Paris, August 24th, 2009

On the scientific work of Professor Imre Simon (1943–2009)

Imre Simon’s work had and still has considerable influence in theoretical computer science, in discrete mathematics and in semigroup theory. I would like to report here on the consequences of some of his most important achievements.

(1) Locally testable languages and graph congruences.
(2) Piecewise testable languages and $\mathcal{J}$-trivial monoids.
(3) Factorization forests.
(4) Burnside problem, limitedness problems and tropical mathematics.

Result (1) is joint work with Janusz B. Brzozowski. It states, in Eilenberg’s terminology, that the variety of locally testable languages corresponds to the variety of finite locally idempotent and commutative semigroups. This result was also obtained independently by McNaughton. It is one of the cornerstones of the theory of finite automata: at the time it was first published (1971), it was, together with Schützenberger’s theorem on star-free languages, the only significant example of an algebraic characterization of a class of rational languages. It also had considerable influence in semigroup theory and gave notably a new impetus to the study of the semidirect product. A key argument of the proof was popularized by Eilenberg as Simon’s theorem on graph congruences and was subsequently generalized by Knast, Thérien, Straubing, Almeida, Steinberg and many others. It was actually the first use of categories as algebraic objects, a key idea in subsequent research, notably to find the right notion of kernel in semigroup theory. It motivated Tilson’s trace delay theorem and was also used by Margolis and myself to prove a covering result for semigroups whose idempotents commute. More surprisingly, it was again a key ingredient of the algebraic proof of the equivalence between Büchi and Muller automata on infinite words (Le Saec, Pin and Weil 1991).

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Result (2) is probably the most cited one and is also a result on varieties. It states that the variety of piecewise testable languages corresponds to the variety of finite $J$-trivial monoids. This result, published in 1975, had a major impact in language theory but also in three other areas: combinatorics on words, semigroup theory and logic.

**Combinatorics on words**: The proof includes a series of elegant combinatorial results around the notion of subword (in the sense of subsequence): Imre developed later this point of view together with Jacques Sakarovitch in a reference chapter of Lothaire’s book *Combinatorics on words*. Imre was also interested in algorithms and wrote a very nice survey entitled *Sequence comparison: some theory and some practice*.

**Semigroup theory**: It is not difficult to establish that Imre’s theorem is equivalent to a statement of pure semigroup theory: a finite monoid is $J$-trivial if and only if it is the quotient of an ordered monoid in which 1 is the top element. This statement served as a prototype to subsequent covering results and several potential extensions of Imre’s result are still open problems.

**Logic**: Wolfgang Thomas has shown that a language is piecewise testable if and only if it is expressible by a Boolean combination of $\Sigma_1[<]$-formulas. Now, Imre’s theorem shows that it is decidable to know whether a first order formula is equivalent to $\Sigma_1[<]$-formula.

All of this explains why so many different approaches were used to prove Imre’s result: model theory (Stern, 1985), ordered monoids (Straubing and Thérien, 1988), profinite topology (Almeida, 1990), endomorphisms of linear orders (Higgins, 1997), factorization trees (Henckell and Pin, 2000).

Imre’s result (3) on factorization forests is a combinatorial result in the spirit of Ramsey’s theorem, but adapted to finite semigroups. Imre published several proofs of his result and other proofs were proposed by Chalopin and Leung (2004), Colcombet (2007), Kuftleitner (2008) and Bojańczyk (2009), who presented an invited lecture entitled *Factorization forests* at the conference DLT’09 last July. Imre originally used his result in connection with his study of tropical semirings, but it also had a number of other applications: polynomial operations on rational languages (Pin-Weil, Branco-Pin), $\omega$-languages (Bojańczyk/Colcombet), trees and infinite words (Colcombet), decidability of the logic $\Sigma_2[<]$ (Pin-Weil, Bojańczyk).

The Burnside problem was originally stated for groups, but it has a natural extension to semigroups: what is the structure of the free semigroup satisfying the equation $x^n = x^{n+m}$? To my knowledge, Imre published only one article on the Burnside problem, a survey paper he wrote jointly with his student Alair Pereira do Lago. But at the end of this survey, one finds the following reference: I. Simon, Notes on non-counting languages of order 2, manuscript, 1970. In fact, Imre was extremely proud and happy of the brilliant results obtained by Alair on the Burnside problem, because he worked

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himself hard on it. But he also noted with humour: “I worked in the seventies on the case \( x^2 = x^3 \), thinking it was the easiest one, but it turned out to be the most difficult one, you know…” Imre was right, as usual, and this case is still open…

Burnside problem is perhaps the reason why Imre got interested into the limitedness problem: can one decide whether a rational language satisfies \( L^* = L^n \) for some \( n \)? The solution that Imre gave to this problem is probably more important than the problem itself. Imre actually solved a kind of Burnside problem for the semiring \( (\mathbb{N} \cup \{+\infty\}, \min, +) \). This semiring and its cousins are now known as tropical semirings in the honour of Imre. The term became so popular that the new branch of geometry using these semirings has been baptized tropical geometry.

Imre Simon was an outstanding mathematician and computer scientist. I just mentioned four of his leading works, but I could also refer to his work on the shuffle operation (a joint paper with Z. Ésik), on nondeterministic complexity of finite automata, on string matching algorithms, on compression and entropy, etc. The impact of his results is enormous and still at the heart of ongoing research. In view of the depth and influence of Imre’s research work, I fully support his nomination to the title of Professor Emeritus.

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