MAT121 - Cálculo II - IOUSP
3ª Lista de Exercícios - 2º semestre de 2014
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1. Calcule:

a) $\displaystyle\int \frac{x-3}{(x-1)^2\,(x+2)^2}\,dx$

b) $\displaystyle\int \frac{x+1}{x(x-2)(x+3)^2}\,dx$

c) $\displaystyle\int \frac{x^4+x+1}{x^3-x}\,dx$

d) $\displaystyle\int \frac{x+3}{x^3-2x^2-x+2}\,dx$

e) $\displaystyle\int \frac{x^2+1}{(x-2)^3}\,dx$

f) $\displaystyle\int \frac{x^5+3}{x^3-4x}\,dx$

g) $\displaystyle\int \frac{4x^2+17x+13}{(x-1)(x^2+6x+10)}\,dx$

h) $\displaystyle\int \frac{3x^2+5x+4}{x^3+x^2+x-3}\,dx$

i) $\displaystyle\int \frac{x^3+4x^2+6x+1}{x^3+x^2+x-3}\,dx$

j) $\displaystyle\int \frac{x^4+2x^2-8x+4}{x^3-8}\,dx$

2. Calcule:

a) $\displaystyle\int_1^{+\infty} \frac{1}{x^3}\,dx$

b) $\displaystyle\int_0^{+\infty} e^{-x}\,dx$

c) $\displaystyle\int_0^{+\infty} e^{-sx}\,dx \quad (s>0)$

d) $\displaystyle\int_1^{+\infty} \frac{1}{\sqrt{x}}dx$

e) $\displaystyle\int_0^{+\infty} te^{-t}\,dt$

f) $\displaystyle\int_0^{+\infty} te^{-st}\,dt \quad (s>0)$

g) $\displaystyle\int_0^{+\infty} xe^{-x^2}\,dx$

h) $\displaystyle\int_0^{+\infty} \frac{1}{1+x^2}\,dx$

i) $\displaystyle\int_0^{+\infty} \frac{1}{s^2+x^2}\,dx \quad (s>0)$

j) $\displaystyle\int_1^{+\infty} \frac{1}{x^4}\,dx$

l) $\displaystyle\int_2^{+\infty} \frac{1}{x-1}\,dx$

m) $\displaystyle\int_2^{+\infty} \frac{1}{x^2-1}\,dx$

n) $\displaystyle\int_0^{+\infty} \frac{x}{1+x^4}\,dx$

o) $\displaystyle\int_1^{+\infty} \frac{1}{\sqrt[3]{x^4}}\,dx$

p) $\displaystyle\int_0^{+\infty} e^{-t}\sin t\,dt$

q) $\displaystyle\int_1^{+\infty} \frac{1}{x^3+x}\,dx$

3. Calcule

a) $\displaystyle\int_{-\infty}^{0} e^x \, dx$

b) $\displaystyle\int_{-\infty}^{-1} \frac{1}{x^5} \, dx$

c) $\displaystyle\int_{-\infty}^{-1} \frac{1}{\sqrt[3]{x}} \, dx \quad (s > 0)$

d) $\displaystyle\int_{-\infty}^{0} xe^{-x^2} dx$

e) $\displaystyle\int_{-\infty}^{+\infty} f(x) \, dx, \text{ com } f(x) = \begin{cases} 1 \text{ se } |x| \leq 1 \\ 0 \text{ se } |x| > 1 \end{cases}$

f) $\displaystyle\int_{-\infty}^{+\infty} e^{-|x|} \, dx$

g) $\displaystyle\int_{-\infty}^{+\infty} \frac{1}{4 + x^2} \, dx$

h) $\displaystyle\int_{-\infty}^{+\infty} f(x) \, dx, \text{ com } f(x) = \begin{cases} 1 \text{ se } |x| \leq 1 \\ \frac{1}{x^2} \text{ se } |x| > 1 \end{cases}$

4. Sejam α e s, com $s > 0$, números reais dados. Verifique as fórmulas abaixo.

a) $\displaystyle\int_{0}^{+\infty} e^{-st} \sin \alpha t \, dt = \frac{\alpha}{s^2 + \alpha^2} \quad (\alpha \neq 0)$

b) $\displaystyle\int_{0}^{+\infty} e^{-st} \cos \alpha t \, dt = \frac{s}{s^2 + \alpha^2}$

c) $\displaystyle\int_{0}^{+\infty} e^{-st} e^{\alpha t} \, dt = \frac{1}{s - \alpha} \quad (s > \alpha)$

d) $\displaystyle\int_{0}^{+\infty} e^{-st} \, dt = \frac{1}{s}$

e) $\displaystyle\int_{0}^{+\infty} e^{-st} te^{\alpha t} \, dt = \frac{1}{(s - \alpha)^2} \quad (s > \alpha)$

f) $\displaystyle\int_{0}^{+\infty} te^{-st} \, dt = \frac{1}{s^2}$

5. Calcule:

a) $\displaystyle\int_{0}^{1} \frac{1}{\sqrt[3]{x}} \, dx$

b) $\displaystyle\int_{0}^{1} \frac{1}{x} \, dx$

c) $\displaystyle\int_{1}^{3} \frac{x^2}{\sqrt{x^3 - 1}} \, dx$

d) $\displaystyle\int_{0}^{1} \ln x \, dx$

e) $\displaystyle\int_{0}^{1} \frac{1}{\sqrt{1 - x^2}} \, dx$

f) $\displaystyle\int_{0}^{2} \frac{1}{\sqrt{2 - x}} \, dx$

g) $\displaystyle\int_{-1}^{2} \frac{1}{4 - x^2} \, dx$

h) $\displaystyle\int_{0}^{1} \frac{x}{\sqrt{1 - x^2}} \, dx$

i) $\displaystyle\int_{0}^{2} \frac{1}{\sqrt[3]{x - 1}} \, dx$

j) $\displaystyle\int_{-1}^{1} \frac{1}{|x|} \, dx$

6. Decida se as integrais abaixo são convergentes ou divergentes

a) $\displaystyle\int_1^{+\infty} \frac{1}{x^5+3x+1}\,dx$

b) $\displaystyle\int_1^{+\infty} \frac{x^2+1}{x^3+1}\,dx$

c) $\displaystyle\int_2^{+\infty} \frac{1}{\sqrt[3]{x^4+2x+1}}\,dx$

d) $\displaystyle\int_1^{+\infty} \frac{e^{\frac{1}{x}}}{x^2}\,dx$

e) $\displaystyle\int_1^{+\infty} \frac{\cos 3x}{x^3}\,dx$

f) $\displaystyle\int_1^{+\infty} \frac{\cos 2x}{x}\,dx$

g) $\displaystyle\int_4^{+\infty} \frac{2x-3}{x^3-3x^2+1}\,dx$

h) $\displaystyle\int_2^{+\infty} \frac{1}{x^2\ln x}\,dx$

i) $\displaystyle\int_0^{+\infty} e^{-x}\cos\sqrt{x}\,dx$

j) $\displaystyle\int_0^{+\infty} \frac{xe^{-x}}{\sqrt{x^2+x+1}}\,dx$

l) $\displaystyle\int_1^{+\infty} \frac{\sqrt{x+1}}{\sqrt[3]{x^6+x+1}}\,dx$

m) $\displaystyle\int_{-\infty}^{+\infty} \frac{1}{\sqrt{x^4+x^2+1}}\,dx$

n) $\displaystyle\int_2^{+\infty} \frac{x^6-x+1}{x^7-2x^2+3}\,dx$

o) $\displaystyle\int_{10}^{+\infty} \frac{x^5-3}{\sqrt{x^{20}+x^{10}-1}}\,dx$

7. Calcule o comprimento da curva dada

a) $\gamma(t) = (t\cos t, t\sin t)$, $t \in [0, 2\pi]$

b) $\gamma(t) = (2t-1, t+1)$, $t \in [1, 2]$

c) $\gamma(t) = (\cos t, \sin t, e^{-t})$, $t \in [0, \pi]$

d) $\gamma(t) = (e^{-t}\cos t, e^{-t}\sin t, e^{-t})$, $t \in [0, 1]$

e) $\gamma(t) = (t, \ln t)$, $t \in [1, e]$

f) $\gamma(t) = (1-\cos t, t-\sin t)$, $t \in [0, \pi]$

g) $\gamma(t) = \left(t, \frac{1}{2}(e^t + e^{-t})\right)$, $t \in [-1, 0]$

8. Calcule o comprimento do gráfico da função dada.

a) $y = \frac{2}{3}x^{\frac{3}{2}}$, $0 \le x \le 1$

b) $y = \frac{4}{3}x + 3$, $0 \le x \le 2$

c) $y = \ln x$, $1 \le x \le e$

d) $y = \sqrt{x}$, $\frac{1}{4} \le x \le \frac{3}{4}$

9. Calcule o comprimento da curva dada em forma paramétrica.

a) b) $x = 3t$ e $y = t^{\frac{3}{2}}$, $0 \le t \le 1$

b) $x = \frac{t^2}{2}$ e $y = \frac{2}{5}t^{\frac{5}{2}}$, $0 \le t \le 1$