MAT 1352 - CÁLCULO II - IFUSP

Professor Oswaldo Rio Branco de Oliveira

Período: Segundo Semestre de 2023

LISTA 7 DE EXERCÍCIOS

1. Mostre que para quaisquer $x \neq 1$, $n \in \mathbb{N}$ e $N \in \mathbb{N}$, com $N \geq n$, temos
$$\sum_{j=n}^{N} x^j = \frac{x^n - x^{n+N+1}}{1-x} \ .$$

2. Verifique as fórmulas abaixo.

   (a) $\sum\limits_{j=1}^{n} j = \frac{n(n+1)}{2}$.

   (b) $\sum\limits_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$.

   (c) $\sum\limits_{j=1}^{n} j^3 = \left(\frac{n(n+1)}{2}\right)^2$.

3. Mostre que
$$\sum_{j=1}^{n}\sum_{k=1}^{m} x_j y_k = \sum_{k=1}^{m}\sum_{j=1}^{n} x_j y_k = \left(\sum_{j=1}^{n} x_j\right)\left(\sum_{k=1}^{m} y_k\right).$$

4. Sejam $(x_j)_{1\leq j\leq n}$ e $(y_k)_{1\leq k\leq n}$ duas sequências finitas em $\mathbb{C}$. Verifique
$$\left(\sum_{j=1}^{n} x_j y_j\right)^2 = \left(\sum_{j=1}^{n} x_j^2\right)\left(\sum_{k=1}^{n} y_k^2\right) - \sum_{1\leq j<k\leq n} (x_j y_k - x_k y_j)^2 \ .$$

5. Verifique a Propriedade Telescópica:
$$\sum_{k=m}^{n} (z_{k+1} - z_k) = z_{n+1} - z_m.$$

6. Calcule, aplicando a propriedade telescópica,

   (a) $\sum\limits_{k=1}^{n} [\,(k+1)^3 - k^3\,]$.

   (b) $\sum\limits_{j=2}^{n} \frac{1}{j(j-1)}$

   (c) $\sum\limits_{j=100}^{500} \frac{1}{j(j+1)(j+2)}$

   Sugestão para (c): verique que
$$\frac{1}{j(j+1)(j+2)} = \frac{1}{2}\left(\frac{1}{j(j+1)} - \frac{1}{(j+1)(j+2)}\right).$$

7. Calcule a soma da série dada.

(a) $\sum_{k=0}^{+\infty} \left(\frac{1}{10}\right)^k$.

(b) $\sum_{k=0}^{+\infty} \pi^{-k}$.

(c) $\sum_{k=0}^{+\infty} \frac{1}{(4k+1)(4k+5)}$.

(d) $\sum_{k=1}^{+\infty} \frac{1}{(k+1)(k+2)(k+3)}$.

8. Calcule a soma da série dada.

(a) $\sum_{n=1}^{+\infty} \frac{1}{n(n+1)(n+2)(n+3)}$

(b) $\sum_{n=1}^{+\infty} n\alpha^n$, $0 < \alpha < 1$.

9. Determine a convergência ou divergência das séries (v. Guidorizzi, Vol. 4).

(a) $\sum_{k=0}^{+\infty} \frac{1}{k^2+1}$.

(b) $\sum_{k=3}^{+\infty} \frac{1}{k^2 \log(k)}$.

(c) $\sum_{k=0}^{+\infty} \frac{\sqrt{k}}{1+k^4}$

(d) $\sum_{p=4}^{+\infty} \log \frac{2p}{p+1}$

(e) $\sum_{n=5}^{+\infty} \frac{n^2-3n+1}{n^2+4}$.

10. Determine se convergem ou não as séries abaixo.

(a) $\sum_{k=2}^{+\infty} \frac{k}{4k^3-k+10}$.

(b) $\sum_{k=2}^{+\infty} \frac{(k+1)e^{-k}}{2k+3}$.

(c) $\sum_{k=2}^{+\infty} \frac{\sqrt{k}+\sqrt[3]{k}}{k^2+7k+11}$.

(d) $\sum_{k=20}^{+\infty} \frac{2^k}{k^5}$.

(e) $\sum_{k=1}^{+\infty} \frac{2^k}{k!}$

(f) $\sum_{k=3}^{+\infty} \frac{1}{k(\log k)^{10}}$

(g) $\sum_{n=2}^{+\infty} \frac{1}{n\sqrt[3]{n^2+3n+1}}$.

11. Determine se convergem ou não as séries abaixo.

(a) $\sum_{n=0}^{+\infty} \frac{3^n}{1+4^n}$.

(b) $\sum_{n=1}^{+\infty} \frac{n!\,2^n}{n^n}$.

(c) $\sum_{n=3}^{+\infty} [\sqrt{n+1} - \sqrt{n}]$.

(d) $\sum_{n=4}^{+\infty} \frac{n^3+4}{2^n}$

12. Estude, com relação à convergência ou divergência:

(a) $\sum_{k=0}^{+\infty} \frac{k}{k^2+1}$

(b) $\sum_{n=1}^{+\infty} \frac{n}{\sqrt[n]{n}}$.

13. A série

$$\sum_{n=1}^{+\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots 2n}$$

é convergente ou divergente? Justifique.

14. Determine se é convergente ou divergente a série dada abaixo.

(a) $\sum\limits_{n=1}^{+\infty} \frac{\cos^2 n+n^2}{n^4}$

(b) $\sum\limits_{n=1}^{+\infty} n^2(1-\cos\frac{1}{n^2})$

(c) $\sum\limits_{n=1}^{+\infty} \log(1+\frac{1}{n^2})$

(d) $\sum\limits_{n=3}^{+\infty} \frac{\sqrt[3]{n^5+3n+1}}{n^3(\log n)^2}$

(e) $\sum\limits_{n=3}^{+\infty} \frac{(\log n)^3}{n^2}$

(f) $\sum\limits_{n=1}^{+\infty} \arctan\left(\frac{1}{n\sqrt[3]{n^2+3}}\right)$

(g) $\sum\limits_{n=1}^{+\infty} \left(\frac{n^2+5}{n^2+3}-1\right)$

(h) $\sum\limits_{n=1}^{+\infty} \log\left(\frac{n^2+5}{n^2+3}\right)$.

15. Determine se é convergente ou divergente a série dada abaixo.

(a) $\sum\limits_{n=0}^{+\infty} \frac{2^n}{1+3^n}$

(b) $\sum\limits_{n=1}^{+\infty} \frac{1}{2^n}(1+\frac{1}{n})^n$

(c) $\sum\limits_{n=0}^{+\infty} \frac{n^2}{n!}$

(d) $\sum\limits_{n=1}^{+\infty} \frac{n!}{n^n}$

(e) $\sum\limits_{n=1}^{+\infty} 3^n \frac{n!}{n^n}$

(f) $\sum\limits_{n=1}^{+\infty} \frac{n!}{3.5.7.....(2n+1)}$

(g) $\sum\limits_{n=1}^{+\infty} \frac{2.4.6.....(2n)}{n^n}$.

16. Determine se é convergente ou divergente a série dada abaixo.

(a) $\sum\limits_{n=0}^{+\infty} \frac{(n!)^2}{(2n)!}$

(b) $\sum\limits_{n\geq p}^{+\infty} \frac{n^{n-p}}{n!}$, com p fixo em $\mathbb{N}$

(c) $\sum\limits_{n=1}^{+\infty} \frac{1.3.5.....(2n+1)}{4.6.8......(2n+4)}$

(d) $\sum\limits_{n=1}^{+\infty} \sqrt{\frac{1.3.5......(2n-1)}{2.4.6....(2n)}}$.

17. Nos exercícios abaixo determine se a série $\sum\limits_{n=3}^{+\infty} a_n$ é convergente ou divergente. No caso de convergência, verifique se a convergência é absoluta ou condicional.

(a) $a_n = \frac{\sin(2n+1)}{n^{20}}$

(b) $a_n = (-1)^{n-1}\frac{n-3}{10n+4}$

(c) $a_n = (-1)^{n-1}\frac{1}{\log n}$

(d) $a_n = (-1)^n \frac{\log n}{n}$

(e) $a_n = (-1)^n \left[\frac{1.3.5.....(2n-1)}{2.4.6....(2n)}\right]^3$

(f) $a_n = \frac{(-1)^{n-1}}{\log(e^n+e^{-n})}$.

18. Determine $z \in \mathbb{C}$ para que a série dada seja convergente:

(a) $\sum_{n=1}^{+\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots 2n} \; z^{2n}$

(b) $\sum_{n=1}^{+\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots 2n} \; \frac{z^{2n+1}}{2n+1}$

(c) $\sum_{n=1}^{+\infty} 2^n z^n$

(d) $\sum_{n=1}^{+\infty} \frac{z^n}{n}$.

(e) $\sum_{n=2}^{+\infty} \frac{z^n}{n^2}$

(f) $\sum_{n=1}^{+\infty} \frac{z^n}{2^n}$.

(g) $\sum_{n=3}^{+\infty} \frac{z^n}{\log n}$

(h) $\sum_{n=1}^{+\infty} \frac{z^n}{1.3.5 \ldots\ldots (2n+1)}$.

(i) $\sum_{n=1}^{+\infty} \frac{(2n+1) z^n}{n!}$.