3ª Lista de Cálculo I - MAT111 - IAG
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1. Prove:

   a) $\cos(\alpha - \beta) = \cos 2\ \cos\beta + \text{sen}\alpha\ \text{sen}\beta$.

   b) $\text{sen}(\alpha - \beta) = \text{sen}\alpha\ \cos\beta - \text{sen}\beta\ \cos\alpha$.

   c) $\cos(\alpha + \beta) = \cos\alpha\ \cos\beta - \text{sen}\alpha\ \text{sen}\beta$.

   d) $\text{sen}(\alpha + \beta) = \text{sen}\alpha\ \cos\beta + \text{sen}\beta\ \cos\alpha$.

   e) $tg(\alpha - \beta) = \dfrac{tg\alpha - tg\beta}{1 + tg\alpha \cdot tg\beta}$, se $tg\alpha \cdot tg\beta \neq -1$.

2. Prove:

   a) $\cos^2\alpha = \dfrac{1}{2} + \dfrac{1}{2}\cos 2\alpha$.

   b) $\text{sen}^2\alpha = \dfrac{1}{2} - \dfrac{1}{2}\cos 2\alpha$.

3. Prove:

   a) $\cos 2\alpha = \cos^2\alpha - \text{sen}^2\alpha$.

   b) $\text{sen}2\alpha = 2\text{sen}\alpha\ \cos\alpha$.

4. Prove:

   a) $\text{sen}\alpha\ \cos\beta = \dfrac{1}{2}\ [\text{sen}(\alpha + \beta) + \text{sen}(\alpha - \beta)]$.

   b) $\cos\alpha\ \cos\beta = \dfrac{1}{2}\ [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$.

   c) $\text{sen}\alpha\ \text{sen}\beta = \dfrac{1}{2}\ [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$.

5. Prove:

   a) $\text{sen}p - \text{sen}q = 2\text{sen}\left(\dfrac{p-q}{2}\right)\ \cos\left(\dfrac{p+q}{2}\right)$.

   b) $\cos p - \cos q = -2\text{sen}\left(\dfrac{p+q}{2}\right)\ \text{sen}\left(\dfrac{p-q}{2}\right)$.

6. Esboce o gráfico de:

   a) $\cos x$, $x \in \mathbb{R}$

   b) $\text{sen}x$, $x \in \mathbb{R}$

   c) tgx , $\dfrac{-\pi}{2} < x < \dfrac{\pi}{2}$

   d) tgx , se $\cos x \neq 0$

   e) $cotgx$, se $0 < x < \pi$

   f) $cotgx$, se $\text{sen}x \neq 0$

7. Verifique que $Im(f) \subset Dom(g)$ e determine a composta $h(x) = g(f(x))$.

a) $g(x) = 3x + 1$ e $f(x) = x + 2$.

b) $g(x) = \sqrt{x}$ e $f(x) = 2 + x^2$.

c) $g(x) = \dfrac{2}{x-2}$ e $f(x) = x + 1$, $x \neq 1$.

d) $g(x) = \dfrac{x+1}{x-1}$ e $f(x) = \dfrac{x}{x+1}$.

8. Determine o domínio maximal de f tal que $Im(f) \subset Dom(g)$. Cosntrua a composta $h(x) = g(f(x))$.

a) $g(x) = \dfrac{2}{x+2}$ e $f : Dom(f) \to I\!R$, $f(x) = x + 3$.

b) $g(x) = \sqrt{x-1}$ e $f : Dom(f) \to I\!R$, $f(x) = x^2$.

c) $g(x) = \dfrac{1}{x}$ e $f : Dom(f) \to I\!R$, $f(x) = x^3 - x^2$.

d) $g(x) = \sqrt{x^2 - 1}$ e $f : Dom(f) \to I\!R$, $f(x) = x^2 - 2$.

9. Determine $f[f = g^{-1}]$ tal que $g(f(x)) = x$, $\forall x \in Dom(f)$.

a) $g(x) = \dfrac{1}{x}$

b) $g(x) = \dfrac{x+2}{x+1}$

c) $g(x) = x^2$, $x \geq 0$

d) $g(x) = x^2 - 4x + 3$, $x \geq 2$