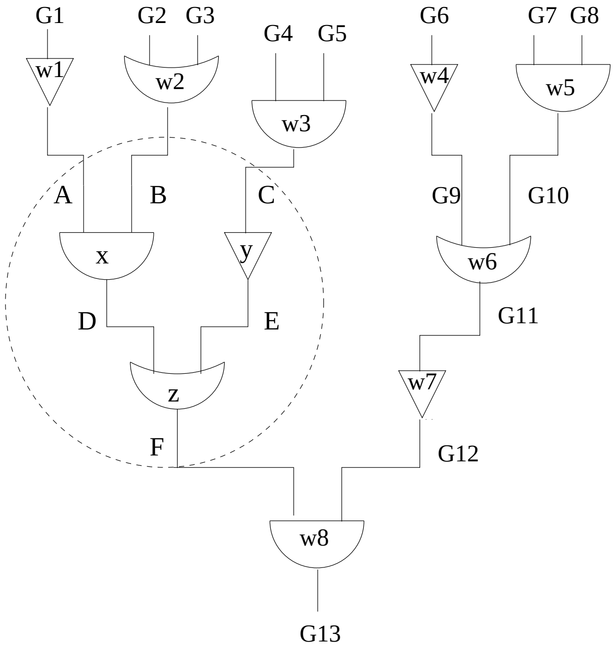

Figure 2: Larger Circuit

together with the assumption that all components are working well, there must be some faulty component. Moreover, the faulty component must be in the path between C and F (of course, there may be other faulty components, but we are only searching for the abnormality that explains the observation). We only need to consider the descriptions of components y and z in order to find the diagnosis.

In the next section we will show how to use the framework described in (Wassermann 1999) in order to find diagnoses without having to check the entire system description for consistency.

Local Kernel Diagnosis

As we have seen, diagnosis problems fit very well in the framework for local change that we proposed in (Hansson & Wassermann 1999) and (Wassermann 1999). Besides the fact that the traditional method for finding diagnosis based on the notion of consistency is almost identical to the construction of kernel semi-revision, in most diagnosis problems there is a very natural notion of relatedness that can be used to structure the belief base so that the search for diagnoses becomes more efficient.

In this section we formalize the example in Figure 2 in order to show how to derive a concrete relatedness relation from the given database.

We will use a relatedness relation between atoms, as illustrated in Figure 3. The relation is not symmetric. We can easily adapt the definitions presented in (Wassermann 1999) to deal with a directed graph.

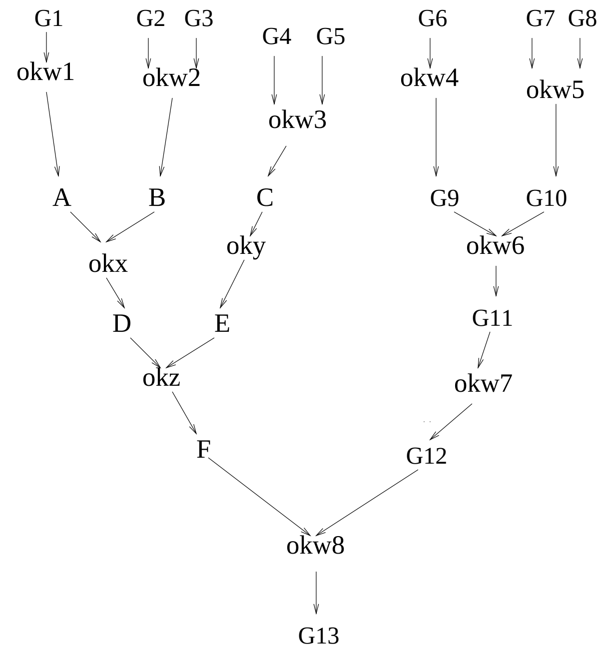

Figure 3: Relatedness relation between atoms

The basic algorithm is as follows: we start from the propositional variables that occur in the observation and spread the activation in the graph, following the direction of the arcs. The spreading finishes either when the end of the paths are reached or when we run out of resources (time or memory). This is done by the algorithm `Retrieve` below, an adaptation of the algorithm given in (Wassermann 1999). The algorithm uses the function `Adjacent` to collect all nodes related to a given node, i.e., given a relatedness relation R, $\texttt{Adjacent}(x)=\{y \in \text{Var}(\text{SD}) \cup \text{ASS} | R(x,y)\}$, where $\text{Var}(X)$ is the set of propositional variables ocurring in the formulas of set X. For a set Y of propositional variables, $\texttt{Adjacent}(Y)= \bigcup\{\texttt{Adjacent}(y) | y \in Y\}$.

`Retrieve`(OBS,ASS,Relevant):
1. For all $p \in$ Var(OBS), mark(p)
2. Δ^1(OBS) := Adjacent(Var(OBS))
3. Relevant := Var(OBS)∩ASS
4. i := 1; stop := false
5. While not stop do
   5.1. For all $p \in \Delta^i$(OBS), mark(p)
   If $p \in$ASS,
   then Relevant := Relevant $\cup\{p\}$
   5.2. i := i+1; $\Delta^i(\text{OBS})=\emptyset$
   5.3 For all $p \in \Delta^{i-1}$(OBS),
   Δ^i(OBS) := Δ^i(OBS) $\cup\{q \in$Adjacent(p) s.t. not marked(q)$\}$
   5.4 If Δ^i(OBS)= $\emptyset$, then stop := true

After we have retrieved the relevant assumables, the relevant compartment is taken to be the observation together with all formulas in SD∪ASS which mention the relevant assumables.

```
Compartment(OBS,SD,ASS,Comp):
1. Retrieve(OBS,ASS,Relevant)
2. Comp=OBS
3. For all p ∈Relevant,
        Comp:= Comp∪{α ∈SD∪ASS|p ∈Var(α)}.
```

As we have seen in (Wassermann 1999), the algorithm for `Retrieve` is an anytime algorithm. The algorithm for `Compartment` is not, at least in principle. But if one keeps the order in which the relevant atoms are retrieved and uses them in this order in line 3 of algorithm `Compartment`, one can be sure that the description of the most relevant components will be retrieved first.

For the circuit in Figure 2, we have:

$$\begin{aligned}
\text{SD}=\{&(A \wedge B) \wedge okX \rightarrow D,\ \neg(A \wedge B) \wedge okX \rightarrow \neg D,\\
&C \wedge okY \rightarrow \neg E,\ \neg C \wedge okY \rightarrow E,\\
&(D \vee E) \wedge okZ \rightarrow F,\ \neg(D \vee E) \wedge okZ \rightarrow \neg F,\\
&G1 \wedge okW1 \rightarrow \neg A,\ \neg G1 \wedge okW1 \rightarrow A,\\
&(G2 \vee G3) \wedge okW2 \rightarrow B,\ \neg(G2 \vee G3) \wedge okW2 \rightarrow \neg B,\\
&(G4 \wedge G5) \wedge okW3 \rightarrow C,\ \neg(G4 \wedge G5) \wedge okW3 \rightarrow \neg C,\\
&G6 \wedge okW4 \rightarrow \neg G9,\ \neg G6 \wedge okW4 \rightarrow G9,\\
&(G7 \wedge G8) \wedge okW5 \rightarrow G10,\\
&\neg(G7 \wedge G8) \wedge okW5 \rightarrow \neg G10,\\
&(G9 \vee G10) \wedge okW6 \rightarrow G11,\\
&\neg(G9 \vee G10) \wedge okW6 \rightarrow \neg G11,\\
&G11 \wedge okW7 \rightarrow G12,\ \neg G11 \wedge okW7 \rightarrow \neg G12,\\
&(F \wedge G12) \wedge okW8 \rightarrow G13,\\
&\neg(F \wedge G12) \wedge okW8 \rightarrow \neg G13\}
\end{aligned}$$

$$\begin{aligned}
\text{ASS} =\{&okX, okY, okZ, okW1, okW2, okW3, okW4,\\
&okW5, okW6, okW7, okW8\}
\end{aligned}$$

If we apply the algorithm `Retrieve`($\neg C \wedge \neg F$,ASS,Relevant) to the graph depicted in Figure 3, we get Relevant={okY,okZ,okW8}. For `Compartment`(OBS, SD, ASS, Comp) we get

$$\begin{aligned}
\text{Comp}=\{&\neg C \wedge \neg F, C \wedge okY \rightarrow \neg E,\\
&\neg C \wedge okY \rightarrow E, (D \vee E) \wedge okZ \rightarrow F,\\
&\neg(D \vee E) \wedge okZ \rightarrow \neg F,\\
&(F \wedge G12) \wedge okW8 \rightarrow G13,\\
&\neg(F \wedge G12) \wedge okW8 \rightarrow \neg G13, okY, okZ, okW8\}.
\end{aligned}$$

The diagnosis can be searched using only the formulas in Comp. Note that the component w8 was not really relevant for the diagnosis but, nevertheless, we have reduced the set to be semi-revised.

This is a very general method for focusing on a small part of the system description. One can add to it some domain specific heuristics to improve its efficiency. The system IDEA (Sanseverino & Cascio 1997), used by FIAT repair centers works on dependence graphs that show graphically the relation between the several components of a device.

In (Wassermann 2000) we have shown that Reiter's algorithm for consistency-based diagnosis can be used for kernel semi-revision. The algorithm for kernel operations can be easily combined with the algorithm `Compartment` presented in this section.

Applying Reiter's algorithm to Comp, given the observation $\neg C \wedge \neg F$, we get as possible diagnoses: $\{okY\}$ and $\{okZ\}$.

Conclusions

In this paper we have shown how to combine Reiter's algorithm for consistency-based diagnosis with the algorithm for finding the relevant compartment of a database. The result is a method for finding diagnosis which focuses on the relevant part of the system description.

Making clear the similarities between diagnosis and belief revision can be very profitable for both areas of research. As shown in (Wassermann 2000), the computational tools developed in the field of diagnosis can be adapted to be used for belief revision. And as we show in this paper, theories developed for belief revision can be applied on diagnosis for obtaining more efficient methods.

Future work includes the study of other approaches to diagnosis as well as the study of the computational complexity of the method proposed.

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