MAT 1352 - Cálculo II - IFUSP
Lista 6 de Exercícios - Segundo semestre de 2016
Prof. Oswaldo Rio Branco de Oliveira

1. Calcule as integrais abaixo.

   a) $\displaystyle\int_1^{+\infty} \frac{1}{x^3} dx$

   b) $\displaystyle\int_0^{+\infty} e^{-x}\, dx$

   c) $\displaystyle\int_0^{+\infty} e^{-sx}\, dx \quad (s > 0)$

   d) $\displaystyle\int_1^{+\infty} \frac{1}{\sqrt{x}}\, dx$

   e) $\displaystyle\int_0^{+\infty} te^{-t}\, dt$

   f) $\displaystyle\int_0^{+\infty} te^{-st}\, dt \quad (s > 0)$

   g) $\displaystyle\int_0^{+\infty} xe^{-x^2}\, dx$

   h) $\displaystyle\int_0^{+\infty} \frac{1}{1+x^2}\, dx$

   i) $\displaystyle\int_0^{+\infty} \frac{1}{s^2+x^2}\, dx \quad (s > 0)$

   j) $\displaystyle\int_1^{+\infty} \frac{1}{x^4}\, dx$

   l) $\displaystyle\int_2^{+\infty} \frac{1}{x-1}\, dx$

   m) $\displaystyle\int_2^{+\infty} \frac{1}{x^2-1}\, dx$

   n) $\displaystyle\int_0^{+\infty} \frac{x}{1+x^4}\, dx$

   o) $\displaystyle\int_1^{+\infty} \frac{1}{\sqrt[3]{x^4}}\, dx$

   p) $\displaystyle\int_0^{+\infty} e^{-t} \sin t\, dt$

   q) $\displaystyle\int_1^{+\infty} \frac{1}{x^3+x}\, dx$

2. Calcule $\displaystyle\int_1^{+\infty} \frac{1}{x^\alpha}\, dx$, onde α é um real dado.

3. Calcule as integrais abaixo.

   a) $\displaystyle\int_{-\infty}^{0} e^{x}\, dx$

   b) $\displaystyle\int_{-\infty}^{-1} \frac{1}{x^5}\, dx$

   c) $\displaystyle\int_{-\infty}^{-1} \frac{1}{\sqrt[3]{x}}\, dx$

   d) $\displaystyle\int_{-\infty}^{0} xe^{-x^2}\, dx$

   e) $\displaystyle\int_{-\infty}^{+\infty} f(x) dx$ onde $f(x) = \begin{cases} 1 \text{ se } |x| \leq 1 \\ 0 \text{ caso contrário.} \end{cases}$

   f) $\displaystyle\int_{-\infty}^{+\infty} e^{-|x|}\, dx$

   g) $\displaystyle\int_{-\infty}^{+\infty} \frac{1}{4+x^2}\, dx$

4. Sejam dados um real $s > 0$ e um natural $n \neq 0$. Verifique as fórmulas abaixo.

$$(a) \quad \int_0^{+\infty} e^{-st}t^n\,dt = \frac{n}{s}\int_0^{+\infty} t^{n-1}e^{-st}dt \qquad (b) \quad \int_0^{+\infty} e^{-st}t^n dt = \frac{n!}{s^{n+1}}.$$

5. Sejam um real $s > 0$ e um real α. Verifique as fórmulas abaixo.

a) $\displaystyle\int_0^{+\infty} e^{-st}\sin(\alpha t)\,dt = \frac{\alpha}{s^2+\alpha^2} \quad (\alpha \neq 0)$

b) $\displaystyle\int_0^{+\infty} e^{-st}\cos(\alpha t)\,dt = \frac{s}{s^2+\alpha^2}$

c) $\displaystyle\int_0^{+\infty} e^{-st}e^{\alpha t}\,dt = \frac{1}{s-\alpha} \quad (s > \alpha)$

d) $\displaystyle\int_0^{+\infty} e^{-st}\,dt = \frac{1}{s}$

e) $\displaystyle\int_0^{+\infty} e^{-st}t\,dt = \frac{1}{s^2}$

f) $\displaystyle\int_0^{+\infty} e^{-st}te^{\alpha t}\,dt = \frac{1}{(s-\alpha)^2} \quad (s > \alpha)$

6. Calcule as integrais abaixo.

a) $\displaystyle\int_0^1 \frac{1}{\sqrt[3]{x}}\,dx$ b) $\displaystyle\int_0^1 \frac{1}{x}\,dx$

c) $\displaystyle\int_1^3 \frac{x^2}{\sqrt{x^3-1}}\,dx$ d) $\displaystyle\int_0^1 \ln(x)\,dx$

7. Seja $f : (a,b) \to \mathbb{R}$ não limitada e integrável em $[a,t]$ para todo $a < t < b$. Defina a integral imprópria $\int_a^b f(x)\,dx$.

8. Calcule as integrais abaixo.

a) $\displaystyle\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$ b) $\displaystyle\int_0^2 \frac{1}{\sqrt{2-x}}\,dx$

c) $\displaystyle\int_{-1}^2 \frac{1}{4-x^2}\,dx$ d) $\displaystyle\int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx$

9. Seja f não limitada e contínua nos intervalos em $[a, c)$ e $(c, b]$. Defina a integral imprópria $\int_a^b f(x)\,dx$.

10. Calcule as integrais abaixo.

a) $\displaystyle\int_0^2 \frac{1}{\sqrt[3]{x-1}}\,dx$ b) $\displaystyle\int_{-1}^1 \frac{1}{|x|}\,dx$

11. Seja f contínua em (a, b) e não limitada em $(a, c]$ e em $[c, b)$. Defina a integral imprópria $\int_a^b f(x)\,dx$.

12. Decida se as integrais abaixo são convergentes ou divergentes.

a) $\displaystyle\int_1^{+\infty} \frac{1}{x^5+3x+1}\,dx$ b) $\displaystyle\int_1^{+\infty} \frac{x^2+1}{x^3+1}\,dx$

c) $\displaystyle\int_2^{+\infty} \frac{1}{\sqrt[3]{x^4+2x+1}}\,dx$ d) $\displaystyle\int_1^{+\infty} \frac{e^{\frac{1}{x}}}{x^2}\,dx$

e) $\displaystyle\int_1^{+\infty} \frac{\cos 3x}{x^3}\,dx$ f) $\displaystyle\int_1^{+\infty} \frac{\cos 2x}{x}\,dx$

g) $\displaystyle\int_4^{+\infty} \frac{2x-3}{x^3-3x^2+1}\,dx$ h) $\displaystyle\int_2^{+\infty} \frac{1}{x^2\ln(x)}\,dx$

i) $\displaystyle\int_0^{+\infty} e^{-x}\cos\sqrt{x}\,dx$ j) $\displaystyle\int_0^{+\infty} \frac{xe^{-x}}{\sqrt{x^2+x+1}}\,dx$

l) $\displaystyle\int_1^{+\infty} \frac{\sqrt{x+1}}{\sqrt[3]{x^6+x+1}}\,dx$ m) $\displaystyle\int_{-\infty}^{+\infty} \frac{1}{x^4+x^2+1}\,dx$

13. Determine se são convergentes ou divergentes as seguintes integrais

a) $\displaystyle\int_2^{+\infty} \frac{x^6-x+1}{x^7-2x^2+3}\,dx$ b) $\displaystyle\int_{10}^{+\infty} \frac{x^5-3}{\sqrt{x^{20}+x^{10}-1}}\,dx$

c) $\displaystyle\int_1^{+\infty} \frac{2x^3+x^2+1}{x^5+x+2}\,dx$ d) $\displaystyle\int_1^{+\infty} \frac{\ln x}{x\,\ln(x+1)}\,dx$