MAT 1352 - Cálculo II - IFUSP - Semestre 2 de 2016

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Dúvidas.

1. Compute
$$\int \sqrt{1-e^x}dx.$$

Solução.

Seja $t = \sqrt{1-e^x}$. Logo, $e^x = 1-t^2$. Isto é, $x = x(t) = \ln(1-t^2)$ e
$$x'(t) = \frac{-2t}{1-t^2}.$$

Computemos então
$$\begin{aligned}
\int t\frac{dx}{dt}(t)dt = \int tx'(t)dt = \int \frac{-2t^2}{1-t^2}dt = \int \frac{2(1-t^2)-2}{1-t^2}dt \\
= \int \left(2 - \frac{2}{1-t^2}\right)dt \\
= 2t - 2\int \frac{1}{(1-t)(1+t)}dt \\
= 2t - 2\int \left(\frac{1/2}{1-t} + \frac{1/2}{1+t}\right)dt \\
= 2t + \ln|1-t| - \ln|1+t| + c.
\end{aligned}$$

Segue então
$$\int \sqrt{1-e^x}dx = 2\sqrt{1-e^x} + \ln|1-\sqrt{1-e^x}| - \ln|1+\sqrt{1-e^x}| + c \clubsuit$$

2. Compute o comprimento do gráfico de

$$y = \ln(x),\ 1 \leq x \leq e.$$

Resposta.

O comprimento é

$$\int_1^e \sqrt{1+\frac{1}{x^2}}dx = \int_1^e \frac{\sqrt{1+x^2}}{x}dx.$$

Efetuemos a substituição $t = \sqrt{1+x^2}$. Logo, $x = \sqrt{t^2-1}$ e

$$x'(t) = \frac{t}{\sqrt{t^2-1}}.$$

Logo,

$$\int_1^e \frac{\sqrt{1+x^2}}{x}dx = \int_{\sqrt{2}}^{\sqrt{e^2+1}} \frac{t}{\sqrt{t^2-1}}\frac{t}{\sqrt{t^2-1}}dt$$

$$= \int_{\sqrt{2}}^{\sqrt{e^2+1}} \frac{t^2}{t^2-1}dt$$

$$= \int_{\sqrt{2}}^{\sqrt{e^2+1}} \left(1+\frac{1}{t^2-1}\right)dt$$

$$= \sqrt{e^2+1}-\sqrt{2}+\int_{\sqrt{2}}^{\sqrt{e^2+1}} \left(\frac{1/2}{t-1}-\frac{1/2}{t+1}\right)dt$$

$$= \sqrt{e^2+1}-\sqrt{2}+\frac{1}{2}\ln\left(\frac{t-1}{t+1}\right)\Big|_{\sqrt{2}}^{\sqrt{e^2+1}}$$

$$= \sqrt{e^2+1}-\sqrt{2}+\frac{1}{2}\ln\left(\frac{\sqrt{e^2+1}-1}{\sqrt{e^2+1}+1}\right)-\frac{1}{2}\ln\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)$$

$$= \sqrt{e^2+1}-\sqrt{2}+\frac{1}{2}\ln\frac{e^2}{(\sqrt{e^2+1}+1)^2}-\frac{1}{2}\ln\frac{1}{(\sqrt{2}+1)^2}$$

$$= 1+\sqrt{e^2+1}-\sqrt{2}-\ln(\sqrt{e^2+1}+1)+\ln(\sqrt{2}+1)$$

$$= 1+\sqrt{e^2+1}-\sqrt{2}+\ln\left(\frac{\sqrt{2}+1}{\sqrt{e^2+1}+1}\right) \clubsuit$$

3. Compute o comprimento do gráfico de

$$y = e^x,\ 0 \leq x \leq 1.$$

Resposta.

O comprimento é

$$C = \int_0^1 \sqrt{1+e^{2x}}dx.$$

Façamos a substituição $t = \sqrt{1+e^{2x}}$. Então,

$$x = \frac{\ln(t^2-1)}{2}.$$

Donde segue

$$x'(t) = \frac{t}{t^2-1}.$$

Assim,

$$\begin{aligned}
C &= \int_0^1 \sqrt{1+e^{2x}}dx \\
&= \int_0^1 tx'(t)dt \\
&= \int_{\sqrt{2}}^{\sqrt{e^2+1}} \frac{t^2}{t^2-1}dt \\
&= \int_{\sqrt{2}}^{\sqrt{e^2+1}} \left(1+\frac{1}{t^2-1}\right) dt \\
&= \sqrt{e^2+1}-\sqrt{2}+\int_{\sqrt{2}}^{\sqrt{e^2+1}} \left(\frac{1/2}{t-1}-\frac{1/2}{t+1}\right) dt. \\
&= \sqrt{e^2+1}-\sqrt{2}+\frac{1}{2}\left[\ln(t-1)-\ln(t+1)\right]\Big|_{\sqrt{2}}^{\sqrt{e^2+1}}. \\
&= \sqrt{e^2+1}-\sqrt{2}+\frac{1}{2}\ln\left(\frac{\sqrt{e^2+1}-1}{\sqrt{e^2+1}+1}\right)-\frac{1}{2}\ln\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right). \\
&= \sqrt{e^2+1}-\sqrt{2}+\frac{1}{2}\ln\frac{e^2}{(\sqrt{e^2+1}+1)^2}-\frac{1}{2}\ln\frac{1}{(\sqrt{2}+1)^2} \\
&= 1+\sqrt{e^2+1}-\sqrt{2}-\ln(\sqrt{e^2+1}+1)+\ln(\sqrt{2}+1) \\
&= 1+\sqrt{e^2+1}-\sqrt{2}+\ln\left(\frac{\sqrt{2}+1}{\sqrt{e^2+1}+1}\right) \clubsuit
\end{aligned}$$