**Curso: MAT 220- CÁLCULO DIFERENCIAL E INTEGRAL IV - IFUSP**
**Professor Oswaldo Rio Branco de Oliveira**
**Período: Segundo Semestre de 2009**

## LISTA DE EXERCÍCIOS 3 - SÉRIES

1. Sejam $(x_n)$ e $(y_n)$ sequências limitadas em $\mathbb{R}$. Mostre que

   (a) $\limsup(x_n + y_n) \le \limsup x_n + \limsup y_n$ $\qquad \liminf(x_n + y_n) \ge \liminf x_n + \liminf y_n$.

   (b) $\limsup(-x_n) = -\liminf x_n$ $\qquad \liminf(-x_n) = -\limsup x_n$.

   (c) $\limsup(x_n y_n) \le (\limsup x_n)(\limsup y_n)$ $\qquad \liminf(x_n y_n) \ge (\liminf x_n)(\liminf y_n)$.

2. Calcule: $(a)$ $\lim \sqrt[n]{n}$ $\qquad (b)$ $\lim \sqrt[n]{n!}$ .

3. Calcule $\lim\limits_{n\to+\infty} \int\int_{A_n} e^{-\sqrt{x^2+y^2}} dxdy$, onde $A_n$ é o círculo $x^2 + y^2 \le n^2$, $n \ge 1$.

4. Calcule: (a) $\lim\limits_{n\to+\infty} \left(\frac{n+2}{n+1}\right)^n$ $\qquad$ (b) $\lim\limits_{n\to+\infty} \left(1 + \frac{x}{n}\right)^n$ $(x \in \mathbb{R})$ $\qquad$ (c) $\lim\limits_{n\to-\infty} \left(1 + \frac{x}{n}\right)^n$ $(x \in \mathbb{R})$.

5. Calcule a soma da série dada (seção 2.1, Guidorizzi, H., Um Curso de Cálculo, vol 4).

   (a) $\sum\limits_0^\infty \left(\frac{1}{3}\right)^k$. $\qquad$ (b) $\sum\limits_0^\infty e^{-k}$.

   (c) $\sum\limits_0^\infty \frac{1}{(4k+1)(4k+5)}$. $\qquad$ (d) $\sum\limits_1^\infty \frac{1}{(k+1)(k+2)(k+3)}$.

6. Determine a convergência ou divergência das séries abaixo (seção 3.1, mesmo livro).

   (a) $\sum\limits_0^\infty \frac{1}{k^2+1}$ . $\qquad$ (b) $\sum\limits_2^\infty \frac{1}{k^2 \log(k)}$ .

   (c) $\sum\limits_2^\infty \frac{1}{k^\alpha \log(k)}$. $\qquad$ (d) $\sum\limits_0^\infty \frac{k}{1+k^4}$ .

   (e) $\sum\limits_1^\infty \log \frac{2p}{p+1}$. $\qquad$ (f) $\sum\limits_1^\infty \frac{n^2-3n+1}{n^2+4}$.

   (g) $\sum\limits_2^\infty \frac{1}{p(\log p)^\alpha}$. $\qquad$ (h) $\sum\limits_2^\infty \frac{1}{(\log p)^\alpha}$.

7. Determine se convergem ou não (seção 3.2 - mesmo livro):

   (a) $\sum\limits_2^\infty \frac{k}{2k^3-k+1}$ . $\qquad$ (b) $\sum\limits_2^\infty \frac{(k+1)e^{-k}}{2k+3}$ .

   (c) $\sum\limits_2^\infty \frac{\sqrt{k}+\sqrt[3]{k}}{k^2+3k+1}$. $\qquad$ (d) $\sum\limits_0^\infty \frac{2^k}{k^5}$ .

   (e) $\sum\limits_1^\infty \frac{2^k}{k!}$. $\qquad$ (f) $\sum\limits_1^\infty \frac{1}{k(\log k)^{10}}$.

   (g) $\sum\limits_2^\infty \frac{1}{k^\alpha(\log k)^\beta}$. $\qquad$ (h) $\sum\limits_2^\infty \frac{1}{n\sqrt[3]{n^2+3}}$.

8. Determine se convergem ou não (seção 3.4 - mesmo livro):

   (a) $\sum\limits_0^\infty \frac{3^n}{1+4^n}$. $\qquad$ (b) $\sum\limits_2^\infty \frac{n!\,2^n}{n^n}$ .

   (c) $\sum\limits_1^\infty [\sqrt{n+1} - \sqrt{n}]$. $\qquad$ (d) $\sum\limits_0^\infty \frac{n^3+4}{2^n}$ .